GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2009
Problem Set #8

Assigned: 12-Oct-09
Due Date: Week of 19-Oct-09

Quiz #2 will be held in lecture on Friday 23-Oct-09. It will cover material from Chapter 3 (sections 3-3 through 3-9) and Chapters 4-6 as represented in Problem Sets #4–#8.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides).

Reading: In *SP First*, Chapter 6: *Frequency Response of FIR Filters*.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

Turn in all **STARRED** problems. Some subset of these problems will be randomly selected for grading.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM. After this assignment is handed in by everyone, solutions will be posted to the web.

Two-Part Format for HW Solutions: For each homework problem, two distinct pieces of information are required for a complete solution:

(a) *Approach:* Write a clear explanation of ***how*** you are going to solve the problem. Write in complete sentences. This explanation should be written with little or no mathematical formulas, and it should also be written so that it is independent of the specific numerical values in the problem.

(b) *Details:* Carry out the solution of the particular problem. Details mean getting the algebra correct, making precise plots, and doing the numerical calculations are the key.

PROBLEM 8.1*:

Consider the linear time-invariant system given by the difference equation

$$y[n] = 0.2x[n] + 0.2x[n-1] + 0.2x[n-2] + 0.2x[n-3] + 0.2x[n-4] = \sum_{k=0}^{4} \tfrac{1}{5}x[n-k]$$

(a) Find an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(b) Show that your answer in (a) can be expressed in the form

$$H(e^{j\hat{\omega}}) = \frac{\sin(5\hat{\omega}/2)}{5\sin(\hat{\omega}/2)} e^{-j2\hat{\omega}}$$

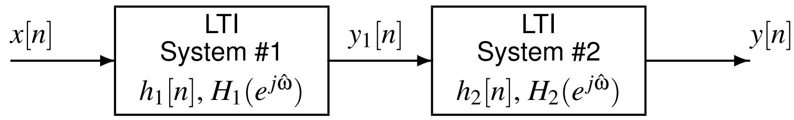
(c) Sketch the frequency response (magnitude only) as a function of frequency from the formula above. You might want to check your plot by doing it in MATLAB with `freekz( )` or `freqz( )`.

(d) Suppose that the input is $x[n] = 7 + 13\cos(\hat{\omega}_0 n)$, for $-\infty < n < \infty$. Find all possible nonzero frequencies $0 < \hat{\omega}_0 \le \pi$ for which the output $y[n]$ is a constant for all n, i.e.,

$$y[n] = c \qquad \text{for } -\infty < n < \infty$$

and find the value for c. (In other words, the sinusoid is removed by the filter.)

PROBLEM 8.2*:
The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



Suppose that System #1 is a filter described by the impulse response

$$h_1[n] = \sum_{k=0}^{4} \delta[n-k]$$

and System #2 is described by the difference equation

$$y_2[n] = y_1[n-1] - y_1[n-2],$$

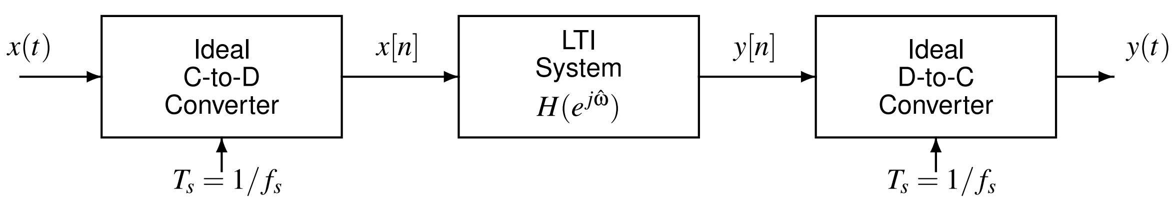
(a) Determine the frequency response, $H_2(e^{j\hat{\omega}})$, of the second system.

(b) Determine the frequency response, $H(e^{j\hat{\omega}})$, of the overall cascade system.

(c) Plot the magnitude of the overall frequency response of the cascaded system.

(d) When the input to this system is $x[n] = 7\cos(0.2\pi n - 0.1\pi)$. Use the frequency response to determine $y[n]$ over the range $-\infty < n < \infty$.

PROBLEM 8.3*:
Consider the following system for discrete-time filtering of a continuous-time signal:



In this problem, assume that the frequency response of the discrete-time system is

$$H(e^{j\hat{\omega}}) = \tfrac{1}{2} - \tfrac{1}{2}e^{-j4\hat{\omega}}$$

(a) Make a plot of the frequency response magnitude for $H(e^{j\hat{\omega}})$ over the frequency range $-\pi < \hat{\omega} \le \pi$.

(b) In this part, assume that the input is

$$x(t) = 40 + 10\cos(300\pi t) \quad \text{for} \quad -\infty < t < \infty$$

For a sampling rate of $f_s = 800$ samples/sec, draw the spectrum of $x[n]$, the discrete-time signal after the C-to-D converter.

(c) For the same $x(t)$ as in the previous part, and the same sampling rate, determine a simple formula for the output $y(t)$ for $-\infty < t < \infty$.

PROBLEM 8.4*:
A discrete-time system is defined by the input/output relation

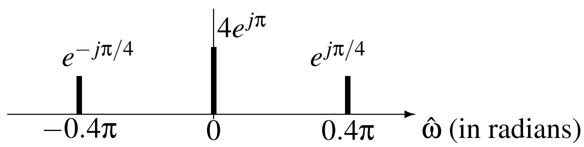
$$y[n] = b_0 x[n] + b_4 x[n-4]$$

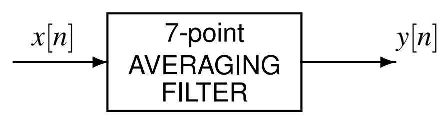
where b_0 and b_4 are constants to be determined.

(a) When the input is the signal, $x_1[n] = 60\cos(0.25\pi n)$, the output is zero, i.e., $y_1[n] = 0$. Also, when the input is the DC signal, $x_2[n] = 50$, the output is $y_2[n] = 50$. Determine the values of b_0 and b_4.

(b) Suppose that the input signal is $x[n] = \cos(\hat{\omega}_1 n)$. Determine all values of $\hat{\omega}_1$ in the range $0 \le \hat{\omega}_1 \le \pi$ for which the output will be equal to the input, i.e., $y[n] = \cos(\hat{\omega}_1 n)$.

PROBLEM 8.5*:
A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.