

GEORGIA INSTITUTE OF TECHNOLOGY
 SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
FINAL EXAM

DATE: 27-Apr-09

COURSE: ECE-2025

NAME:

LAST,

FIRST

GT username:

(ex: gpburdell3)

3 points

3 points

3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L05:Tues-Noon (Bhatti)

L06:Thur-Noon (Barry)

L07:Tues-1:30pm (Bhatti)

L08:Thur-1:30pm (Barry)

L01:M-3pm (Chang)

L09:Tues-3pm (Lee)

L02:W-3pm (Fekri)

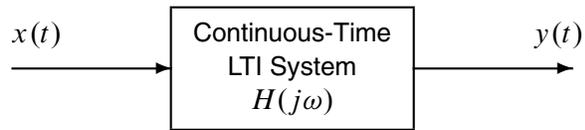
L11:Tues-4:30pm (Lee)

L04:W-4:30pm (Fekri)

- Write your name on the front page **ONLY**. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning **clearly** to receive partial credit.
 Explanations are also **required** to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
 Only these answers will be graded. Circle your answers, or write them in the boxes provided.
 If space is needed for scratch work, use the backs of previous pages.

<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	

PROBLEM s-09-F.1:



Suppose that the frequency response $H(j\omega)$ could be written in terms of a few parameters, e.g.,

$$H(j\omega) = \frac{j\omega d}{a + j\omega}$$

where a and d are *real-valued parameters*.

- (a) Determine the impulse response of the system above when $a = 2\pi$ and $d = 100$. Determine the *simplest possible* formula for $h(t)$.

$h(t) =$

- (b) Determine the output signal $y(t)$ when the input signal has a Fourier transform given by

$$X(j\omega) = 2\pi\delta(\omega - 4\pi)$$

Once again assume that the parameters of the system are $a = 2\pi$ and $d = 100$. Determine a *simple* formula for the output $y(t)$, which is *complex-valued* in this case.

$y(t) =$

PROBLEM s-09-F.2:

The two subparts of this problem are completely independent of one another.

- (a) When two finite-duration signals are convolved, the result is a finite-duration signal. In this part,

$$h(t) = e^{-t}[u(t) - u(t + 20)] \quad \text{and} \quad x(t) = 4[u(t + 35) - u(t + 15)]$$

Determine the starting and ending times of the output signal $y(t) = x(t) * h(t)$, i.e., find T_1 and T_2 so that $y(t) = 0$ for $t < T_1$ and for $t > T_2$. *Hint:* Visualize the flip-and-slide, but don't integrate.

$T_1 =$	sec.	$T_2 =$	sec.
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- (b) Determine the result of the following convolution:

$$y(t) = \{6e^{-3t}u(t-1)\} * u(t-11)$$

$y(t) =$

PROBLEM s-09-F.3:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. *Write each answer in the box provided.* (The operator * denotes convolution.)

$$(a) x(t) = \left(\frac{\sin(\pi(t+1))}{\pi(t+1)} \delta(t+1) \right) * \delta(t-5)$$

$$(b) x(t) = \frac{d}{dt} \left\{ \frac{\sin(5\pi t)}{\pi t} \right\}$$

$$(c) x(t) = 2 \frac{\sin(\pi t)}{\pi t} \cos(4\pi t)$$

$$(d) x(t) = \frac{\sin(5\pi t)}{\pi t} * \frac{\sin(3\pi t)}{\pi t}$$

$$(e) x(t) = 2 \frac{\sin(\pi t)}{\pi t} * \cos(4\pi t)$$

Each of the time signals above has a Fourier transform that should be in the list below.

[0] $X(j\omega) = u(\omega + 3\pi) - u(\omega - 3\pi)$

[1] $X(j\omega) = 2u(\omega + \pi) - 2u(\omega - \pi) + \pi\delta(\omega + 4\pi) + \pi\delta(\omega - 4\pi)$

[2] $X(j\omega) = e^{-j4\omega} [u(\omega + 5\pi) - u(\omega - 5\pi)]$

[3] $X(j\omega) = e^{-j4\omega} [j\pi\delta(\omega + \pi) - j\pi\delta(\omega - \pi)]$

[4] $X(j\omega) = e^{-j4\omega}$

[5] $X(j\omega) = u(\omega + 5\pi) - u(\omega - 5\pi)$

[6] $X(j\omega) = 0$

[7] $X(j\omega) = u(\omega + 5\pi) - u(\omega + 3\pi) + u(\omega - 3\pi) - u(\omega - 5\pi)$

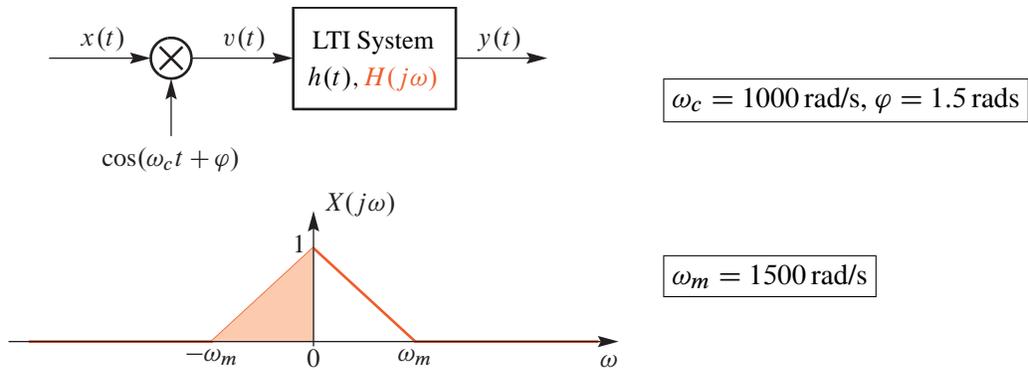
[8] $X(j\omega) = [j\pi u(\omega + \pi) - j\pi u(\omega - \pi)] * \delta(\omega - 4\pi)$

[9] $X(j\omega) = j\omega [u(\omega + 5\pi) - u(\omega - 5\pi)]$

[None] $X(j\omega)$ not in the list above.

PROBLEM s-09-F.4:

The system below involves a multiplier followed by a filter:



Suppose the frequency of the cosine in the multiplier above is $\omega_c = 1000$ rad/s, and its phase is $\varphi = 1.5$ rads. The Fourier transform of the input, $X(j\omega)$, is also shown above.

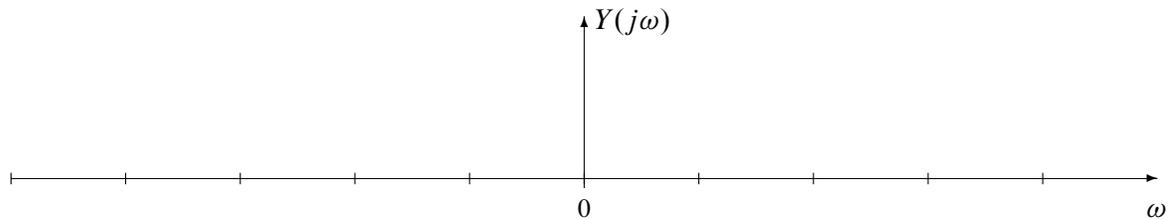
- (a) Determine the DC value of $V(j\omega)$, the Fourier transform of the signal $v(t)$ after the multiplier.

$V_{\text{DC}} =$

- (b) Suppose the LTI system is an *ideal* HPF defined by

$$H(j\omega) = \begin{cases} 0 & |\omega| < 1000 \text{ rad/s} \\ 1 & |\omega| \geq 1000 \text{ rad/s} \end{cases}$$

Make a sketch of the Fourier transform of $y(t)$, called $Y(j\omega)$, when the input is $X(j\omega)$ shown above.



PROBLEM s-09-F.5:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

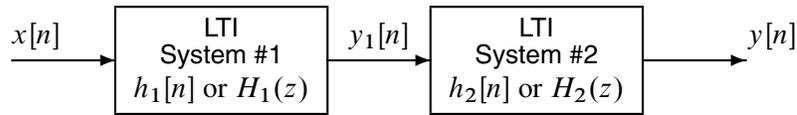


Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that $H_1(z) = 5 - 5z^{-20}$ is the system function for System #1, and

System #2 is an FIR filter described by a difference equation: $y[n] = 100y_1[n - 20] + 100y_1[n - 30]$

- (a) Determine the impulse response $h[n]$ of the overall system. **Give your answer as a stem plot.**



- (b) Determine the frequency response of System #2. Express your answer in the following form by finding values for α , β , and μ :

$$H_2(e^{j\hat{\omega}}) = \alpha e^{-j\beta\hat{\omega}} \cos(\mu\hat{\omega})$$

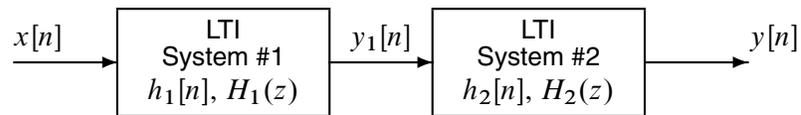
$\alpha =$

$\beta =$

$\mu =$

PROBLEM s-09-F.6:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



Suppose that System #1 is a filter described by its impulse response: $h_1[n] = -100\delta[n-2] + 400\delta[n-3]$

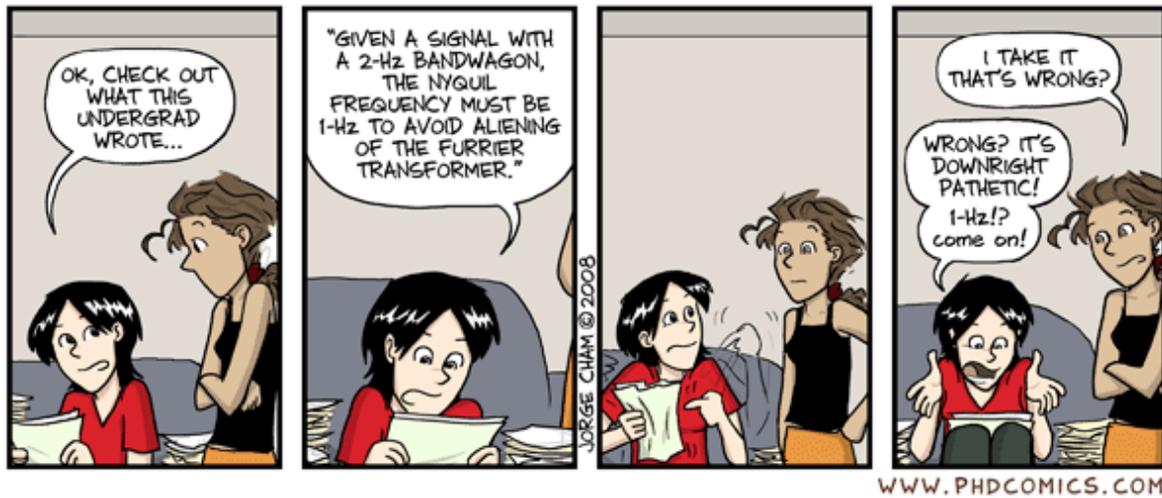
and System #2 is an FIR *running sum* filter whose system function is: $H_2(z) = \frac{1-z^{-5}}{1-z^{-1}}$

- (a) When the input to the *second* system is $y_1[n] = 100\cos(0.2n - 0.2)$, for all n , determine the output of the *second* system, $y[n]$, over the range $-\infty < n < \infty$. **Explain your work to receive credit.**

- (b) When the input to the *first* system is $x[n] = 100$, for $-\infty < n < \infty$, determine the *overall* output, $y[n]$, over the range $-\infty < n < \infty$. **Explain your work to receive credit.**

PROBLEM s-09-F.7:

In ECE-2025 you've learned new vocabulary. Thus, the following comic strip (produced by a 1997 ME grad from Georgia Tech) now qualifies as humor:



(a) What is the correct answer that the GTA is looking for? In addition, *explain why 1-Hz is wrong*.

(b) *Four* technical terms from DSP are morphed into other words in the comic strip. Identify these *four*, and give the correct DSP technical word for each.

PROBLEM s-09-F.8:

Circle the correct answer in both parts.

- (a) A sinusoidal signal $x(t)$ is defined by the vector xx in the following MATLAB code:

```
tt = 0:0.001:10; xx = real( (2+j)*exp(j*pi*tt) ); plot(tt,xx)
```

When xx is plotted versus time (tt), its maximum value will be:

- (a) $A = \sqrt{5}$
 - (b) $A = 2$
 - (c) $A = 2 + j$
 - (d) $A = 1$
 - (e) None of the above
- (b) Determine the amplitude (A) and phase (ϕ) of the sinusoid that is the sum of the following three sinusoids, expressed in MATLAB:

```
10*cos(6*tt + pi/2) + 10*cos(6*tt + 4*pi/3) + 10*cos(6*tt - pi/3)
```

- (a) $A = 10$ and $\phi = \pi/2$.
- (b) $A = 7$ and $\phi = \pi/2$.
- (c) $A = 0$ and $\phi = 0$.
- (d) $A = 3$ and $\phi = \pi/2$.
- (e) $A = 24$ and $\phi = \pi/2$.
- (f) None of the above

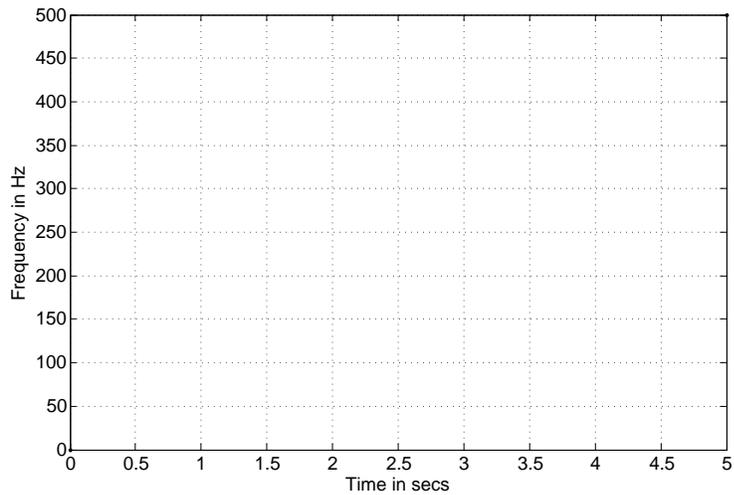
PROBLEM s-09-F.9:

Consider the following snippet of code:

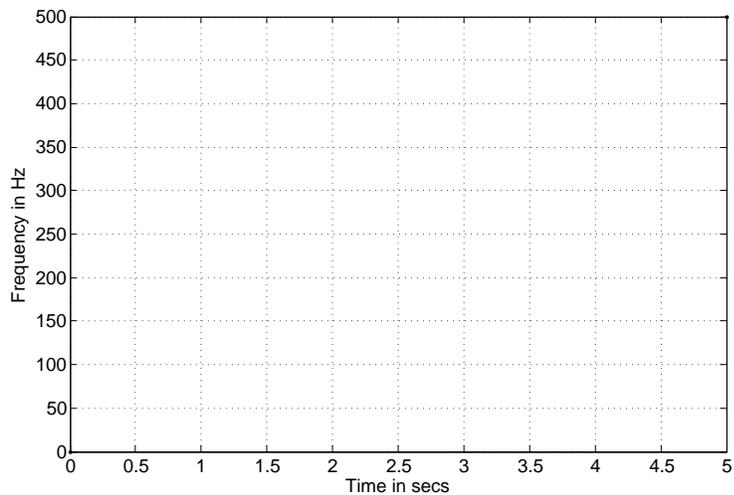
```
nn = 0:5000; fs = 1000; tt = nn/fs;  
xx = ... %<-- this line is given below for each part  
plotspec(xx,fs,128) %-- same as: specgram(xx,128,fs);
```

For two variations of the MATLAB code on the second line, sketch the resulting spectrogram.

(a) `xx = cos(2*pi*50*tt) .* cos(2*pi*400*tt);`

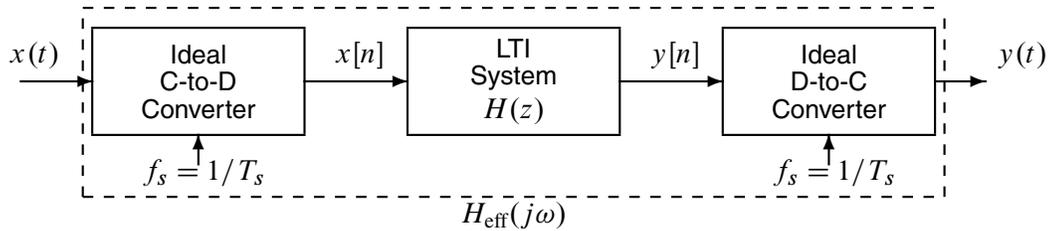


(b) `xx = cos(2*pi*650*tt);`



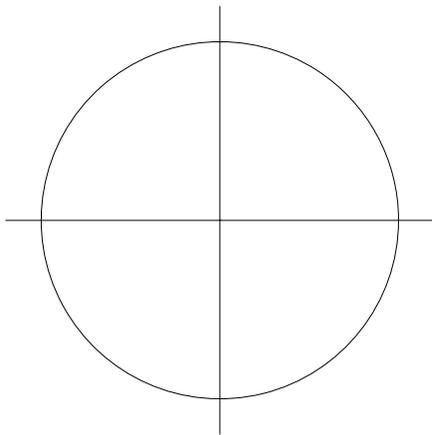
PROBLEM s-09-F.10:

Consider the following system for discrete-time filtering of a continuous-time signal:



Assume that the discrete-time FIR system has a system function $H(z)$ defined as: $H(z) = 1 + b_1 z^{-1} + z^{-2}$

- (a) For the case where $b_1 = 1.2$, determine the poles and zeros of the system, and then give your answer as a pole-zero plot. Account for **all** poles and zeros.



- (b) The effective frequency response of this system (using the $H(z)$ above) is able to null out one sinusoid. It is similar to the system used in the lab¹ to remove a sinusoidal interference from an EKG signal. The value of the filter coefficient b_1 controls the (frequency) location of the null. If the sampling rate is $f_s = 600$ Hz, determine the value of b_1 so that the overall effective frequency response has a null at 60 Hz.

$b_1 =$

¹This is *not* an IIR notch filter; it is an FIR *nulling* filter.