

Y

ECE 2025 Homework #8 Solutions

- a) Find impulse response and use text eqⁿ (6.4).
 b) derive formula for finite sum
 c) Evaluate $H(e^{j\hat{\omega}})$ and plot.
 d) Solve by using frequency response function $H(e^{j\hat{\omega}})$

$$a) h[n] = \sum_{k=0}^4 \frac{1}{5} s[n-k]$$

$$H(e^{j\hat{\omega}}) = \frac{1}{5} \sum_{k=0}^4 e^{-j\hat{\omega}k}$$

$$b) H(e^{j\hat{\omega}}) = \frac{1}{5} \sum_{k=0}^4 x^k, \text{ where } x = e^{-j\hat{\omega}}$$

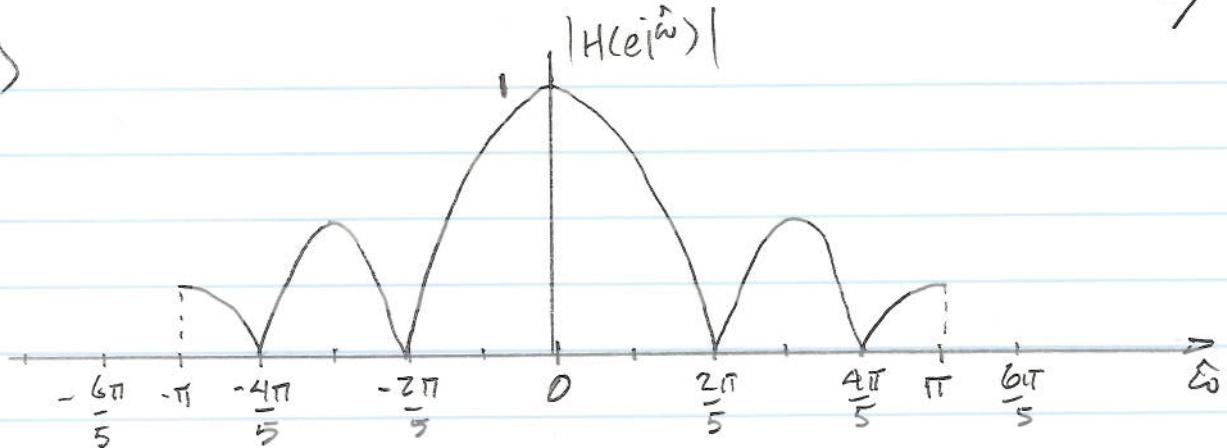
However in second term

$$\begin{aligned} \sum_{k=0}^M x^k &= \sum_{k=0}^{\infty} x^k - \sum_{k=M+1}^{\infty} x^k && l = k-M-1 \\ &= \frac{1}{1-x} - \sum_{l=0}^{\infty} x^{l+M+1} && k = l+M+1 \\ &= \frac{1}{1-x} - x^{M+1} \sum_{l=0}^{\infty} x^l \\ &= \frac{1}{1-x} - \frac{x^{M+1}}{1-x} = \frac{1-x^{M+1}}{1-x} \end{aligned}$$

$$\begin{aligned} \therefore H(e^{j\hat{\omega}}) &= \frac{1}{5} \cdot \frac{1 - e^{-j5\hat{\omega}}}{1 - e^{-j\hat{\omega}}} \\ &= \frac{j2e^{-j\frac{5}{2}\hat{\omega}} \left(\frac{e^{j\frac{5}{2}\hat{\omega}} - e^{-j\frac{5}{2}\hat{\omega}}}{2} \right)}{5 \cdot j2e^{-j\frac{\hat{\omega}}{2}} \left(\frac{e^{j\frac{\hat{\omega}}{2}} - e^{-j\frac{\hat{\omega}}{2}}}{2} \right)} = \frac{\sin(5\hat{\omega}/2)e^{-j\hat{\omega}}}{5 \sin(\hat{\omega}/2)} \end{aligned}$$

2/

c)



d) $x[n] = 7 + 13 \cos(\hat{\omega}_0 n)$

dc response

$$\begin{aligned} \text{At } \hat{\omega}_0 = 0 \quad y[n] &= x[n] \cdot H(e^{j0}) \\ &= 20 \cdot 1 = 20 \text{ constant} \end{aligned}$$

Also at $\hat{\omega}_0 = \frac{2\pi}{5}$ and $\frac{4\pi}{5}$ $H(e^{j\hat{\omega}}) = 0$

$$y[n] = 7 \cdot 1 = 7 \text{ constant.}$$

since the $13 \cos(\hat{\omega}_0 n)$ term is filtered out.

- 2 a) Find impulse resp. then freq resp.
- b) multiply $H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})$; use #1
- c) Plot over range $-\pi \leq \hat{\omega} \leq \pi$
- d) Find response $H(e^{j\hat{\omega}})$ at 0.2π

a) $h_2[n] = S[n-1] - S[n-2]$

$$\begin{aligned} H(e^{j\hat{\omega}}) &= e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} \\ &= j2e^{-j\frac{3\hat{\omega}}{2}} \left(\frac{e^{j\frac{\hat{\omega}}{2}} - e^{-j\frac{\hat{\omega}}{2}}}{j2} \right) \\ &= 2e^{-j\left(\frac{3\hat{\omega}}{2} - \pi/2\right)} \sin(\hat{\omega}/2) \end{aligned}$$

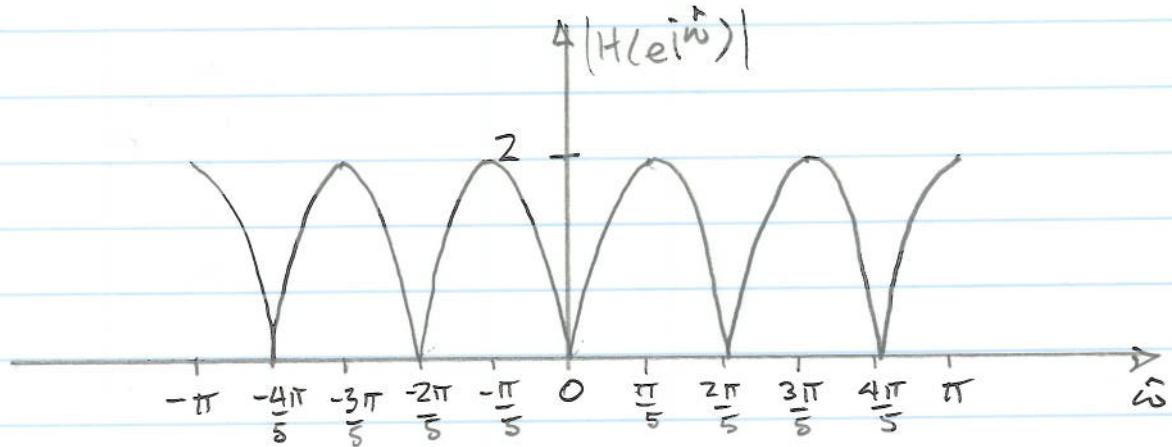
$$b) H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}}) H_2(e^{j\hat{\omega}})$$

Using results from Problem #1, $H_1(e^{j\hat{\omega}}) = \frac{\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j2\hat{\omega}}$

$$\text{Then } H(e^{j\hat{\omega}}) = \frac{\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j2\hat{\omega}} \cdot 2e^{-j(\frac{3\hat{\omega}}{2} - \pi/2)} \sin(\hat{\omega}/2)$$

$$= 2 \sin(5\hat{\omega}/2) e^{-j(\frac{7\hat{\omega}}{2} - \pi/2)}$$

$$c) |H(e^{j\hat{\omega}})| = |2 \sin(5\hat{\omega}/2)|$$



$$d) H(e^{j0.2\pi}) = 2 \sin(\pi/2) e^{-j(\frac{7\pi}{10} - \frac{\pi}{2})}$$

$$= 2 e^{-j\pi/5}$$

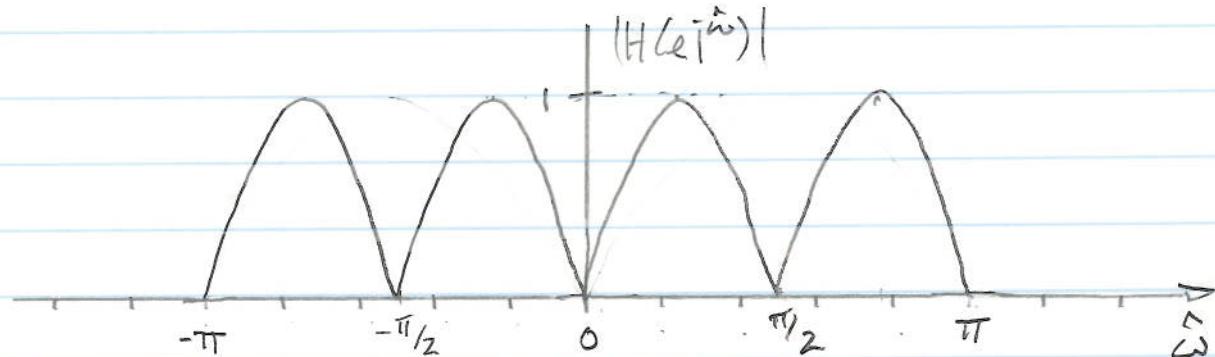
$$\therefore y[n] = 7 \cdot 2 \cdot \cos(0.2\pi n - 0.1\pi - 0.2\pi)$$

$$= 14 \cos(0.2\pi n - 0.3\pi)$$

- 3 a) write $H(e^{j\hat{\omega}})$ as a sine function and plot.
 b) draw spectrum and show all aliases,
 especially those in range $-\pi \leq \omega \leq \pi$
 c) Reconstruct $y[n]$ from components
 having frequencies in range $\pi \leq \omega \leq \pi$.

$$\begin{aligned} a) \quad H(e^{j\hat{\omega}}) &= \frac{1}{2} - \frac{1}{2}e^{-j4\hat{\omega}} \\ &= j e^{j2\hat{\omega}} \left(\frac{e^{j2\hat{\omega}} - e^{-j2\hat{\omega}}}{j2} \right) \\ &= \sin(2\hat{\omega}) e^{j(z\hat{\omega} - \pi/2)} \end{aligned}$$

$$|H(e^{j\hat{\omega}})| = |\sin(2\hat{\omega})|$$



$$b) \quad x[n] = x(t) | t = n/f_s$$

$$= 40 + 10 \cos\left(\frac{3\pi n}{8}\right)$$

• 40



c) use superposition

$$x[n] = x_1[n] + x_2[n]$$

$$x_1[n] = 40$$

$$x_2[n] = 10 \cos\left(\frac{3\pi}{8}n\right)$$

For $x_1[n] = 40$, $\hat{\omega} = 0$ and $H(e^{j0}) = 0$

$$\therefore y_1[n] = 0$$

For $x_2[n] = 10 \cos\left(\frac{3\pi}{8}n\right)$, $\hat{\omega} = \frac{3\pi}{8}$

$$H(e^{j\frac{3\pi}{8}}) = \sin\left(\frac{3\pi}{4}\right) e^{-j\left(\frac{3\pi}{4} - \frac{\pi}{2}\right)}$$

$$= \sin\left(\frac{3\pi}{4}\right) e^{-j\pi/4}$$

$$= \frac{1}{\sqrt{2}} e^{-j\pi/4}$$

$$y_2[n] = 10 |H(e^{j\frac{3\pi}{8}})| \cos\left(\frac{3\pi}{8}n + \angle H(e^{j\frac{3\pi}{8}})\right)$$

$$= \frac{10}{\sqrt{2}} \cos\left(\frac{3\pi}{8}n - \frac{\pi}{4}\right)$$

Finally, $y[n] = y_1[n] + y_2[n]$

$$= \frac{10}{\sqrt{2}} \cos\left(\frac{3\pi}{8}n - \frac{\pi}{4}\right)$$

$$y(t) = y[n]|_{n=tfs} = \frac{10}{\sqrt{2}} \cos(300\pi t - \pi/4)$$

4a) Find the frequency response in terms of b_0 and b_4 . Use the two inputs to create two equations in two unknowns and solve

b) Find all values of ω such that $H(e^{j\omega}) = 1$.

$$a) H(e^{j\omega}) = b_0 + b_4 e^{-j4\omega}$$

$$\text{From } x_2[n] = 50 \rightarrow y_2[n] = 50$$

$$\text{we have } 50 = H(e^{j0}) \cdot 50$$

$$50 = (b_0 + b_4) \cdot 50$$

$$b_0 + b_4 = 1 \quad (1)$$

$$\text{From } x_1[n] = 60 \cos\left(\frac{\pi}{4}n\right) \Rightarrow y_1[n] = 0$$

$$0 = H(e^{j\frac{\pi}{4}}) \cdot 60 \cos\left(\frac{\pi}{4}n\right)$$

$$\therefore H(e^{j\frac{\pi}{4}}) = 0$$

so

$$b_0 + b_4 e^{-j\pi} = 0$$

$$b_0 - b_4 = 0 \quad (2)$$

From (1) and (2)

$$b_0 = b_4 = \frac{1}{2}$$

$$\begin{aligned}
 b) \quad H(e^{j\hat{\omega}}) &= \frac{1 + e^{-j4\hat{\omega}}}{z} \\
 &= e^{-j2\hat{\omega}} \left(\frac{e^{j2\hat{\omega}} + e^{-j2\hat{\omega}}}{z} \right) \\
 &= \cos(2\hat{\omega}) e^{-j2\hat{\omega}}
 \end{aligned}$$

For $y[n] = x[n] = \cos(\hat{\omega}_1 n)$

We need $H(e^{j\hat{\omega}}) = 1$

Hence, $\hat{\omega}_1 = 0$ or π

5. a) use inverse Euler formula for cosines
 b) write $x[n] = x_1[n] + x_2[n]$ and use
 superposition to find $y[n] = y_1[n] + y_2[n]$

$$\begin{aligned}
 a) \quad x[n] &= 4e^{j\pi} + e^{j(0.4\pi n + \pi/4)} + e^{-j(0.4\pi n + \pi/4)} \\
 &= -4 + 2\cos(0.4\pi n + \pi/4)
 \end{aligned}$$

b) From Problem #1 or (7.25) in textbook

$$\begin{aligned}
 H(e^{j\hat{\omega}}) &= \frac{1}{7} \sum_{k=0}^6 e^{-j\hat{\omega} k} \\
 &= \frac{\sin(7\hat{\omega}/2)}{7 \sin(\hat{\omega}/2)} e^{-j3\hat{\omega}}
 \end{aligned}$$

$$\text{For } x_1[n] = -4$$

$$H(e^{j0}) = 1 \text{, so } y_1[n] = -4$$

$$\text{For } x_1[n] = 2 \cos(0.4\pi n + \pi/4)$$

$$H(e^{j\frac{2\pi}{5}}) = \frac{\sin(7\pi/5)}{7\sin(\pi/5)} e^{-j\frac{6\pi}{5}}$$

$$\text{Hence, } y_1[n] = \frac{2\sin(7\pi/5)}{7\sin(\pi/5)} \cos(0.4\pi n + \frac{\pi}{4} - \frac{6\pi}{5})$$

$$= \frac{2\sin(7\pi/5)}{7\sin(\pi/5)} \cos(0.4\pi n - 19\pi/20)$$

Then

$$y[n] = y_1[n] + y_2[n]$$

$$= -4 + \frac{2\sin(7\pi/5)}{7\sin(\pi/5)} \cos(0.4\pi n - 19\pi/20)$$

$$= -4 - 0.462 \cos(0.4\pi n - 19\pi/20)$$