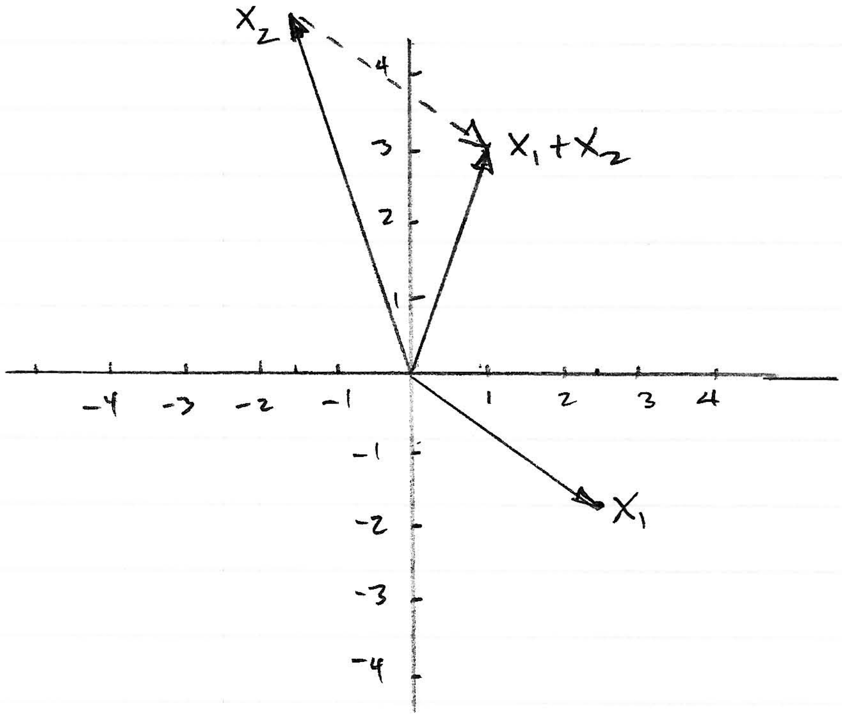
ECE 2025 Problem Set #2 Solutions

2.1 a) $x_1(t) = 3\cos(999t - 0.2\pi) + 5\cos(999t + 0.6\pi)$

i) $X_1 = 3e^{-j0.2\pi}$ $\qquad X_2 = 5e^{j0.6\pi}$



$X_1 = 2.427 - j1.763$

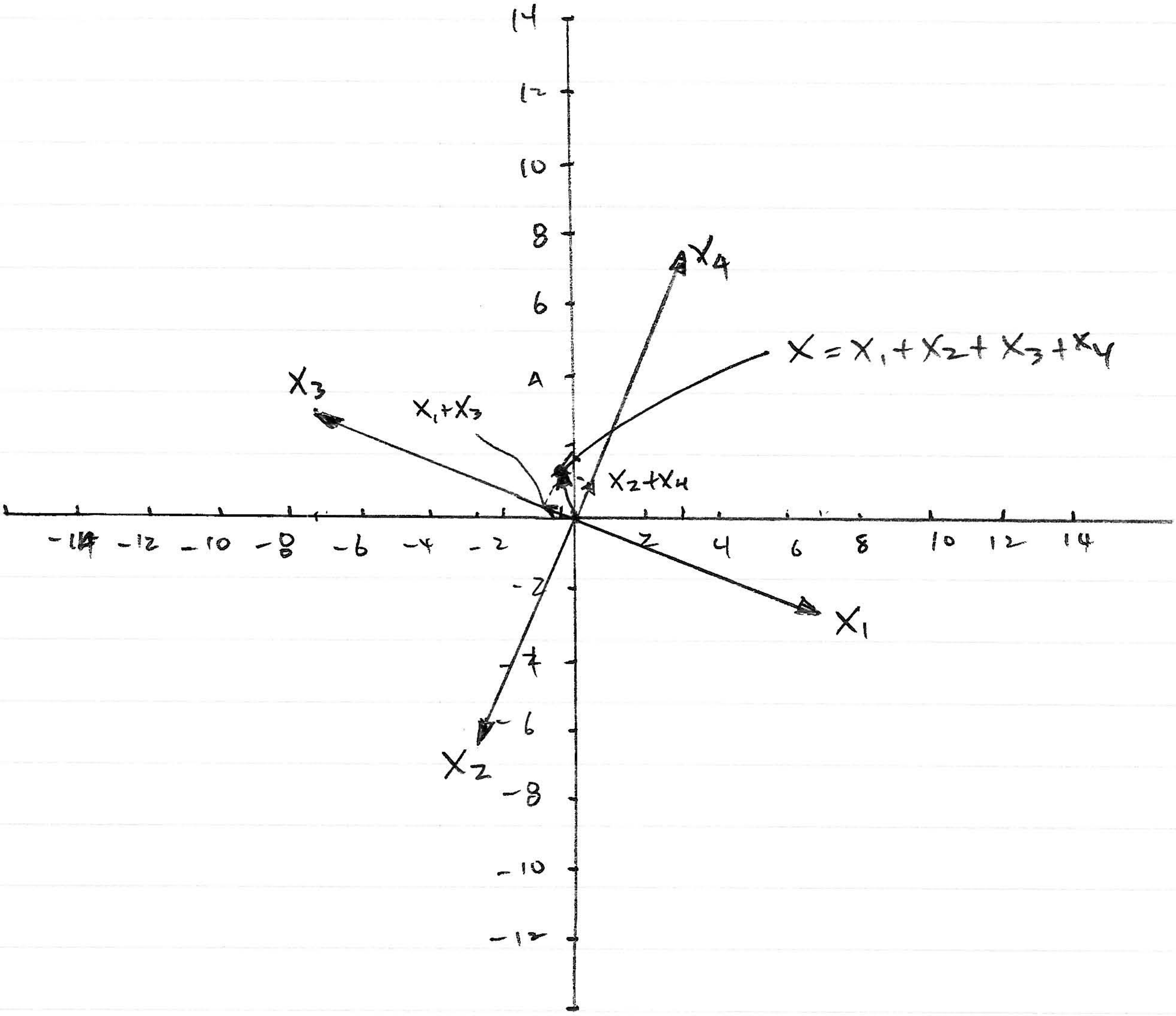
$X_2 = -1.545 + j4.755$

$X = X_1 + X_2 = 0.882 + j2.992$

$= 3.119e^{j0.409\pi}$

$x_1(t) = 3.119\cos(999t + 0.409\pi)$

b) $x_2(t) = 7\cos(\pi t - \pi/8) + 7\cos(\pi t - 5\pi/8)$
$\quad + 8\cos(\pi t - 9\pi/8) + 8\cos(\pi t + 3\pi/8)$



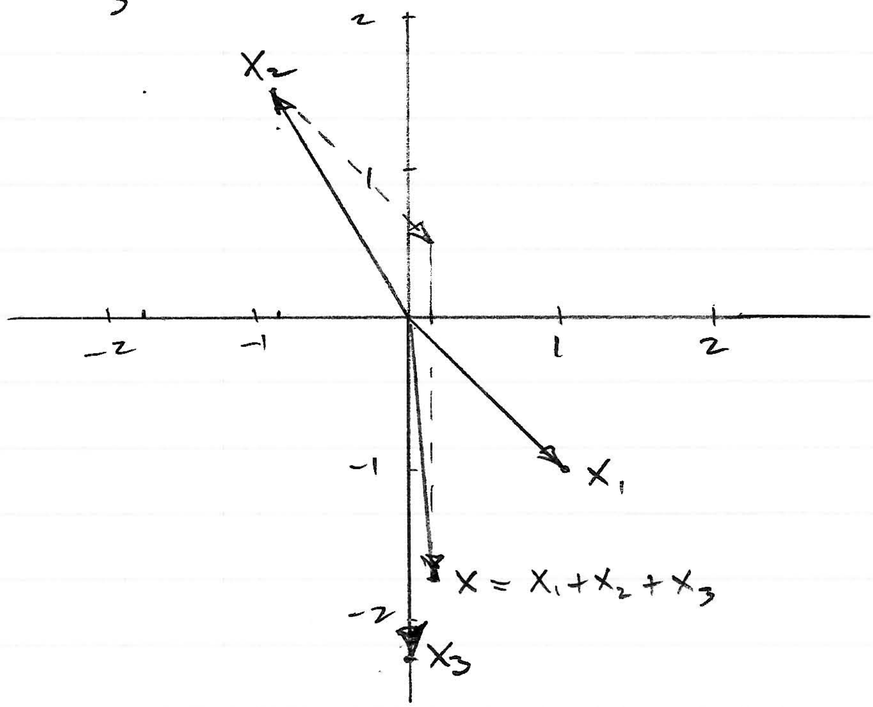
$$X_1 = 7e^{-j\pi/8} = 6.467 - j2.679$$
$$X_2 = 7e^{-j5\pi/8} = -2.679 - j6.467$$
$$X_3 = 8e^{-j9\pi/8} = -7.391 + j3.061$$
$$X_4 = 8e^{j3\pi/8} = 3.061 + j7.391$$

$$X = X_1 + X_2 + X_3 + X_4 = -0.542 + j1.306 = \sqrt{2}e^{j5\pi/8}$$

$$x_2(t) = \sqrt{2}\cos(\pi t + 5\pi/8)$$

c) $x_3(t) = \sqrt{2}\cos(6\pi t - 45) - \sqrt{3}\cos(6\pi t - 60)$
$+ \sqrt{5}\cos(6\pi t - 90)$

$$X_1 = \sqrt{2}\, e^{-j\pi/4}$$
$$X_2 = \sqrt{3}\, e^{j(-\pi/3 + \pi)} = \sqrt{3}\, e^{j2\pi/3}$$
$$X_3 = \sqrt{5}\, e^{-j\pi/2}$$



$$X_1 = 1 - j$$
$$X_2 = -\sqrt{3}/2 + 3/2$$
$$X_3 = -j\sqrt{5}$$

$$X = X_1 + X + X_3 = 0.134 - j1.736$$
$$= 1.741\, e^{-j0.475\pi}$$

$$x_3(t) = 1.741\cos(6\pi t - 0.475\pi)$$
$$= 1.741\cos(6\pi t - 85.586)$$

2.2a)

$$\int_0^{0.5} e^{j\pi t}\,dt$$

$$= \frac{1}{j\pi} e^{j\pi t} \Big|_0^{0.5}$$

$$= \frac{1}{j\pi}\left(e^{j\pi/2} - e^{j0}\right)$$

$$= \frac{j-1}{j\pi} = \frac{-1-j}{\pi} = \frac{\sqrt{2}}{\pi} e^{-j3\pi/4}$$

b)

$$z(t) = (1-j)e^{j\pi t}$$

$$z^*(t) = (1+j)e^{-j\pi t}$$

Hence. $z(t)\,z^*(t) = (1-j)(1+j)$

$$= 1-j^2 = 2$$

and $\int_0^1 z(t) z^*(t)\,dt = \int_0^1 2\,dt = 2t\Big|_0^1 = 2$

2.3) $x(t) = 7\cos(33\pi t + 1.1\pi) + 5\cos(33\pi t + 0.4\pi)$

a) $X_1 = 7e^{j1.1\pi} = -6.657 - j2.163$

$X_2 = 5e^{j0.4\pi} = 1.545 + j4.755$

$X = X_1 + X_2 = -5.112 + j\,2.592$

$= 5.732e^{j0.851\pi}$

$x(t) = 5.732\cos(33\pi t + 0.851\pi)$

$= \mathrm{Re}\{5.732e^{j(33\pi t + 0.851\pi)}\}$

Hence, $z(t) = 5.732e^{j(33\pi t + 0.851\pi)}$

b) $z_1(t) = (Ae^{j\phi})e^{j\omega t}$

$\frac{d}{dt}z_1(t) = (Ae^{j\phi})\,j\omega e^{j\omega t}$

$= A\omega e^{j(\omega t + \phi + \pi/2)}$

$\mathrm{Re}\left\{\frac{d}{dt}z_1(t)\right\} = A\omega\cos(\omega t + \phi + \pi/2)$

$= 7\cos(33\pi t + 1.1\pi)$

Hence, $\omega = 33\pi$ $\quad 33\pi A = 7 \Rightarrow A = \frac{7}{33\pi}$

$\phi + \pi/2 = 1.1\pi \Rightarrow \phi = 0.6\pi$

(cont'd)

$\therefore \quad z_1(t) = \frac{7}{33\pi} e^{j0.6\pi} e^{j33\pi t}$

c) $\quad s(t) = 5\cos(33\pi t + 0.4\pi)$

$s(t-0.01) = 5\cos(33\pi(t-0.01)+0.4\pi)$

$= 5\cos(33\pi t - .33\pi + 0.4\pi)$

$= 5\cos(33\pi t + 0.07\pi)$

Hence, $\quad z_s(t) = 5e^{j(33\pi t + 0.07\pi)}$

2.4)
a) $\displaystyle\sum_{k=0}^{N-2} e^{j\frac{2\pi(k+1/2)}{N}}$

$\displaystyle = \sum_{k=0}^{N-1} e^{j\frac{2\pi(k+1/2)}{N}} - e^{j\frac{2\pi(N-1+1/2)}{N}}$

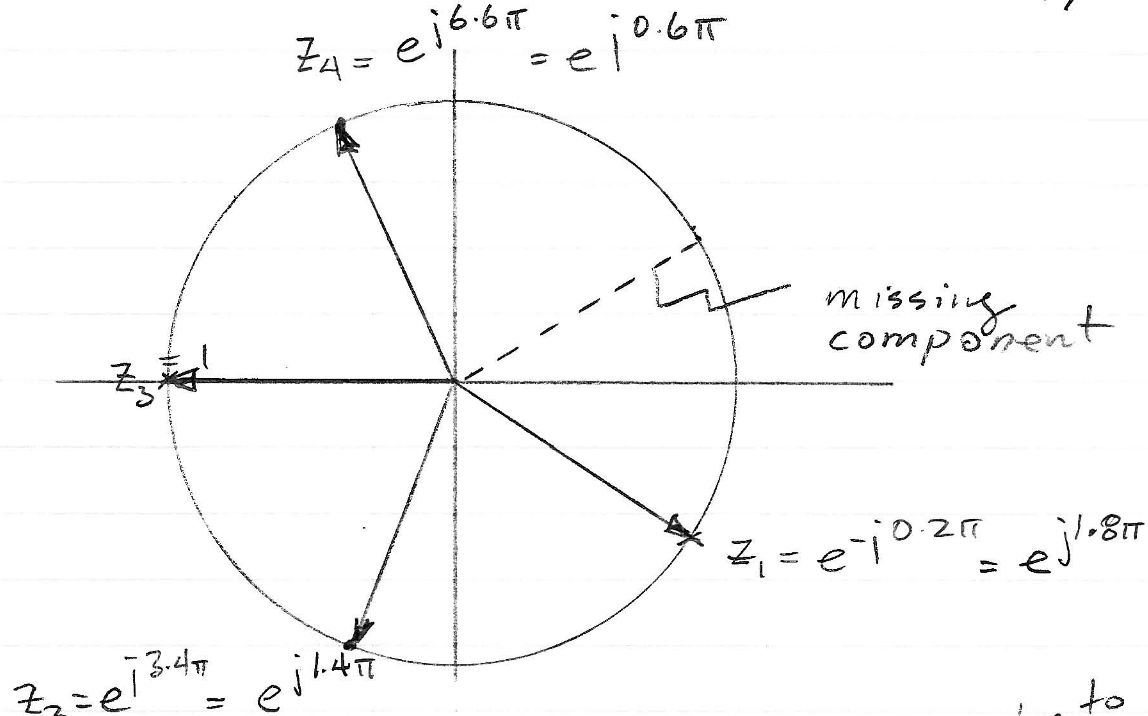
$\displaystyle = e^{j\frac{\pi}{N}}\Big(\cancel{\sum_{k=0}^{N-1} e^{j\frac{2\pi k}{N}}}^{\,0}\Big) - e^{j\frac{2\pi(N-1/2)}{N}}$

$= 0 - e^{j2\pi} \cdot e^{-j\frac{\pi}{N}}$

$= -e^{-j\frac{\pi}{N}}$

or $\quad = e^{-j\left(\frac{N+1}{N}\right)\pi}$

b)



$$Z_1 + Z_2 + Z_3 + Z_4 = e^{j0.2\pi}\Big(\cancel{\sum_{k=0}^{4} e^{j\frac{2\pi k}{5}}}^{\,0} - 1\Big)$$

due to missing component

$$= -e^{j0.2\pi} = e^{j1.2\pi}$$

c) let

$$x(t) = \sum_{k=5}^{11} 99\cos\left(0.006\pi t + \frac{\pi k}{4}\right)$$

let $l = k-5$ or $k = l+5$

$$x(t) = \sum_{l=0}^{6} 99\cos\left(0.006\pi t + \frac{\pi}{4}(l+5)\right)$$

$$= \sum_{l=0}^{6} 99\cos\left(0.006\pi t + \frac{2\pi l}{8} + \frac{5\pi}{4}\right)$$

From this we see $N = 8$

Then

$$x(t) = \sum_{l=0}^{7} 99 \cancelto{0}{\cos\left(0.006\pi t + \frac{2\pi l}{8} + \frac{5\pi}{4}\right)}$$

$$- 99\cos\left(0.006\pi t + \frac{14\pi}{8} + \frac{5\pi}{4}\right)$$

where first term is zero using

$$e^{i\theta} \sum_{k=0}^{N-1} e^{i\frac{2\pi k}{N}} = 0 \quad ; \quad \theta = \frac{5\pi}{4}, \quad N = 8$$

$$\therefore \quad x(t) = -99\cos\left(0.006\pi t + \frac{14}{8}\pi + \frac{5}{4}\pi\right)$$

$$= 99\cos\left(0.006\pi t + \frac{14\pi}{8} + \frac{5\pi}{4} - \pi\right)$$

$$= 99\cos\left(0.006\pi t + 2\pi\right)$$

$$= 99\cos\left(0.006\pi t\right)$$

Hence,

$$A = 99$$

$$\omega_0 = 0.006\pi$$

$$\phi = 0$$

2.5 a)

$$1.1e^{j2} = A_1e^{j\phi_1} + A_2e^{j(\phi_2+\pi/2)} + A_3e^{j(\phi_3-\pi/2)}$$

$$1.2e^{-j2} = A_1e^{j\phi_1} + A_2e^{j(\phi_2-\pi/2)} + A_3e^{j(\phi_3+\pi/2)}$$

$$1.3 = A_1e^{j\phi_1} + A_2e^{j\phi_2} + A_3e^{j\phi_3}$$

let
$$X_1 = A_1e^{j\phi_1}$$
$$X_2 = A_2e^{j\phi_2}$$
$$X_3 = A_3e^{j\phi_3}$$

$$1.1e^{j2} = X_1 + X_2e^{j\pi/2} + X_3e^{-j\pi/2}$$
$$1.2e^{-j2} = X_1 + X_2e^{-j\pi/2} + X_3e^{j\pi/2}$$
$$1.3 = X_1 + X_2 + X_3$$

$$\Rightarrow \begin{bmatrix} 1 & e^{j\pi/2} & e^{-j\pi/2} \\ 1 & e^{-j\pi/2} & e^{j\pi/2} \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1.1e^{j2} \\ 1.2e^{-j2} \\ 1.3 \end{bmatrix}$$

or
$$\begin{bmatrix} 1 & j & -j \\ 1 & -j & j \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1.1\cos 2 + j\,1.1\sin 2 \\ 1.2\cos 2 - j\,1.2\sin 2 \\ 1.3 \end{bmatrix}$$

b) The following MATLAB code should work

```
clear all;
A = [1 j -j ; 1 -j j ; 1 1 1 ];
B = [1.1*cos(2)+j*1.1*sin(2) ;
     1.2*cos(2)-j*1.2*sin(2) ;
     1.3 ];
X = A\B ;
amp = abs(X)
phi = angle(X)
```

This gives

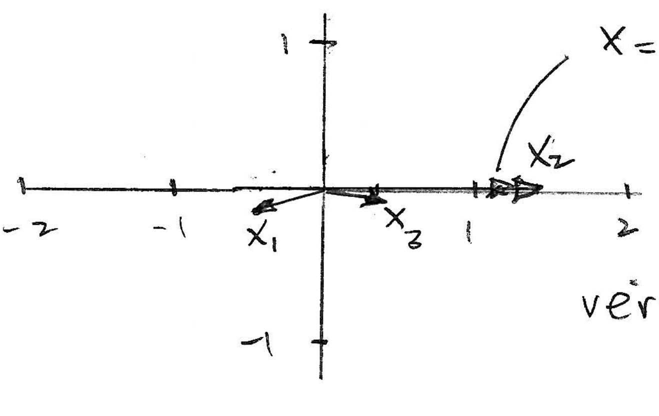
$$A_1 = .4807 \qquad \phi_1 = -3.0469$$
$$A_2 = 1.4122 \qquad \phi_2 = 0.0087$$
$$A_3 = 0.3679 \qquad \phi_3 = 0.0902$$

radians

c)

$$X_1 = -0.4786 - j\,0.0455$$
$$X_2 = 1.4121 + j\,0123$$
$$X_3 = 0.3664 + j\,0331$$

$$X = X_1 + X_2 + X_3 = 1.3$$



$X = X_1 + X_2 + X_3 = 1.3$

verifies 3rd equation