ECE-2025 Spring-2011

LECTURE #3
Phasor Addition Theorem
24-Jan-2011

T-square Info

- Help ? (during the semester)
  - Use OFFICE HOURS
- Check **Chat Room** & Announcements daily
- SP First URL quick link on T-square
  `Username = gt, password = student`
- Lectures are being posted
  - PDF format (4 per page)
- Get PDF files of Labs & HWs from T-square

Lab Info

- **Be prepared for Lab**
  - Work the Pre-Lab section
  - Even better, do some or all of the Exercises for Verification
- Lab Verifications: Turn in at end of Lab
  - Counts as part of the Lab Report score
- Lab Report due one week later
  - Learn **your** Lab TA's format requirements
- Lab FAQs are posted, but relate to old labs
- **Bring your laptop to lab if you want to use your own environment.**

Teams for Lab Projects

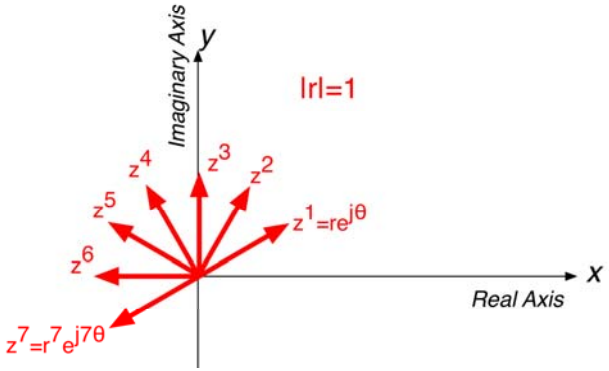
- Two-student teams have been assigned
- One (paper) report per team
  - plus, Electronic submission of all .m & .doc files
- New teams at mid-semester
- Peer Evaluation (on-Line)
  - Diary of your work/meetings
  - Sharing the workload with your partner

Demo of ITS

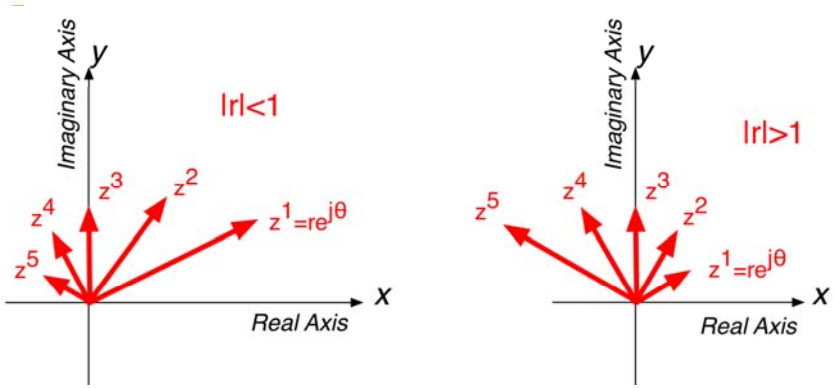
- Login to **its.vip.gatech.edu**
- Answer 2-3 questions during Lab #1
  - Questions are served at random
  - You can skip questions
- Review mode available
  - Check your answers
- Part of the HW points will come from ITS

POWERS of Complex Numbers

- Raising to a power N rotates vector by Nθ and scales vector length by r^N



MORE POWERS



ROOTS OF UNITY

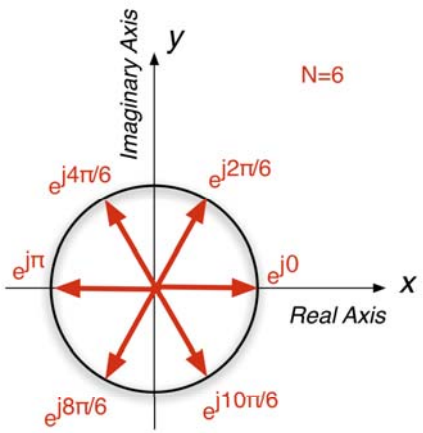
- We often have to solve $z^N=1$
- How many solutions?

$$z^N = r^N e^{jN\theta} = 1 = e^{j2\pi k}$$

$$\Rightarrow r = 1, \quad N\theta = 2\pi k \Rightarrow \theta = \frac{2\pi k}{N}$$

$$z = e^{j2\pi k/N}, \quad k = 0, 1, 2, \ldots N-1$$

ROOTS OF UNITY for N=6



- Solutions to $z^N=1$ are N equally spaced vectors on the unit circle!
- *What happens if we take the sum of all of them?*

Sum the Roots of Unity

- Looks like the answer is zero (for N=6)

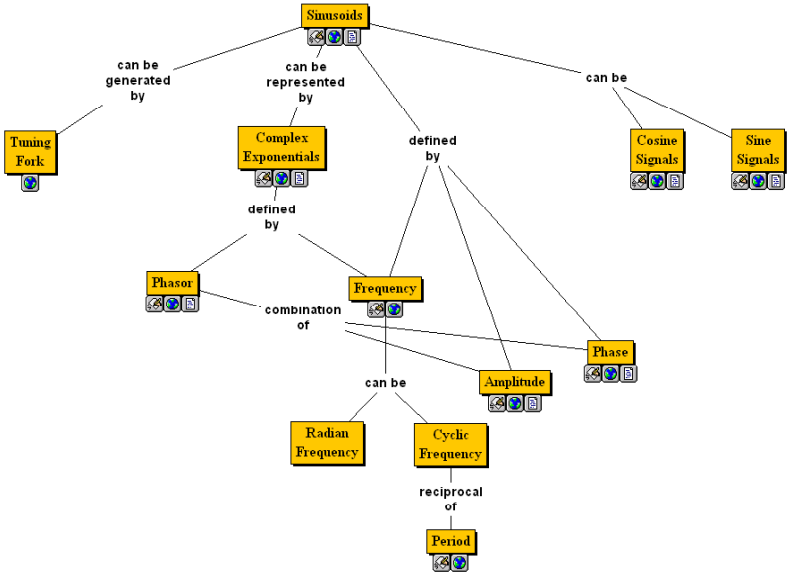
$$\sum_{k=0}^{N-1} e^{j2\pi k/N} = 0 ?$$

- Write as geometric sum

$$\sum_{k=0}^{N-1} r^k = \frac{1-r^N}{1-r} \quad \text{then let } r = e^{j2\pi/N}$$

$$\text{Numerator} \quad 1-r^N = 1-(e^{j2\pi/N})^N = 1-e^{j2\pi} = 0$$

Concept Map for Chapter 2



JUMP

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, Section 2-6
- Other Reading:
  - Appendix A: Complex Numbers
  - Appendix B: MATLAB
  - Next Lecture: start Chapter 3

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LECTURE OBJECTIVES

- Phasors = Complex Amplitude
  - Complex Numbers **represent** Sinusoids

$$A\cos(\omega t+\varphi)=\Re\{(Ae^{j\varphi})e^{j\omega t}\}$$

- *Develop the ABSTRACTION:*
  - *Adding Sinusoids = Complex Addition*
  - ***PHASOR ADDITION THEOREM***

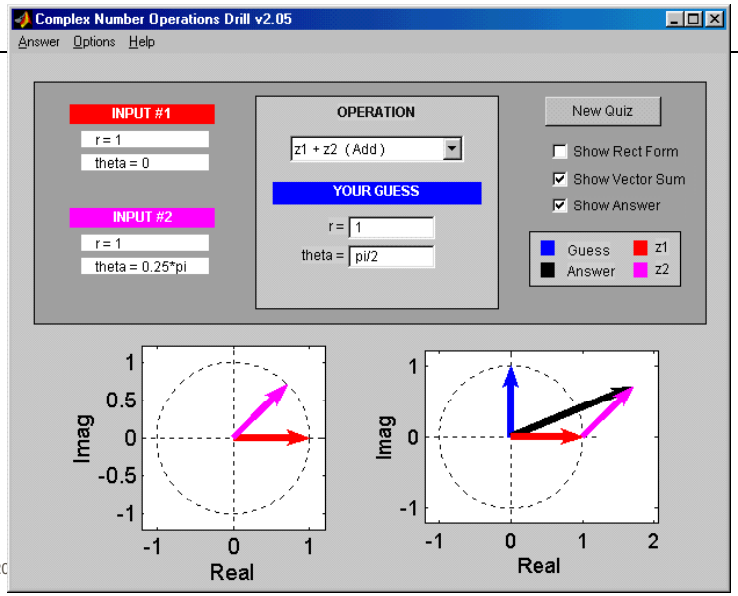
Adding Complex Numbers

- Polar Form
  - Could convert to Cartesian and back out
  - **Use Calculator that does complex ops !**
  - Use MATLAB
  - Visualize the vectors

$$1.7e^{j70\pi/180}+1.9e^{j200\pi/180}=Ae^{j\varphi}\ ?$$

$$1.532e^{j141.79\pi/180}$$

Z DRILL (Complex Arith)



Cos = REAL PART

What about sinusoidal signals over time?
Real part of Euler's

$$\cos(\omega t)=\Re\{e^{j\omega t}\}$$

General Sinusoid

$$A\cos(\omega t+\varphi)=\Re\{Ae^{j(\omega t+\varphi)}\}$$
$$=\Re\{Ae^{j\varphi}e^{j\omega t}\}$$

***Complex Amplitude**: Constant* *Varies with time*

POP QUIZ: Complex Amp

- Find the COMPLEX AMPLITUDE for:

$$x(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- Use EULER's FORMULA:

$$x(t) = \Re\{\sqrt{3}e^{j(77\pi t + 0.5\pi)}\}$$
$$= \Re\{\sqrt{3}e^{j0.5\pi}e^{j77\pi t}\}$$

$$X = \sqrt{3}e^{j0.5\pi}$$

POP QUIZ-2: Complex Amp

- Determine the 60-Hz sinusoid whose COMPLEX AMPLITUDE is:

$$X = \sqrt{3} + j3$$

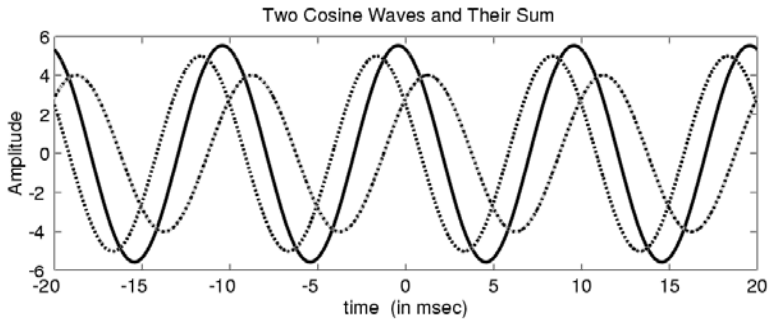
- Convert ***X*** to **POLAR**:

$$x(t) = \Re\{(\sqrt{3} + j3)e^{j(120\pi t)}\}$$
$$= \Re\{\sqrt{12}e^{j\pi/3}e^{j120\pi t}\}$$

$$\Rightarrow x(t) = \sqrt{12}\cos(120\pi t + \pi/3)$$

WANT to ADD SINUSOIDS

- **Main point to remember**: Adding sinusoids of common frequency results in sinusoid with **SAME** frequency



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PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k)$$

$$= A\cos(\omega_0 t + \phi)$$

Get the new complex amplitude by complex addition

$$\sum_{k=1}^{N} A_k e^{j\phi_k} = Ae^{j\phi}$$

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Phasor Addition Proof

$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = \sum_{k=1}^{N} \Re e\left\{A_k e^{j(\omega_0 t+\phi_k)}\right\}$$

$$= \Re e\left\{\sum_{k=1}^{N} A_k e^{j\phi_k} e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(\sum_{k=1}^{N} A_k e^{j\phi_k}\right) e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(A e^{j\phi}\right) e^{j\omega_0 t}\right\} = A\cos(\omega_0 t + \phi)$$

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POP QUIZ: Add Sinusoids

- ADD THESE 2 SINUSOIDS:

$$x_1(t) = \cos(77\pi t - \pi)$$

$$x_2(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- COMPLEX (PHASOR) ADDITION:

$$1e^{-j\pi} + \sqrt{3}e^{j0.5\pi}$$

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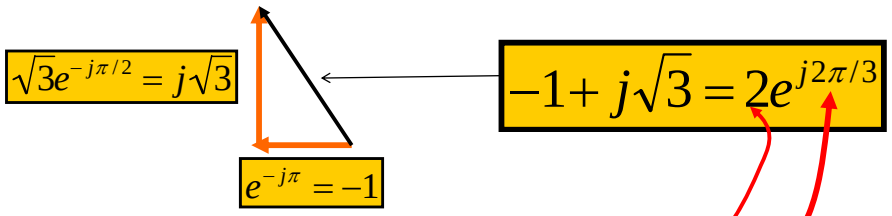
POP QUIZ (answer)

- COMPLEX ADDITION: $1e^{-j\pi} + \sqrt{3}e^{j0.5\pi}$



- CONVERT back to cosine form:

$$x_3(t) = 2\cos(77\pi t + \tfrac{2\pi}{3})$$

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ADD SINUSOIDS EXAMPLE

- ALL SINUSOIDS have SAME FREQUENCY
- HOW to GET {Amp,Phase} of RESULT ?

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t) = A\cos(\omega t + \varphi)$$

$$= \Re\{Ae^{j\varphi}e^{j20\pi t}\}$$

Convert Sinusoids to Phasors

- Each sinusoid → Complex Amp

$$1.7\cos(20\pi t + 70\pi/180) \rightarrow 1.7e^{j70\pi/180}$$

$$1.9\cos(20\pi t + 200\pi/180) \rightarrow 1.9e^{j200\pi/180}$$

$$1.7e^{j70\pi/180} + 1.9e^{j200\pi/180} = ?$$

$$1.532e^{j141.79\pi 180}$$

$$\rightarrow 1.532\cos(20\pi t + 141.79\pi/180)$$

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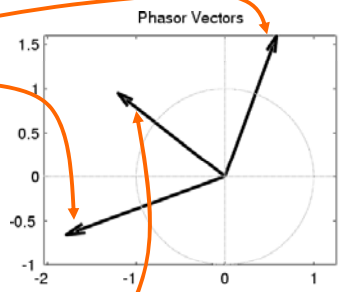
Phasor Add: Numerical

- Convert Polar to Cartesian
  - X_1 = 0.5814 + j1.597
  - X_2 = -1.785 - j0.6498
  - sum =
  - X_3 = -1.204 + j0.9476
- Convert back to Polar
  - X_3 = 1.532 at angle 141.79π/180
  - This is the sum



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ADDING SINUSOIDS IS COMPLEX ADDITION

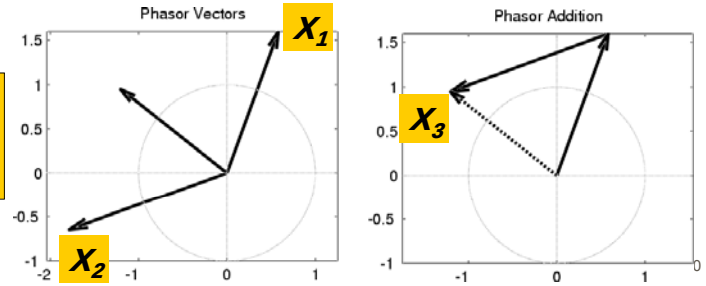
$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

VECTOR (PHASOR) ADD



Add 20 Sinusoids (MATLAB)

$$s(t) = \sum_{k=1}^{20} \sqrt{k}\cos(120\pi(t - 0.002k))$$

- Each sinusoid → Complex Amp

$$S = Ae^{j\varphi} = \sum_{k=1}^{20} \sqrt{k}\, e^{j120\pi(-0.002k)}$$

- *MATLAB*

```
kk=1:20;
SS = sum( sqrt(kk) .* exp(120i*pi*(-0.002)*kk) );
zprint( SS )
```

$$A = 6.949, \quad \varphi = -1.545$$