NAME: _____   GT username: _____

LAST,   FIRST   (**ex:** `gpburdell8`)

| 3 points | | 3 points | | 3 points |

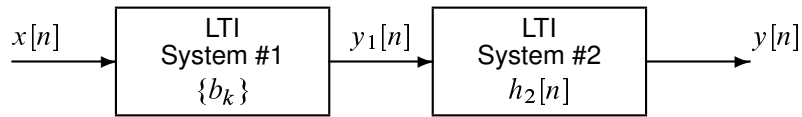Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Stüber)		L06:Thur-Noon (Bhatti)
	L07:Tues-1:30pm (Stüber)		L08:Thur-1:30pm (Bhatti)
L01:M-3pm (McClellan)	L09:Tues-3pm (Lee)	L02:W-3pm (Chang)	L10:Thur-3pm (Madisetti)
L03:M-4:30pm (Lee)	L11:Tues-4:30pm (Lee)	L04:W-4:30pm (Chang)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.

- Closed book, but a calculator is permitted.

- One page $(8\frac{1}{2}'' \times 11'')$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.

- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also REQUIRED to receive **FULL** credit for any answer.

- You must write your answer in the space provided on the exam paper itself.
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  If space is needed for scratch work, use the backs of previous pages.

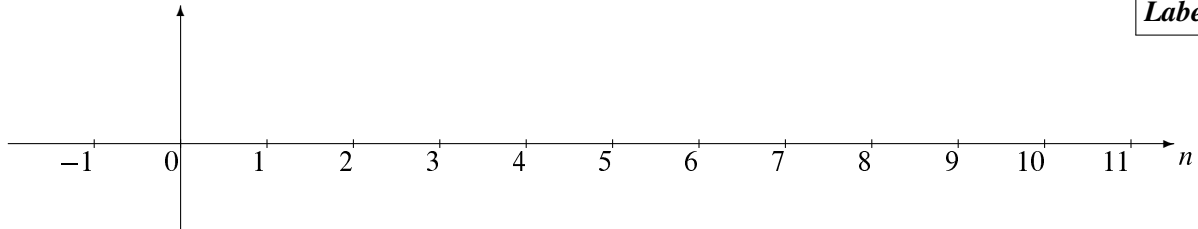| Problem	Value	Score
1	30	
2	30	
3	40	
No/Wrong Rec	−3	

PROBLEM SPR-11-Q.3.1:

The diagram in the figure below depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

$$x[n] \rightarrow \boxed{\begin{array}{c}\text{LTI}\\\text{System \#1}\\\{b_k\}\end{array}} \xrightarrow{y_1[n]} \boxed{\begin{array}{c}\text{LTI}\\\text{System \#2}\\h_2[n]\end{array}} \rightarrow y[n]$$

Suppose that System #1 is an FIR filter whose filter coefficients are $\{b_k\} = \{10, 0, 30, 40\}$ and System #2 is described by the impulse response $h_2[n] = 3\delta[n-2] - 4\delta[n-3]$.

(a) Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a *stem plot*.
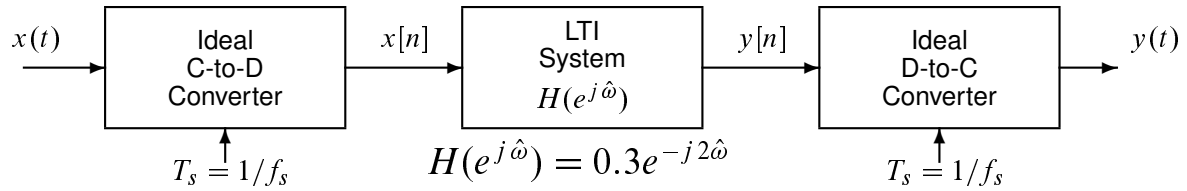
Label Carefully

(b) Determine the impulse response of the overall system; give your answer as a *sum of shifted deltas.*

(c) If the input signal is a DC signal, $x[n] = -2$ (for all n), determine both output signals, $y_1[n]$ and $y[n]$, which are also DC signals, i.e., constants. *Hint: use convolution.*
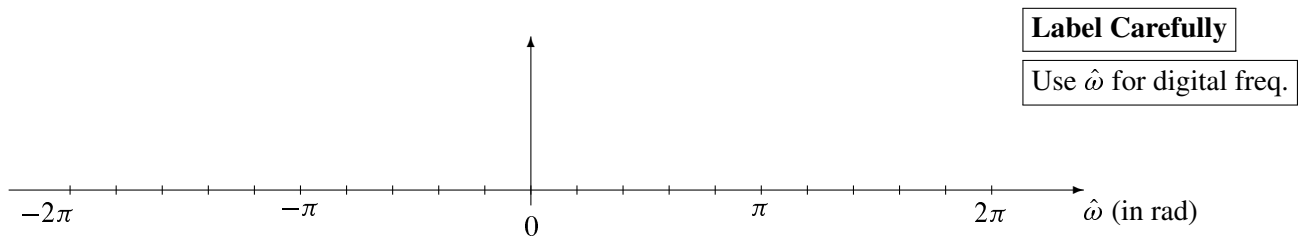
PROBLEM SPR-11-Q.3.2:

Consider the following system for discrete-time filtering of a continuous-time signal:

$x(t)$ → | Ideal C-to-D Converter | → $x[n]$ → | LTI System $H(e^{j\hat{\omega}})$ | → $y[n]$ → | Ideal D-to-C Converter | → $y(t)$

$T_s = 1/f_s$ $\qquad H(e^{j\hat{\omega}}) = 0.3e^{-j2\hat{\omega}} \qquad T_s = 1/f_s$

For all parts below, the input to the ideal C-to-D converter is $x(t) = 50\cos(300\pi t)$.

(a) For the discrete-time LTI system, determine the difference equation relating $y[n]$ to $x[n]$.

(b) If the sampling rate of the C-to-D converter is $f_s = 500$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the interval $-2\pi \le \hat{\omega} \le 2\pi$, **which is a larger range than normal.** Make sure to show **all spectrum lines** and label the frequency, amplitude and phase of each spectral component.

| Label Carefully |
| Use $\hat{\omega}$ for digital freq. |

$-2\pi \qquad -\pi \qquad 0 \qquad \pi \qquad 2\pi \qquad \hat{\omega}$ (in rad)

(c) If the sampling rate of the ideal D-to-C converter is $f_s = 500$ samples/sec, determine the mathematical formula for the continuous-time output signal, $y(t)$. Use $x(t)$ defined above.

PROBLEM SPR-11-Q.3.3:

(a) The frequency response of an LTI system is: $H(e^{j\hat{\omega}}) = 0.5e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}}$.

When the input to this system is $x[n] = \cos(0.6n)$, the frequency response must be evaluated at $\hat{\omega} = 0.6$. Determine the numerical value for $H(e^{j0.6})$ as a complex number in *polar form*.
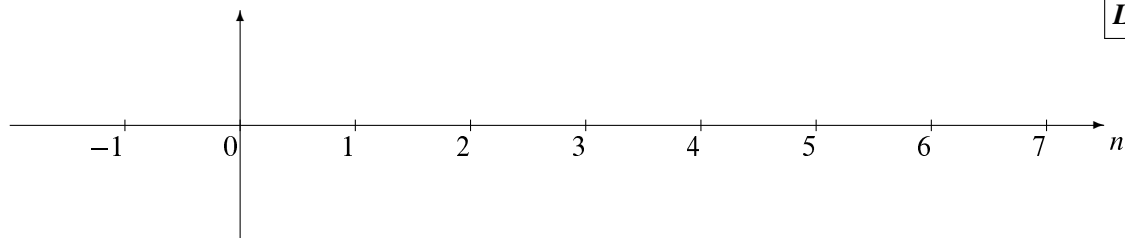
$$H(e^{j0.6}) =$$

(b) The frequency response of an FIR filter is given by the following "Dirichlet-like" form:

$$H(e^{j\hat{\omega}}) = 5\pi \left(\frac{\sin(4.5\hat{\omega})}{\sin(0.5\hat{\omega})} \right) e^{-j4\hat{\omega}}$$

In a MATLAB implementation, the FIR filter would be implemented as **yn = conv(bk,xn)**, where **xn** and **yn** correspond to $x[n]$ and $y[n]$. Write a correct MATLAB statement to define the vector **bk**.

(c) Make a **stem plot** of the signal $s[n] = 0.3u[n-1] - 0.2u[n-5]$ for $-1 \le n \le 7$, where $u[n]$ is the unit-step signal.

Label all points

(d) Suppose that S is a linear, time-invariant system whose exact form is unknown. It can be tested by running some inputs into the system, and then observing the output signals. When the **input signal** is a **unit-step** signal, $x[n] = u[n]$, the observed **output signal** is $y[n] = -0.1\delta[n-3]$. Use this test case to determine the impulse response, $h[n]$, of the system. Write $h[n]$ as a sum of shifted deltas.

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 18-Mar-11   COURSE: ECE-2025

NAME: _____**ANSWER KEY**_____   GT username: **VERSION #2**_____

LAST,   FIRST   (ex: gpburdell8)

| 3 points | 3 points | 3 points |

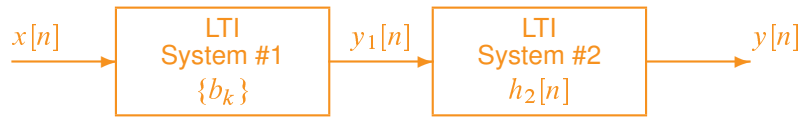Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

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	L07:Tues-1:30pm (Stüber)		L08:Thur-1:30pm (Bhatti)
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- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.

- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also REQUIRED to receive **FULL** credit for any answer.

- You must write your answer in the space provided on the exam paper itself.
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  If space is needed for scratch work, use the backs of previous pages.

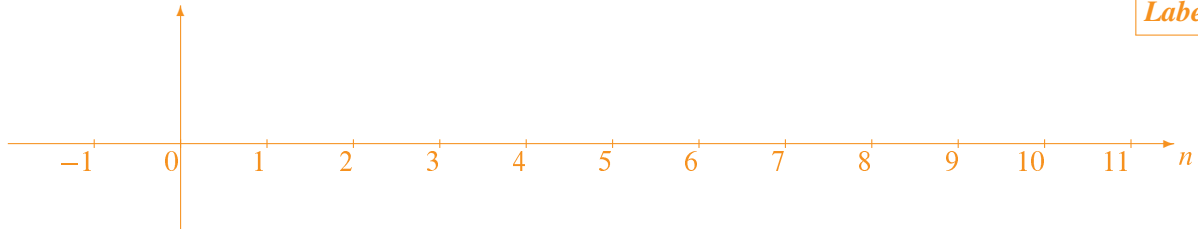| Problem	Value	Score
1	30	
2	30	
3	40	
No/Wrong Rec	−3	

PROBLEM SPR-11-Q.3.1:

The diagram in the figure below depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

$$x[n] \longrightarrow \boxed{\begin{array}{c} \text{LTI} \\ \text{System \#1} \\ \{b_k\} \end{array}} \xrightarrow{\;y_1[n]\;} \boxed{\begin{array}{c} \text{LTI} \\ \text{System \#2} \\ h_2[n] \end{array}} \longrightarrow y[n]$$

Suppose that System #1 is an FIR filter whose filter coefficients are $\{b_k\} = \{10, 0, 30, 40\}$ and System #2 is described by the impulse response $3\delta[n-2] - 4\delta[n-3]$.

(a) Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a **stem plot.**

<div align="right">Label Carefully</div>

Make a plot of $h_1[n] = 10\delta[n] + 30\delta[n-2] + 40\delta[n-3]$.

(b) Determine the impulse response of the overall system; give your answer as a **sum of shifted deltas.**

Convolve: $(10\delta[n] + 30\delta[n-2] + 40\delta[n-3]) * (3\delta[n-2] - 4\delta[n-3])$ to get

$$h[n] = 30\delta[n-2] - 40\delta[n-3] + 90\delta[n-4] - 160\delta[n-6]$$

(c) If the input signal is a DC signal, $x[n] = -2$ (for all n), determine both output signals, $y_1[n]$ and $y[n]$, which are also DC signals, i.e., constants. *Hint: use convolution.*

Note: when convolving a shifted impulse with a constant signal you get a constant, i.e., $A\delta[n - n_d] * (C) = AC$. Furthermore, the shifted unit impulse $\delta[n]$ is the impulse response of a delay filter, and delaying a constant gives that same constant.

Convolve: $(10\delta[n] + 30\delta[n-2] + 40\delta[n-3]) * (-2)$ to get

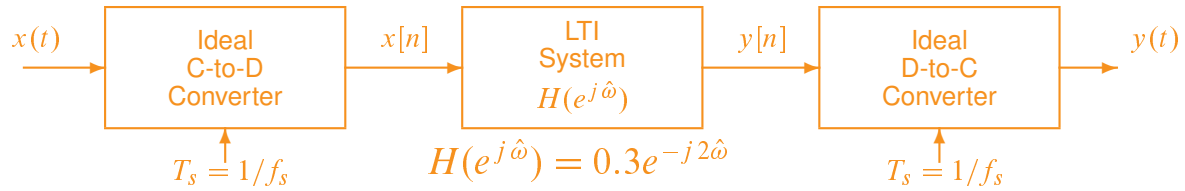$$y_1[n] = 10(-2) + 30(-2) + 40(-2) = -160 \qquad \text{for all } n$$

Note that the constant term (-2) can be factored out when you convolve

$y[n] = (30\delta[n-2] - 40\delta[n-3] + 90\delta[n-4] - 160\delta[n-6]) * (-2)$, so

$$y[n] = (30 - 40 + 90 - 160)(-2) = +160 \qquad \text{for all } n$$

Consider the following system for discrete-time filtering of a continuous-time signal:

$x(t)$ → [Ideal C-to-D Converter] → $x[n]$ → [LTI System $H(e^{j\hat{\omega}})$] → $y[n]$ → [Ideal D-to-C Converter] → $y(t)$

$T_s = 1/f_s$ $H(e^{j\hat{\omega}}) = 0.3e^{-j2\hat{\omega}}$ $T_s = 1/f_s$

For all parts below, the input to the ideal C-to-D converter is $x(t) = 50\cos(300\pi t)$.

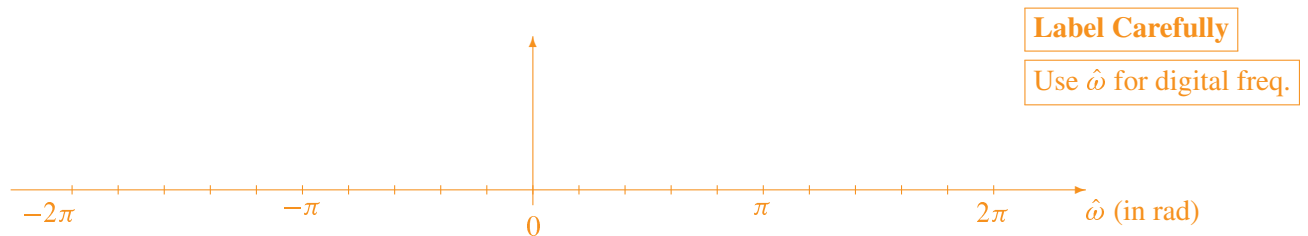(a) For the discrete-time LTI system, determine the difference equation relating $y[n]$ to $x[n]$.

There is only one filter coefficient, because the frequency response has only one complex exponential.
The FIR filter is a pure delay: $y[n] = 0.3x[n-2]$.

(b) If the sampling rate of the C-to-D converter is $f_s = 500$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the interval $-2\pi \leq \hat{\omega} \leq 2\pi$, **which is a larger range than normal.** Make sure to show **all spectrum lines** and label the frequency, amplitude and phase of each spectral component.

The signal $x[n]$ has a discrete-time frequency of $\hat{\omega} = 300\pi/500 = 0.6\pi$, so there will be spectral lines at $\hat{\omega} = \pm 0.6\pi \pm 2\pi$.
In the figure, the spectrum lines will be at $\hat{\omega} = \{0.6\pi, -1.4\pi, -0.6\pi, 1.4\pi\}$. The complex amplitudes will all be $\frac{1}{2}(50) = 25e^{j0}$.

Label Carefully

Use $\hat{\omega}$ for digital freq.

-2π $-\pi$ 0 π 2π $\hat{\omega}$ (in rad)

(c) If the sampling rate of the ideal D-to-C converter is $f_s = 500$ samples/sec, determine the mathematical formula for the continuous-time output signal, $y(t)$. Use $x(t)$ defined above.

The frequency response at $\hat{\omega} = 0.6\pi$ is

$$H(e^{j\hat{\omega}}) = 0.3e^{-j2\hat{\omega}}\Big|_{\hat{\omega}=0.6\pi} = 0.3e^{-j1.2\pi} = 0.3e^{j0.8\pi}$$

Therefore, the output of the FIR filter is $y[n] = 15\cos(0.6\pi n + 0.8\pi)$, which is a discrete-time signal. Then, converting from $\hat{\omega}$ to the output frequency ($\omega_{out} = \hat{\omega} f_s$) gives

$$y(t) = 15\cos(300\pi t + 0.8\pi)$$

(a) The frequency response of an LTI system is: $H(e^{j\hat{\omega}}) = 0.5e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}}$.

When the input to this system is $x[n] = \cos(0.6n)$, the frequency response must be evaluated at $\hat{\omega} = 0.6$. Determine the numerical value for $H(e^{j0.6})$ as a complex number in *polar form*.

$$H(e^{j0.6}) = 1.27e^{-j2.024} = 1.27e^{-j0.644\pi}$$ Evaluate $0.5e^{-j1.2} + e^{-j2.4}$

(b) The frequency response of an FIR filter is given by the following "Dirichlet-like" form:
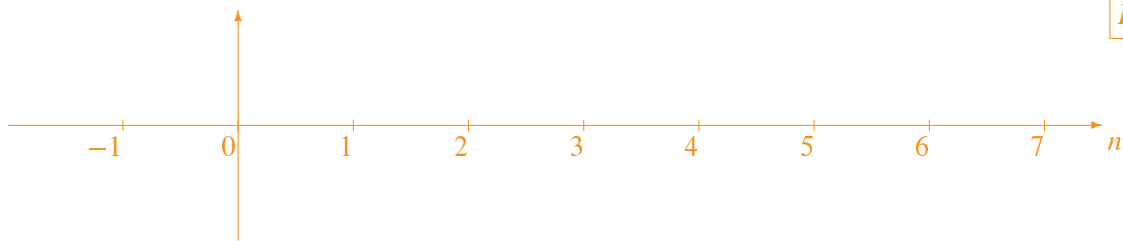
$$H(e^{j\hat{\omega}}) = 5\pi \left(\frac{\sin(4.5\hat{\omega})}{\sin(0.5\hat{\omega})} \right) e^{-j4\hat{\omega}}$$

In a MATLAB implementation, the FIR filter would be implemented as `yn = conv(bk,xn)`, where `xn` and `yn` correspond to $x[n]$ and $y[n]$. Write a correct MATLAB statement to define the vector `bk`.

The Dirichlet form is the frequency response of a filter with L identical filter coefficients; in other words, a scaled running-sum filter. The length is $L = 9$. The scale is 5π.

`bk = 5*pi*ones(1,9)`

(c) Make a *stem plot* of the signal $s[n] = 0.3u[n-1] - 0.2u[n-5]$ for $-1 \leq n \leq 7$, where $u[n]$ is the unit-step signal.

Label all points

$$\text{Make a plot of } s[n] = \begin{cases} 0 & n < 1 \\ 0.3 & 1 \leq n \leq 4 \\ 0.1 & n \geq 5 \end{cases}$$

(d) Suppose that $\mathcal{S}$ is a linear, time-invariant system whose exact form is unknown. It can be tested by running some inputs into the system, and then observing the output signals. When the ***input signal*** is a ***unit-step*** signal, $x[n] = u[n]$, the observed ***output signal*** is $y[n] = -0.1\delta[n-3]$. Use this test case to determine the impulse response, $h[n]$, of the system. Write $h[n]$ as a sum of shifted deltas.

Exploit the properties of linearity and time-invariance to write the unit impulse in terms of shifts, adds and scales of the unit-step signal. For example, if a new input is formed as $x_c[n] = x[n] + cx[n-1]$, then the output will be $y_c[n] = y[n] + cy[n-1]$. In this problem, we can use $c = -1$ because it is true that $\delta[n] = u[n] - u[n-1]$.

Thus, the same operations would be performed on the output signal $y[n]$ to get the response from an impulse at the input: $h[n] = y[n] - y[n-1] = -0.1\delta[n-3] + 0.1\delta[n-4]$