

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2011
Problem Set #1

Assigned: 11-Jan-11

Due Date: Week of 24-Jan-11

Reading: In *SP First: App. A on Complex Numbers*, pp. 430–451; and Ch. 2 on *Sinusoids*, pp. 8–43.

The web site for the course uses **t-square**: <https://t-square.gatech.edu>

The login for **t-square** is your GT login.

⇒ Please check **t-square** daily. All official course announcements will be posted there.

ONLY the **STARRED** problems should be turned in for grading; a random subset of these will be graded.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM. After this assignment is handed in by everyone, a solution to all the starred problems will be posted to the web.

Your homework is due in recitation at the beginning of class. After that, the HW is considered late.

Complex Numbers: A complex number is just an ordered pair of real numbers. Several different mathematical notations can be used to represent complex numbers. In *rectangular form* we will use all of the following notations:

$$\begin{aligned} z &= (x, y) \\ &= x + jy && \text{where } j = \sqrt{-1} \\ &= \Re\{z\} + j\Im\{z\} \end{aligned}$$

Note that $i = \sqrt{-1}$ is typical notation in most math courses. The pair (x, y) can be drawn as a vector, such that x is the horizontal coordinate and y the vertical coordinate in a two-dimensional space. Addition of complex numbers is the same as vector addition; i.e., add the real parts and add the imaginary parts.

In *polar form* we will use the *complex exponential* notation:

$$\begin{aligned} z &= |z|e^{j \arg z} \\ &= re^{j\theta} \end{aligned}$$

where $|z| = r = \sqrt{x^2 + y^2}$ and $\theta = \arctan(y/x) = \arg z$. In a vector drawing, r is the length and θ the direction of the vector measured from the positive x -axis. The angle is often called the *argument* of the complex number. Here is another notation that is much less common:

$$z = r \angle \theta$$

Euler's Formula: $re^{j\theta} = r \cos \theta + jr \sin \theta$ can be used to convert between Cartesian and polar forms.

Some of these problems should be a review of complex numbers learned in high school. In these problems, a calculator will be useful for doing the complex arithmetic, especially if it is one that accepts both polar and cartesian formats. It is essential to learn how to use the polar format feature. However, it is also worthwhile to be able to do the calculations by hand, and *visualize* the calculation to *understand* what your calculator is doing.

PROBLEM 1.1:

Convert the following complex numbers to polar form:

$$\begin{array}{lll} \text{(a) } z = -2\pi & \text{(c) } z = 1 - j\sqrt{3} & \text{(e) } z = (3, -4) \\ \text{(b) } z = (-33 - j33)^2 & \text{(d) } z = (-j8)^3 & \text{(f) } z = (-3 + j4)^7 \end{array}$$

Give numerical values for the magnitude, and the angle (or phase) in radians.

PROBLEM 1.2:

Convert the following complex numbers to rectangular form (by using Euler's formula):

$$\begin{array}{ll} \text{(a) } z = 8e^{j(-5\pi/6)} & \text{(c) } z = \pi \angle (71\pi/2) \\ \text{(b) } z = e^{\pi - j(5\pi/4)} & \text{(d) } z = \pi^e \angle (-41\pi) \end{array}$$

Give numerical values for the real and imaginary parts.

PROBLEM 1.3:

Evaluate the following and give the answer in both rectangular and polar form. In all cases, assume that the complex numbers are $z_1 = -1 - j$ and $z_2 = \sqrt{3}e^{-j(17\pi/6)}$.

$$\begin{array}{lll} \text{(a) Conjugate: } z_1^* & \text{(d) } z_2^2 & \text{(g) } z_1 + z_2^* \\ \text{(b) } jz_2 & \text{(e) } z_1^{-1} = 1/z_1 & \text{(h) } |z_2|^2 = z_2 z_2^* \\ \text{(c) } z_2/z_1 & \text{(f) } z_1 z_2 & \text{(i) } z_2 + z_2^* \end{array}$$

Note: z^* means the "conjugate" of z . Part (h) is the *magnitude-squared*, which can also be written as the product of the complex number and its complex conjugate.

PROBLEM 1.4:

Plot two periods of the following sinusoids over the time-interval $0 \leq t \leq 2T$, where T is the period:

- (a) A cosine wave with a period of 500π secs, an amplitude of three, and a phase of 0.7854 radians.
- (b) $x(t) = \ln(e^7) \cos(\frac{7\pi}{50}t - \frac{\pi}{4})$
- (c) $x(t) = \pi^2 \cos(2\pi(t - 0.1))$

PROBLEM 1.5*:

Solve the following complex number problems:

- (a) Find the *real* numbers p and q such that the following equation is true:

$$\frac{jp}{q + j36} = 0.1e^{j\pi/6}$$

- (b) Find *real* numbers α and β such that the following equation is true:

$$(5e^{j\alpha})(3 + j\beta) = -25$$

Note: there are two different answer sets for $\{\alpha, \beta\}$, so find both of them.

- (c) One of the answers in the previous part has $\beta > 0$. For that case, plot the complex numbers $z_1 = 5e^{j\alpha}$ and $z_2 = 3 + j\beta$ as vectors. This will be relatively easy to do if you express z_2 polar form.
- (d) Explain why the magnitudes in the previous part, $|z_1|$ and $|z_2|$, must be equal.

PROBLEM 1.6*:

The waveform in the following figure (generated from the MATLAB GUI sindrill) can be expressed as

$$x(t) = A \cos[\omega_0(t - t_d)] = A \cos(\omega_0 t + \varphi) = A \cos(2\pi f_0 t + \varphi)$$

From the waveform, determine A , ω_0 , f_0 , t_d , and φ . Choose the value of φ such that $-\pi < \varphi \leq \pi$.

