

Problem 7.1

$$1. \quad y[n] = e^{-j\frac{\pi}{4}x[n]}$$

$$2. \quad y[n] = x[z^n]$$

a) for $x[n] = \delta[n]$

$$1. \quad y[n] = e^{-j\frac{\pi}{4}\delta[n]} = h[n]$$

$$h[n] = \begin{cases} e^{-j\frac{\pi}{4}} & n=0 \\ 1 & n \neq 0 \end{cases}$$

$$2. \quad y[n] = \delta[z^n] = h[n]$$

$$h[n] = \begin{cases} 0 & n \geq 0 \\ \text{undefined} & n < 0 \end{cases}$$

b) Are the systems linear?

1. NO.

If $x_2[n] = 2\delta[n]$, a linear system would produce an output $y_2[n] = 2h[n]$ but the output is

$$y_2[n] = \begin{cases} e^{-j\frac{\pi}{2}} & n=0 \\ 0 & n \neq 0 \end{cases}$$

and $y_2[n] \neq 2h[n]$

2. YES (assuming that we only consider $n \geq 0$)

$$\text{assume } x[n] = ax_1[n] + bx_2[n]$$

$$\text{and } x[n] \Rightarrow y[n]$$

$$x_1[n] \Rightarrow y_1[n]$$

$$x_2[n] \Rightarrow y_2[n]$$

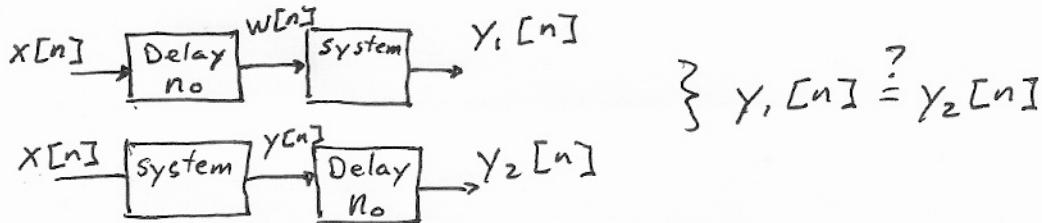
$$\text{then } y[n] = ax_1[z^n] + bx_2[z^n]$$

$$\text{and } ay_1[n] + by_2[n] = ax_1[z^n] + bx_2[z^n] \\ = y[n]$$

Problem 7.1 cont.

c) Are the systems time-invariant?

1. YES



$$w[n] = x[n - n_0]; \quad y_1[n] = e^{-j\frac{\pi}{4}w[n]} = e^{-j\frac{\pi}{4}x[n - n_0]}$$
$$y_2[n] = y[n - n_0] = e^{-j\frac{\pi}{4}x[n - n_0]}$$

2. NO

if $x[n] = \delta[n]$ ~~then~~ and $n_0 = 1$

$$y_1[n] = \begin{cases} 0 & n \geq 0 \\ \text{undefined} & n < 0 \end{cases}$$

$$y_2[n] = \begin{cases} 1 & n = 0 \\ 0 & n > 0 \\ \text{undefined} & \text{else} \end{cases}$$

d) Are the systems causal?

1. Yes, the current input only effects the current output

2. The system is undefined for $n < 0$

Problem 7.2

a) $y_n = \text{conv}([1 \ 0 \ 0 \ 1 \ 1], \cos(\pi/4 \cdot \rho i \cdot (0:8)))$;

$$\cos(\pi/4 \cdot (0:8)) = [1 \ \sqrt{2}/2 \ 0 \ -\sqrt{2}/2 \ -1 \ -\sqrt{2}/2 \ 0 \ \sqrt{2}/2 \ 1]$$

n	0	1	2	3	4	5	6	7	8	9	10	11	12
y	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1				
x	1	0	0	1	1								
	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1				
	0	0	0	0	0	0	0	0	0				
	0	0	0	0	0	0	0	0	0				
	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1				
$+$		1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1			
	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$1 + \frac{\sqrt{2}}{2}$

Matlab produces the same thing:

$$\text{ans} = \begin{bmatrix} 1.0000 & 0.7071 & 0.0000 & 0.2929 & 0.7071 & 0.0000 \\ -0.7071 & -1.0000 & -0.7071 & -0.7071 & 0.7071 & \\ 1.7071 & 1.0000 \end{bmatrix}$$

Problem 7.2 cont

b) $y[n] = p[n] * p[n]$ where $p[n] = u[n-7] - u[n]$

This may be done numerically or analytically

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
---	---	---	---	---	---	---	---	---	---	---	----	----	----	----	----

$p(n)$	-1	-1	-1	-1	-1	-1	-1							
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$p(n)$	-1	-1	-1	-1	-1	-1	-1						
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+	1	1	1	1	1	1	1						
---	---	---	---	---	---	---	---	--	--	--	--	--	--

	1	1	1	1	1	1	1					
--	---	---	---	---	---	---	---	--	--	--	--	--

	1	0	1	1	1	1	1				
--	---	---	---	---	---	---	---	--	--	--	--

	1	1	1	1	1	1	1			
--	---	---	---	---	---	---	---	--	--	--

	1	1	1	1	1	1	1		
--	---	---	---	---	---	---	---	--	--

	1	0	1	1	1	1	1	
--	---	---	---	---	---	---	---	--

+	1	2	3	4	5	6	7	6	5	4	3	2	1	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

- or -

$$y[n] = (u[n-7] - u[n]) * (u[n-7] - u[n])$$

$$= u[n-7] * u[n-7] - 2u[n] * u[n-7] + u[n] * u[n]$$

we find $u[n] * u[n-n_0] = \sum_{k=-\infty}^{\infty} u[k-n_0] u[n-k]$

$$= \left(\sum_{k=n_0}^n 1 \right) u[n-n_0] = (n-n_0) u[n-n_0]$$

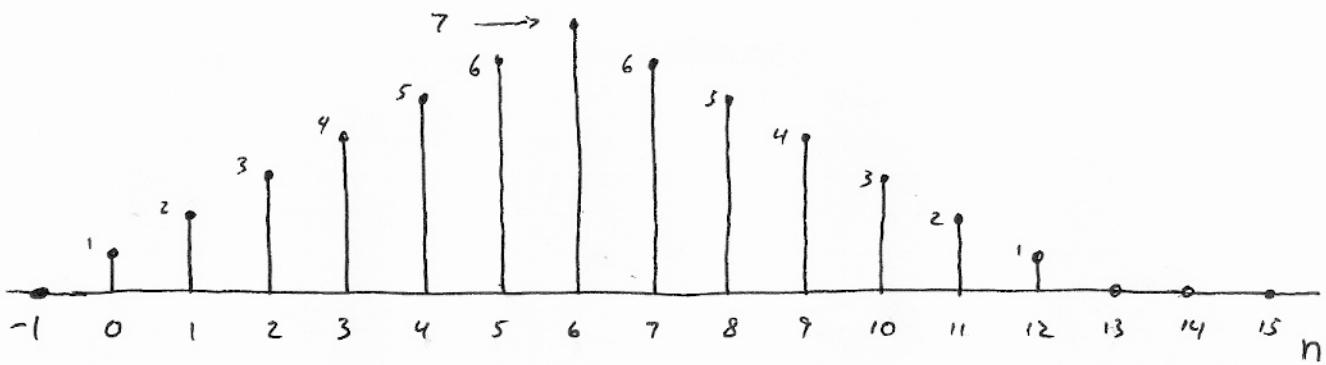
also, note that $u[n-7] * u[n-7] = u[n] * u[n-14]$

then $y[n] = (n-14) u[n-14] - 2(n-6) u[n-7] + (n+1) u[n]$

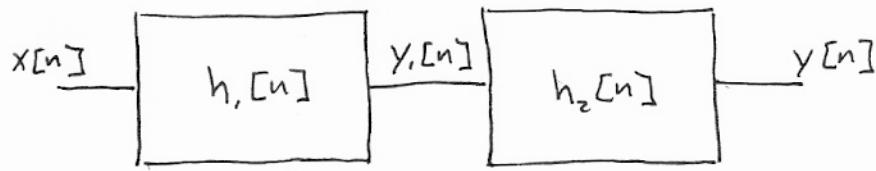
$$= \begin{cases} 0 & n < 0 \\ n+1 & 0 \leq n < 7 \\ 13-n & 7 \leq n < 14 \\ 0 & 14 \leq n \end{cases}$$

Problem 7.2 cont.

b) cont.



Problem 7.3



a) $y_1[n] = \sum_{k=1}^7 \beta^k x[n-k]$ $h_2[n] = \delta[n-1] - \beta \delta[n-2]$

if $x[n] = \delta[n]$ then $h_1[n] = \sum_{k=1}^7 \beta^k \delta[n-k]$
 $h_1[n] = \beta^n (\delta[n-1] - \delta[n-8])$

b) $h[n] = h_1[n] * h_2[n]$

$$= (\beta^n u[n-1] - \beta^n u[n-8]) * (\delta[n-1] - \beta \delta[n-2])$$

$$= (\beta^n u[n-1]) * \delta[n-1] - (\beta^n u[n-1]) * (\beta \delta[n-2]) - \\ (\beta^n u[n-8]) * \delta[n-1] + (\beta^n u[n-8]) (\beta \delta[n-2])$$

$$= \beta^{(n-1)} u[n-2] - \beta^{(n-1)} u[n-3] - \beta^{(n-1)} u[n-9] + \beta^{(n-1)} u[n-10]$$

$$h[n] = \beta^{n-1} (\delta[n-2] - \delta[n-9])$$

c) $y[n] = \beta x[n-2] - \beta^8 x[n-9]$

if $\beta = 0.85$ then

$$y[n] = 0.85 x[n-2] - 0.2725 x[n-8]$$

Problem 7.4

$$y[n] = \sum_{k=2}^7 (0.6)^{k-3} x[n-k]$$

a) filter order $M = 7$

$$\text{filter length } L = 7 - 2 + 1 = 6$$

$$\text{filter coefficients } b_k = \{0 \ 0 \ 0.6^{-1} \ 0.6 \ 0.6^2 \ 0.6^3 \ 0.6^4\}$$

b) $x[n] \neq 0$ iff $19 \leq n \leq 33$

$y[n] \neq 0$ for $21 \leq n \leq 40$

c)

$$d) L_y = 40 - 21 + 1 = 20 \quad (\text{from (b)})$$

$$L_y = L + L_x - 1 = 6 + 15 - 1 = 20$$