

GEORGIA INSTITUTE OF TECHNOLOGY
 SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #3

DATE: 15-Mar-10

COURSE: ECE-2025

NAME:

LAST,

FIRST

GT username:

(ex: gpburdell8)

3 points

3 points

3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L05:Tues-Noon (Michaels)

L06:Thur-Noon (Bhatti)

L07:Tues-1:30pm (Michaels)

L08:Thur-1:30pm (Bhatti)

L01:M-3pm (Lee)

L09:Tues-3pm (Fekri)

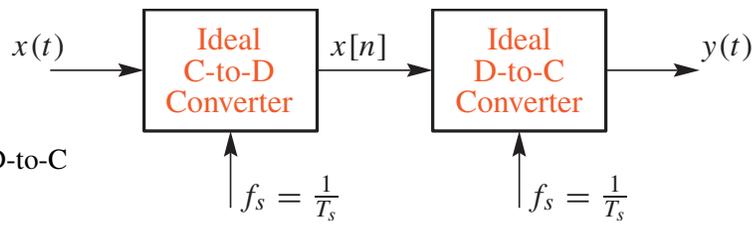
L03:M-4:30pm (Lee)

L11:Tues-4:30pm (Fekri)

- Write your name on the front page **ONLY**. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
 Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
 Only these answers will be graded. Circle your answers, or write them in the boxes provided.
 If space is needed for scratch work, use the backs of previous pages.

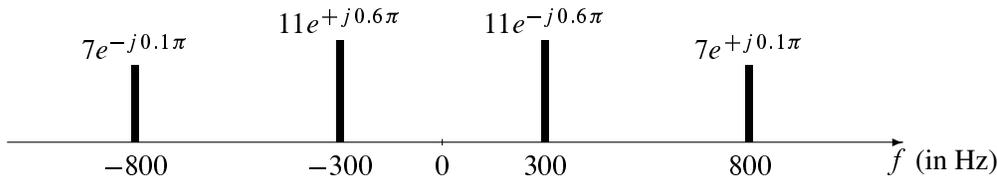
<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	30	
2	40	
3	30	
eXtra	7	
No/Wrong Rec	-3	

PROBLEM sp-10-Q.3.1:



Note: Same f_s for C-to-D and D-to-C

Suppose that the input $x(t)$ to the system above has the spectrum representation shown below:

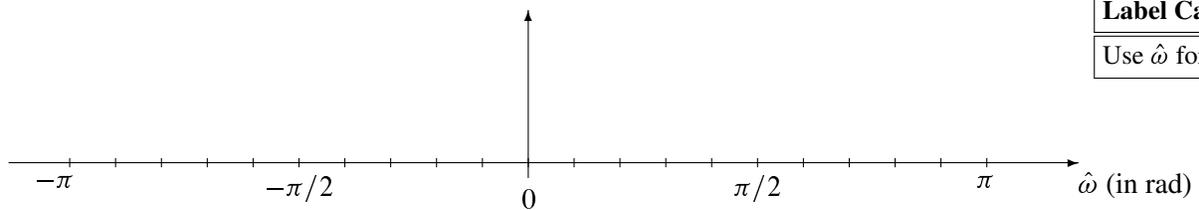


- (a) *Sampling Rate:* Determine the lower bound on *minimum* sampling rate f_s such that the output $y(t)$ will be equal to the input $x(t)$.

$f_s =$ Hz

- (b) *Spectrum of Discrete-Time Signal:* Using the signal $x(t)$ above, form the signal $x[n]$ by sampling at a rate $f_s = 1000$ samples/sec. Determine the spectrum for $x[n]$ and make a plot for your answer over the interval $-\pi \leq \hat{\omega} \leq \pi$. Make sure all the aliased components are included.

Simplify your answer as much as possible, and then label the frequency and complex amplitude (in polar form) of each spectral component.



- (c) *Unknown f_s :* Use the input $x(t)$ defined above by its spectrum. If the output of the ideal D-to-C converter is $y(t) = A \cos(2\pi(300)t + \varphi)$, determine the value of the sampling frequency f_s . It is not necessary to find A and φ .

$f_s =$ Hz

PROBLEM sp-10-Q.3.2:

Consider a system implemented by the following MATLAB program:

```
% xn.mat is a binary file containing the vector of input samples
%   called "xn" which represents the input signal x[n]
load xn
vn = conv(ones(1,3),xn);    %<-- First subsystem, output is v[n]
wn = conv([-7,7],vn);      %<-- Second subsystem, output is w[n]
yn = wn + conv([7, 1, 7, 1],xn); %<-- "yn" represents the output y[n]
```

The overall system from input x_n to output y_n is an LTI system composed of three LTI subsystems.

- (a) From the MATLAB code, draw a block diagram showing how the three component subsystems are connected. There should be three blocks, labelled S_1 , S_2 and S_3 , one for each LTI subsystem.

- (b) For the first subsystem, with input x_n and output v_n , write the difference equation.

- (c) For the second subsystem, with output w_n , determine the impulse response; call it $h_2[n]$. Write $h_2[n]$ as a sum of weighted, shifted impulses.

- (d) The overall system is an LTI system. Hence, a single MATLAB convolution will suffice to obtain the output vector y_n from the input vector x_n . Give the correct MATLAB definition of the vector bb .

$$y_n = \text{conv}(bb, x_n);$$

PROBLEM sp-10-Q.3.3:

- (a) Evaluate the expression $G = \sum_{n=0}^{20} (7\delta[n-5] + 7\delta[n-6]) e^{-j0.5\pi n}$. Express the numerical answer for G as a complex number in *polar form*. $G =$

- (b) Consider a linear time-invariant system defined by the difference equation

$$y[n] = 0.2x[n-1] + 0.2x[n-2] + 0.2x[n-3] + 0.2x[n-4] = \sum_{k=1}^4 0.2x[n-k]$$

The frequency response of this system can be expressed in the following “Dirichlet-like” form:

$$H(e^{j\hat{\omega}}) = \alpha \left(\frac{\sin(L\hat{\omega}/2)}{L \sin(\hat{\omega}/2)} \right) e^{-j\beta\hat{\omega}}$$

Determine the parameters L and α . **Explain your answer.**

Hint: Find the DC response, i.e., find $y[n]$ when $x[n] = 1$.

$L =$	$\alpha =$
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- (c) Suppose that \mathcal{S} is a linear, time-invariant system whose exact form is unknown. It is tested by running some inputs into the system, and then observing the output signals (in the table below).

Input: $x_i[n]$	Output: $y_i[n]$
$x_1[n] = 10 \cos(0.2\pi n + 2)$	$y_1[n] = 30.12 \cos(0.2\pi n + 0.913)$
$x_2[n] = 5 + 13 \cos(0.5\pi n - 1.23)$	$y_2[n] = 20$
$x_3[n] = \cos(0.2\pi n + 2) + \cos(0.5\pi n)$	$y_3[n] = 3.012 \cos(0.2\pi n + 0.913)$

Using this data it is possible to determine three values of the frequency response $H(e^{j\hat{\omega}_i})$. Find the frequency and the value; express complex numbers in polar form.

$\hat{\omega}_i$	$H(e^{j\hat{\omega}_i})$ (in polar form)

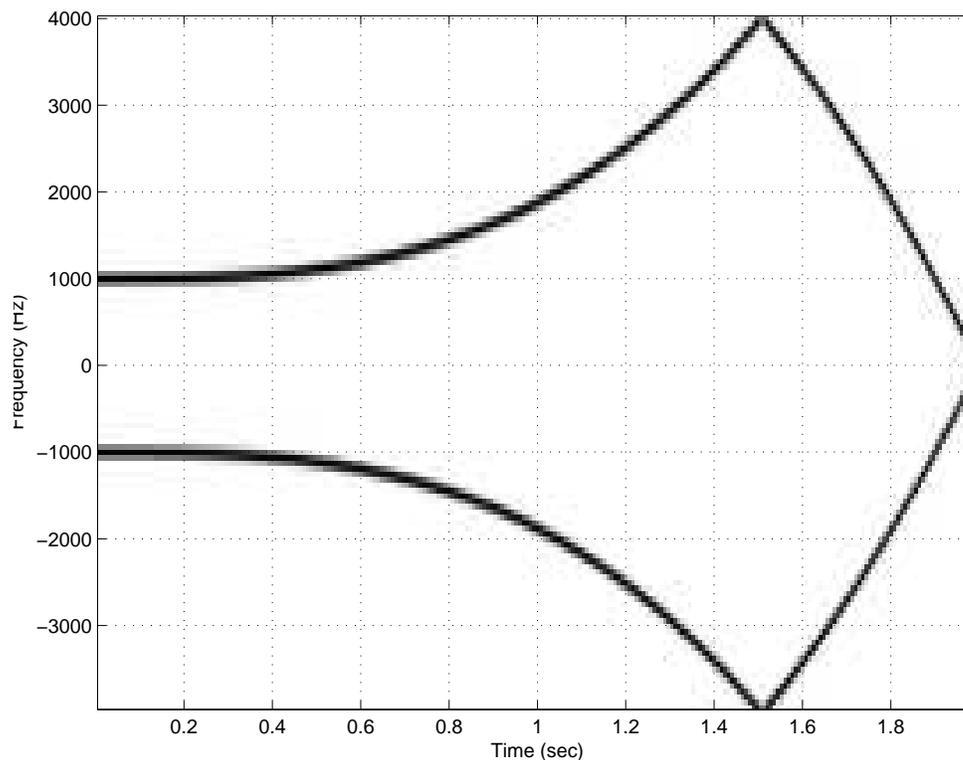
PROBLEM sp-10-Q.3.4:

This extra credit problem is not easy, and it's only worth 7 points. Thus, you shouldn't spend time working it unless you've completed the rest of the quiz, and checked your answers.¹

In the MATLAB code below, determine the value of **ALPHA** in the definition of the FM signal whose spectrogram is produced by the **plotspec** function. The final value of the frequency plot in the spectrogram is zero at $t = 2$ sec. It is likely that values for f_s and BETA will also have to be found, but only the value of ALPHA will be graded. ALPHA =

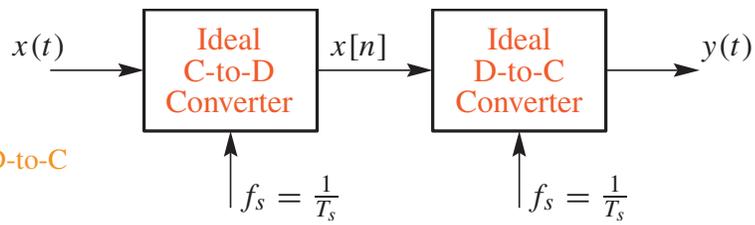
EXPLANATION:

```
%-- make an FM signal and display its spectrogram
fs = ???;
tt = 0:1/fs:2;
psi = ALPHA*pi*tt.^4 + BETA*pi*tt;
xn = cos(psi);
plotspec(xn+j*1e-11,fs,128), grid on %-- spectrogram could be used
```



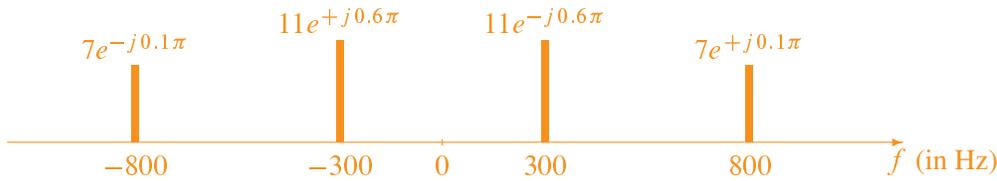
¹Grading Rubric: Completely correct with correct explanation/derivation (+7), Correct explanation/derivation, wrong numerical answer caused by bad arithmetic (+5), Correct answer but wrong or no derivation (0), everything else (0).

PROBLEM sp-10-Q.3.1:



Note: Same f_s for C-to-D and D-to-C

Suppose that the input $x(t)$ to the system above has the spectrum representation shown below:

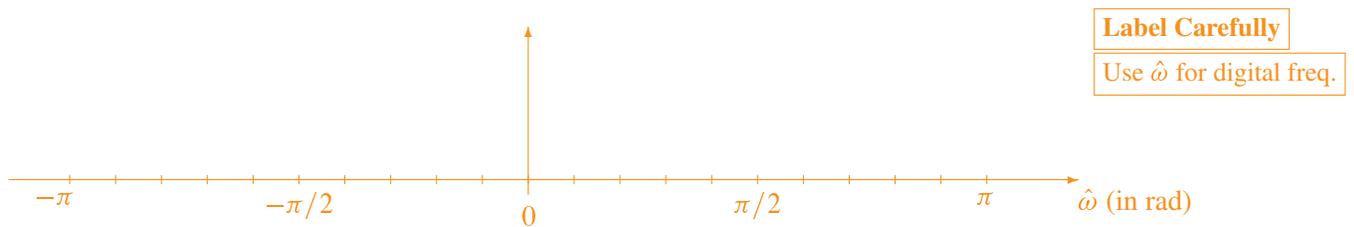


- (a) *Sampling Rate:* Determine the lower bound on *minimum* sampling rate f_s such that the output $y(t)$ will be equal to the input $x(t)$.

$f_s = 1600 \text{ Hz}$

- (b) *Spectrum of Discrete-Time Signal:* Using the signal $x(t)$ above, form the signal $x[n]$ by sampling at a rate $f_s = 1000$ samples/sec. Determine the spectrum for $x[n]$ and make a plot for your answer over the interval $-\pi \leq \hat{\omega} \leq \pi$. Make sure all the aliased components are included.

Simplify your answer as much as possible, and then label the frequency and complex amplitude (in polar form) of each spectral component.



Need to plot four spectrum lines (after aliasing) for:

$$11e^{-j0.6\pi} e^{j0.6\pi n} + 11e^{+j0.6\pi} e^{-j0.6\pi n} + 7e^{-j0.1\pi} e^{j0.4\pi n} + 7e^{+j0.1\pi} e^{-j0.4\pi n}$$

- (c) *Unknown f_s :* Use the input $x(t)$ defined above by its spectrum. If the output of the ideal D-to-C converter is $y(t) = A \cos(2\pi(300)t + \varphi)$, determine the value of the sampling frequency f_s . It is not necessary to find A and φ .

$f_s = 1100 \text{ Hz}$

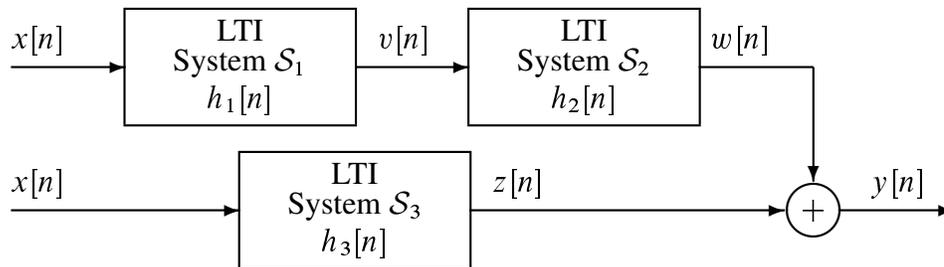
PROBLEM sp-10-Q.3.2:

Consider a system implemented by the following MATLAB program:

```
% xn.mat is a binary file containing the vector of input samples
%   called "xn" which represents the input signal x[n]
load xn
vn = conv(ones(1,3),xn);    %<-- First subsystem, output is v[n]
wn = conv([-7,7],vn);      %<-- Second subsystem, output is w[n]
yn = wn + conv([7, 1, 7, 1],xn); %<-- "yn" represents the output y[n]
```

The overall system from input x_n to output y_n is an LTI system composed of three LTI subsystems.

- (a) From the MATLAB code, draw a block diagram showing how the three component subsystems are connected. There should be three blocks, labelled \mathcal{S}_1 , \mathcal{S}_2 and \mathcal{S}_3 , one for each LTI subsystem.



- (b) For the first subsystem, with input x_n and output v_n , write the difference equation.

$$v[n] = x[n] + x[n-1] + x[n-2]$$

- (c) For the second subsystem, with output w_n , determine the impulse response; call it $h_2[n]$. Write $h_2[n]$ as a sum of weighted, shifted impulses.

$$h_2[n] = -7\delta[n] + 7\delta[n-1]$$

- (d) The overall system is an LTI system. Hence, a single MATLAB convolution will suffice to obtain the output vector y_n from the input vector x_n . Give the correct MATLAB definition of the vector bb .

$$bb = [0, 1, 7, 8] \quad yn = conv(bb, xn);$$

PROBLEM sp-10-Q.3.3:

- (a) Evaluate the expression $G = \sum_{n=0}^{20} (7\delta[n-5] + 7\delta[n-6]) e^{-j0.5\pi n}$. Express the numerical answer for G as a complex number in *polar form*.

$$G = 7\sqrt{2} e^{-j3\pi/4}$$

- (b) Consider a linear time-invariant system defined by the difference equation

$$y[n] = 0.2x[n-1] + 0.2x[n-2] + 0.2x[n-3] + 0.2x[n-4] = \sum_{k=1}^4 0.2x[n-k]$$

The frequency response of this system can be expressed in the following “Dirichlet-like” form:

$$H(e^{j\hat{\omega}}) = \alpha \left(\frac{\sin(L\hat{\omega}/2)}{L \sin(\hat{\omega}/2)} \right) e^{-j\beta\hat{\omega}}$$

Determine the parameters L and α . **Explain your answer.**

Hint: Find the DC response, i.e., find $y[n]$ when $x[n] = 1$.

$$L = 4 \quad \alpha = 0.8$$

- (c) Suppose that \mathcal{S} is a linear, time-invariant system whose exact form is unknown. It is tested by running some inputs into the system, and then observing the output signals (in the table below).

Input: $x_i[n]$	Output: $y_i[n]$
$x_1[n] = 10 \cos(0.2\pi n + 2)$	$y_1[n] = 30.12 \cos(0.2\pi n + 0.913)$
$x_2[n] = 5 + 13 \cos(0.5\pi n - 1.23)$	$y_2[n] = 20$
$x_3[n] = \cos(0.2\pi n + 2) + \cos(0.5\pi n)$	$y_3[n] = 3.012 \cos(0.2\pi n + 0.913)$

Using this data it is possible to determine three values of the frequency response $H(e^{j\hat{\omega}_i})$. Find the frequency and the value; express complex numbers in polar form.

$\hat{\omega}_i$	$H(e^{j\hat{\omega}_i})$ (in polar form)
0.2π	$3.012 e^{-j1.087}$
0.5π	0
0	4

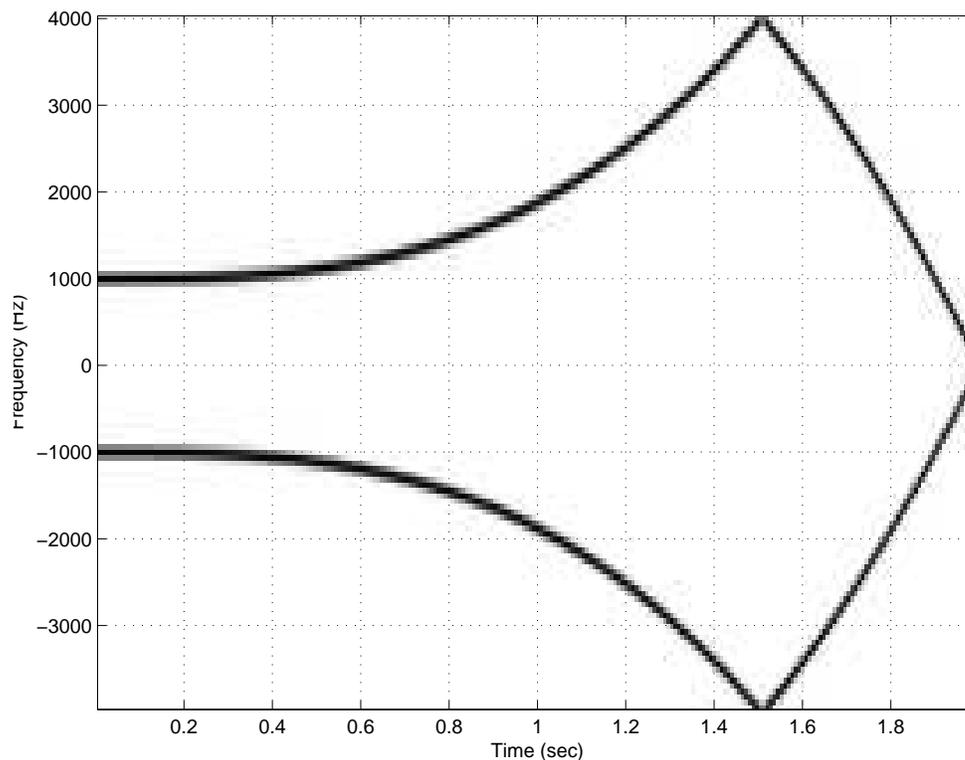
PROBLEM sp-10-Q.3.4:

This extra credit problem is not easy, and it's only worth 7 points. Thus, you shouldn't spend time working it unless you've completed the rest of the quiz, and checked your answers.¹

In the MATLAB code below, determine the value of **ALPHA** in the definition of the FM signal whose spectrogram is produced by the **plotspec** function. The final value of the frequency plot in the spectrogram is zero at $t = 2$ sec. It is likely that values for f_s and BETA will also have to be found, but only the value of ALPHA will be graded. **ALPHA = 437.5**

EXPLANATION:

```
%-- make an FM signal and display its spectrogram
fs = ????.;
tt = 0:1/fs:2;
psi = ALPHA*pi*tt.^4 + BETA*pi*tt;
xn = cos(psi);
plotspec(xn+j*1e-11,fs,128), grid on %-- spectrogram could be used
```



¹Grading Rubric: Completely correct with correct explanation/derivation (+7), Correct explanation/derivation, wrong numerical answer caused by bad arithmetic (+5), Correct answer but wrong or no derivation (0), everything else (0).