GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2009
Problem Set #9

Assigned: 26-Oct-09
Due Date: Week of 2-Nov-09

Reading: In *SP First*, Chapter 7: *z-Transform* and Chapter 8 *IIR Filters.*

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

Turn in all **STARRED** problems. Some subset of these problems will be randomly selected for grading.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM. After this assignment is handed in by everyone, solutions will be posted to the web.

Two-Part Format for HW Solutions: For each homework problem, two distinct pieces of information are required for a complete solution:

(a) *Approach:* Write a clear explanation of ***how*** you are going to solve the problem. Write in complete sentences. This explanation should be written with little or no mathematical formulas, and it should also be written so that it is independent of the specific numerical values in the problem.

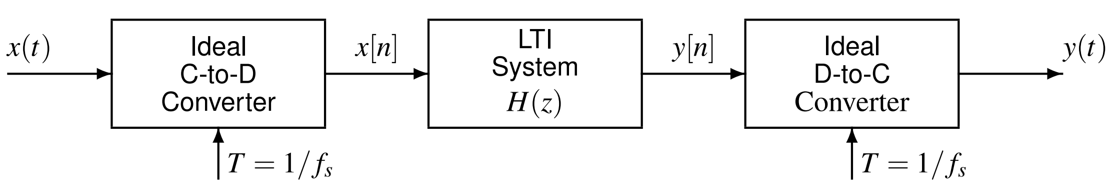
(b) *Details:* Carry out the solution of the particular problem. Details mean getting the algebra correct, making precise plots, and doing the numerical calculations are the key.

PROBLEM 9.1*:
The input to the C-to-D converter in the figure below is

$$x(t) = 3\cos(750\pi t - \pi/3) + 5\cos(1875\pi t - 2\pi/3)$$

The LTI system is an FIR filter that is a six-point running sum filter whose system function is

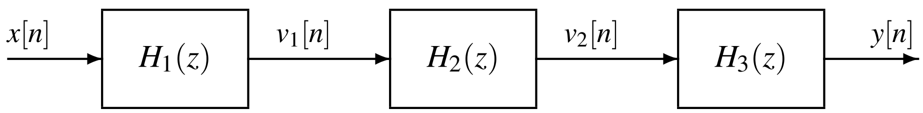
$$H(z) = \frac{1 - z^{-6}}{1 - z^{-1}}$$



(a) Determine the values of the frequency response at $\hat{\omega} = \pi$ and $\frac{1}{2}\pi$ by converting $\hat{\omega}$ into z and evaluating $H(z)$ directly. In other words, do not use the formula for $H(e^{j\hat{\omega}})$

(b) If $f_s = 1500$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter. *Hint:* Recall that the frequency response $H(e^{j\hat{\omega}})$ can be obtained directly from $H(z)$.

PROBLEM 9.2*:
In the following cascade of systems, all of the individual system functions, $H_i(z)$, are known.



$H_1(z) = 1 - z^{-2}$ $\qquad$ $H_2(z) = 2z^{-1} + z^{-2}$ $\qquad$ $y[n] = a_1 y[n-1] + a_2 y[n-2] + a_3 y[n-3] + b_0 v_2[n]$

(a) Determine $H_3(z)$, the z-transform of the last system.

(b) When $a_1 = a_2 = \frac{1}{2}$, $a_3 = 0$ and $b_0 = 12$, determine the system function, $H(z)$ for the cascaded system.

(c) Consider the impulse response $h[n]$ of the cascaded system, i.e., the output when the input is $x[n] = \delta[n]$. Determine values for $\{a_1, a_2, a_3, b_0\}$ so that the impulse response will be $h[n] = 6\delta[n - n_d]$. Also determine the value of the time shift n_d.

PROBLEM 9.3*:
We now have ***four ways*** of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of the four, and you must find the other three.

(a) $H(z) = \dfrac{5z - 6}{z^2}$

(b) $h[n] = \delta[n-3] * \delta[n-5]$

(c) $y[n] = \frac{1}{3}\left(x[n-6] - x[n-9]\right)$

(d) $h[n] = \frac{1}{3}\left(u[n-7] - u[n-11]\right)$

(e) $H(z) = \dfrac{z^{-2} + z^{-4}}{7 - 4z^{-1}}$

PROBLEM 9.4*:
The system function of a linear time-invariant system is given by the formula

$$H(z) = \frac{(1 - z^{-1})(1 - e^{j\pi/2}z^{-1})(1 - e^{-j\pi/2}z^{-1})}{(1 - 0.9e^{j2\pi/3}z^{-1})(1 - 0.9e^{-j2\pi/3}z^{-1})}$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.
Hint: Multiply out the factors of $H(z)$.

(b) Plot all the poles and zeros of $H(z)$ in the complex z-plane.
Hint: Express $H(z)$ as a ratio of factored polynomials in z instead of z^{-1}.

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}_0 n}$, for what values of $-\pi \leq \hat{\omega}_0 \leq \pi$ will $y[n] = 0$?

PROBLEM 9.5*:
We have developed several concepts that are useful in solving problems involving LTI systems. The main concepts are the *difference equation*, the *impulse response*, the *system function*, and the *frequency response* function. Most problem solving demands that you be able to go back and forth among these different mathematical representations of the LTI system because, as simple as it seems, the z-transform is *not* always the best tool for solving problems. Indeed, for a specific problem, one of these representations may be more convenient than the others, or we may need to use more than one of these representations in solving a given problem. The following is a simple problem that might be posed about an LTI system:

Given the input sequence $x[n]$ find the output sequence for all n when the system is an IIR filter:

$$y[n] = -0.75y[n-1] + x[n-2] - x[n-3]$$

The following is a partial list of possible approaches to solving this problem:

1. *Time-Domain:* Use the difference equation representation of the system to compute the output $y[n]$ for all required values of n. *For example, you could do this using* MATLAB.

2. *Z-Domain:* Multiply the z-transform of the input by the system function and determine $y[n]$ as the inverse z-transform of $Y(z)$. Sometimes it is possible to do a pole-zero cancellation to simplify the algebra.

3. *Frequency-Domain:* Break the input into a sum of complex exponential signals, use the frequency response function to determine the output due to each complex exponential signal separately, and finally, add the individual outputs together to get $y[n]$.

In each of these solution methods you would use one or more of the basic representations of the first-order IIR filter. Which method is easiest will have a lot to do with the nature of the input signal. For example, if you are given the difference equation and you want to use approach #2, you will have to determine the system function $H(z)$ from the difference equation coefficients.

Now in each of the following cases, the input will be given. In each case, determine which representation of the system and which of the above approaches will lead to the easiest solution of the problem, and detail the steps in using that approach to solve the problem. For example, if you choose approach #2 to solve the problem, your answer should be something like the following:

Step 1: Find $X(z)$, the z-transform of $x[n]$.

Step 2: Find $H(z)$, the system function of the first-order IIR filter.

Step 3: Multiply $X(z)H(z)$ to get $Y(z)$.

Step 4: Take the inverse z-transform of $Y(z)$ to get $y[n]$.

Now here are some possible inputs. In each case, state which of the approaches above (#1, #2, or #3) you would use. There may not be a clear cut answer. Give the approach that you *think* will yield the solution with least effort. Then carry out the method to get the output.

(a) $x[n] = u[n]$.

(b) $x[n] = \cos((\pi/3)n - \pi/3) + 2\cos((2\pi/3)n - \pi/4)$ for $-\infty < n < \infty$.

(c) $x[n] = 7\delta[n-4]$.

(d) $x[n]$ is a sampled speech signal. It is represented by a vector of 10000 numbers. In this case, you do not have to find the actual output.