GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #3

DATE: 21-Nov-11   COURSE: ECE-2025

NAME: _____   GT username: _____

LAST,   FIRST   (ex: `gtbuzz7`)

| 3 points | | 3 points | | 3 points |

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-9:30am (Richards)	L06:Thur-9:30am (Casinovi)
	L07:Tues-Noon (Richards)	L08:Thur-Noon (Casinovi)
	L09:Tues-1:30pm (Chang)	L10:Thur-1:30pm (Coyle)
L01:M-3pm (Barry) L11:Tues-3pm (Chang)	L02:W-3pm (Clements)	L12:Thur-3pm (Baxley)
L03:M-4:30pm (Barry)	L04:W-4:30pm (Clements)	L14:Thur-4:30pm (Baxley)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.

- Closed book, but a calculator is permitted.

- One page $(8\frac{1}{2}'' \times 11'')$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.

- JUSTIFY your reasoning CLEARLY to receive partial credit.

- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	-3	

PROBLEM Fa-11-Q.3.1:

We've seen several equivalent ways to describe an LTI system. Below are a list of time domain descriptions (e.g., impulse response, difference equation, MATLAB code or filter coefficients) of LTI systems. For each one, choose from the list on the right the corresponding system function or frequency response. Note that there are more entries on the right than you will need to fill in the blanks on the left.

Time-Domain Description   **System Function/Frequency Response**

(a) $y[n] = 0.9y[n-1] + 0.8y[n-2] + x[n] - x[n-3]$

ANS =

$\boxed{1}$ $H(z) = z^{-2}\left(1 + 0.707z^{-1}\right)$

$\boxed{2}$ $H(e^{j\hat{\omega}}) = (1 + 2\cos(\hat{\omega}))e^{-j3\hat{\omega}}$

(b) $y[n] = \sum_{k=1}^{3} \left(\sqrt{k} - 1\right)x[n-k]$

ANS =

$\boxed{3}$ $H(e^{j\hat{\omega}}) = \dfrac{1 - e^{-j3\hat{\omega}}}{1 + 0.9e^{-j\hat{\omega}} + 0.8e^{-j2\hat{\omega}}}$

$\boxed{4}$ $H(z) = \dfrac{z^{-2}(1 - z^{-1})}{1 - 0.8z^{-1}}$

(c) $h[n] = 0.8^{n-2}u[n-2] - 0.8^{n-3}u[n-3]$

ANS =

$\boxed{5}$ $H(e^{j\hat{\omega}}) = \dfrac{1 - e^{-j3\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}} - 0.8e^{-j2\hat{\omega}}}$
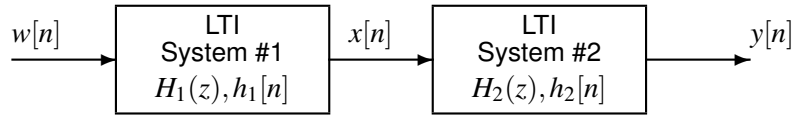
(d) $h[n] = u[n-2] - u[n-5]$

ANS =

$\boxed{6}$ $H(e^{j\hat{\omega}}) = 2\cos(\hat{\omega})\,e^{-j3\hat{\omega}}$

(e) `yy=filter([0,0,1,0,1],1,xx);`

ANS =

PROBLEM Fa-11-Q.3.2:

The diagram in the figure below depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

$$w[n] \longrightarrow \boxed{\begin{array}{c} \text{LTI} \\ \text{System \#1} \\ H_1(z), h_1[n] \end{array}} \xrightarrow{x[n]} \boxed{\begin{array}{c} \text{LTI} \\ \text{System \#2} \\ H_2(z), h_2[n] \end{array}} \longrightarrow y[n]$$

System #1 is the IIR filter described by the system function

$$H_1(z) = \frac{z^{-4}}{1 - 0.9z^{-1}},$$

and System #2 is the FIR filter described by the system function $H_2(z) = z^{-1} - z^{-2}$. Each part of this question can be worked independently.

(a) Determine the poles and zeros of the first system $H_1(z)$ and list them below. If there are no finite poles or zeros, write **NONE** in the corresponding space. If a pole or zero occurs multiple times, be sure to list it multiple times in your answer.

ZEROS at $z =$

POLES at $z =$

(b) Determine the impulse response of the second system $h_2[n]$.
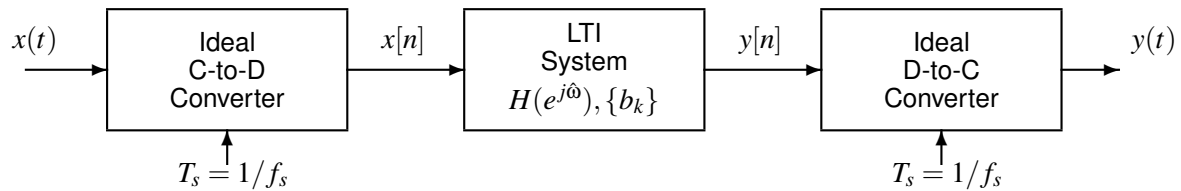
$h_2[n] =$

(c) Determine the overall output $y[n]$ when the input is a delayed unit step function $w[n] = u[n-4]$.

$y[n] =$

PROBLEM Fa-11-Q.3.3:

Consider the following system for discrete-time filtering of continuous-time signals:

$$x(t) \rightarrow \boxed{\begin{array}{c}\text{Ideal}\\ \text{C-to-D}\\ \text{Converter}\end{array}} \xrightarrow{x[n]} \boxed{\begin{array}{c}\text{LTI}\\ \text{System}\\ H(e^{j\hat{\omega}}), \{b_k\}\end{array}} \xrightarrow{y[n]} \boxed{\begin{array}{c}\text{Ideal}\\ \text{D-to-C}\\ \text{Converter}\end{array}} \rightarrow y(t)$$

$$T_s = 1/f_s \qquad\qquad\qquad\qquad T_s = 1/f_s$$

Both parts of this question should be worked independently.

(a) For this part, assume that the LTI system in the figure is defined by the frequency response $H(e^{j\hat{\omega}}) = \cos(\hat{\omega})e^{-j\hat{\omega}}$, and the sampling rate is $f_s = 1600$. Determine the overall system output $y(t)$ if the input is $x(t) = 4 + \cos(2\pi 600t) + \cos(2\pi 2000t + \pi/4)$.
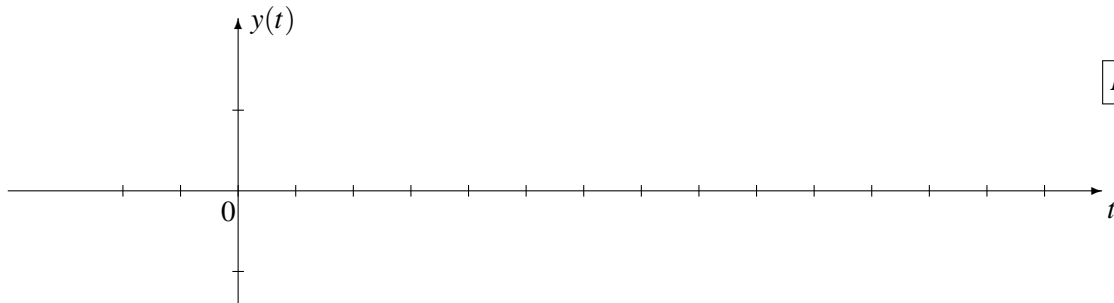
$y(t) =$

(b) This part deals with the design of FIR nulling filters similar to the filters you designed in Lab 8. Given the following three conditions, determine the values of the FIR filter coefficients b_0, b_1 and b_2. First, when the input to the FIR filter is $x[n] = 75\cos(\pi n/4 + 2\pi/3)$ the output is $y[n] = 0$. Second, the filter coefficients should be scaled so that when the input is the DC signal $x[n] = 50$ the output is $y[n] = 200$. Finally, assume that $b_2 = b_0$.

$b_0 =$ $b_1 =$ $b_2 =$

PROBLEM Fa-11-Q.3.4:

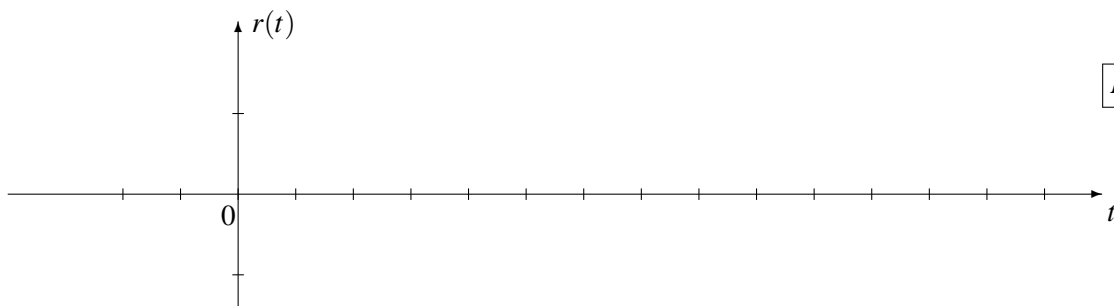The following two questions about continuous-time convolution should be worked independently.

(a) If the input to an LTI system is $x(t) = u(t+3) - u(t-1)$, determine the output signal $y(t) = x(t) * h(t)$ when $h(t) = 2\delta(t-5) - 2\delta(t-6)$. Give your answer as a carefully labeled sketch showing numerical values on both the time and amplitude axes.

$y(t)$

Label carefully

0

t

(b) If the signal $r(t)$ is a rectangular pulse, then $w(t) = r(t) * r(t)$ is a triangle. Suppose that

$$w(t) = r(t) * r(t) = (9t)[u(t) - u(t-6)] + (108 - 9t)[u(t-6) - u(t-12)].$$

On the axes below, give carefully labeled sketches of both $w(t)$ and $r(t)$.

$w(t)$

Label carefully

0

t

$r(t)$

Label carefully

0

t

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Time-Domain Description

System Function/Frequency Response

(a) $y[n] = 0.9y[n-1] + 0.8y[n-2] + x[n] - x[n-3]$

ANS = **5**

$\boxed{1}$ $H(z) = z^{-2}\left(1 + 0.707z^{-1}\right)$

$\boxed{2}$ $H(e^{j\hat{\omega}}) = (1 + 2\cos(\hat{\omega}))e^{-j3\hat{\omega}}$

(b) $y[n] = \sum_{k=1}^{3} \left(\sqrt{k} - 1\right) x[n-k]$

ANS = **1**

$\boxed{3}$ $H(e^{j\hat{\omega}}) = \dfrac{1 - e^{-j3\hat{\omega}}}{1 + 0.9e^{-j\hat{\omega}} + 0.8e^{-j2\hat{\omega}}}$

(c) $h[n] = 0.8^{n-2}u[n-2] - 0.8^{n-3}u[n-3]$

ANS = **4**

$\boxed{4}$ $H(z) = \dfrac{z^{-2}(1 - z^{-1})}{1 - 0.8z^{-1}}$

$\boxed{5}$ $H(e^{j\hat{\omega}}) = \dfrac{1 - e^{-j3\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}} - 0.8e^{-j2\hat{\omega}}}$

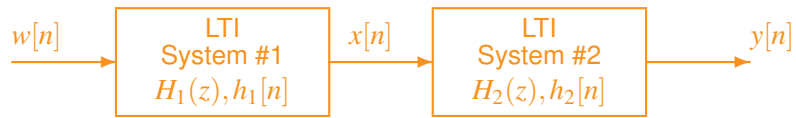(d) $h[n] = u[n-2] - u[n-5]$

ANS = **2**

$\boxed{6}$ $H(e^{j\hat{\omega}}) = 2\cos(\hat{\omega})\, e^{-j3\hat{\omega}}$

(e) `yy=filter([0,0,1,0,1],1,xx);`

ANS = **6**

The diagram in the figure below depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

$$w[n] \longrightarrow \boxed{\begin{array}{c} \text{LTI} \\ \text{System \#1} \\ H_1(z), h_1[n] \end{array}} \xrightarrow{x[n]} \boxed{\begin{array}{c} \text{LTI} \\ \text{System \#2} \\ H_2(z), h_2[n] \end{array}} \longrightarrow y[n]$$

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(a) Determine the poles and zeros of the first system $H_1(z)$ and list them below. If there are no finite poles or zeros, write **NONE** in the corresponding space. If a pole or zero occurs multiple times, be sure to list it multiple times in your answer.

$$\boxed{\text{ZEROS at } z = \text{NONE}}$$

$$\boxed{\text{POLES at } z = 0, 0, 0, 0.9}$$

(b) Determine the impulse response of the second system $h_2[n]$.

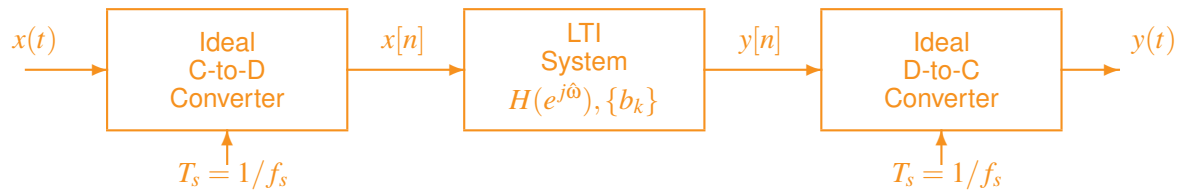$$\boxed{h_2[n] = \delta[n-1] - \delta[n-2]}$$

(c) Determine the overall output $y[n]$ when the input is a delayed unit step function $w[n] = u[n-4]$.

$$\boxed{y[n] = 0.9^{n-9}u[n-9]}$$

PROBLEM Fa-11-Q.3.3:

Consider the following system for discrete-time filtering of continuous-time signals:

$x(t)$ → [Ideal C-to-D Converter] → $x[n]$ → [LTI System $H(e^{j\hat{\omega}}), \{b_k\}$] → $y[n]$ → [Ideal D-to-C Converter] → $y(t)$

$T_s = 1/f_s$ $T_s = 1/f_s$

Both parts of this question should be worked independently.

(a) For this part, assume that the LTI system in the figure is defined by the frequency response $H(e^{j\hat{\omega}}) = \cos(\hat{\omega})e^{-j\hat{\omega}}$, and the sampling rate is $f_s = 1600$. Determine the overall system output $y(t)$ if the input is $x(t) = 4 + \cos(2\pi 600t) + \cos(2\pi 2000t + \pi/4)$.

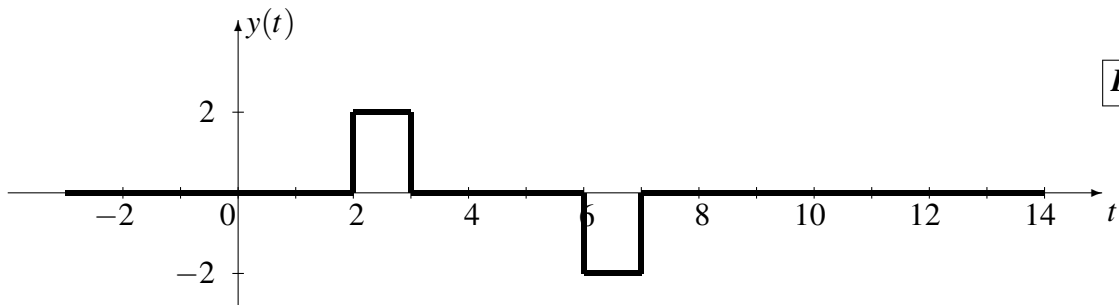$$y(t) = 4 - \tfrac{\sqrt{2}}{2}\cos(2\pi 600t - 3\pi/4) = 4 + \tfrac{\sqrt{2}}{2}\cos(2\pi 600t + \pi/4)$$

(b) This part deals with the design of FIR nulling filters similar to the filters you designed in Lab 8. Given the following three conditions, determine the values of the FIR filter coefficients b_0, b_1 and b_2. First, when the input to the FIR filter is $x[n] = 75\cos(\pi n/4 + 2\pi/3)$ the output is $y[n] = 0$. Second, the filter coefficients should be scaled so that when the input is the DC signal $x[n] = 50$ the output is $y[n] = 200$. Finally, assume that $b_2 = b_0$.

$b_0 = 6.828$ $b_1 = -9.657$ $b_2 = 6.828$

The following two questions about continuous-time convolution should be worked independently.

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Label carefully

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On the axes below, give carefully labeled sketches of both $w(t)$ and $r(t)$.

Label carefully

Label carefully