GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2009
Problem Set #7

Assigned: 5-Oct-09
Due Date: Week of 12-Oct-09

Reading: In *SP First*, Chapter 5: *FIR Filters*.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

Turn in all **STARRED** problems. Some subset of these problems will be randomly selected for grading.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM. After this assignment is handed in by everyone, solutions will be posted to the web.

Two-Part Format for HW Solutions: For each homework problem, two distinct pieces of information are required for a complete solution:

(a) *Approach:* Write a clear explanation of ***how*** you are going to solve the problem. Write in complete sentences. This explanation should be written with little or no mathematical formulas, and it should also be written so that it is independent of the specific numerical values in the problem.

(b) *Details:* Carry out the solution of the particular problem. Details mean getting the algebra correct, making precise plots, and doing the numerical calculations are the key.

PROBLEM 7.1*:
For each of the following systems, the signal $x[n]$ is the input and $y[n]$ is the output.

1. $y[n] = e^{-j0.25\pi x[n]}$ (Complex Exponential)
2. $y[n] = x[2^n]$ (Time Distortion)

(a) Find the impulse response for both systems. Give your answers as plots.

(b) Determine if the systems are (1) linear; give a proof or counterexample.

(c) Determine if the systems are (2) time-invariant; give a proof or counterexample.

(d) Determine if the systems are (3) causal; give a proof or counterexample.

PROBLEM 7.2*:
Evaluate the following

(a) `yn = conv( [1 0 0 1 1], cos(0.25*pi*(0:8)) );`
Do the calculation by hand, and then check the result with MATLAB.

(b) $y[n] = p[n] * p[n]$, where $p[n] = u[n-7] - u[n]$ and $u[n]$ is the unit-step function as defined in problem P-5.2 in the book (p. 126). Make a plot of $y[n]$ versus n.
Hint: Use the MATLAB GUI `dconvdemo` to check your work.

PROBLEM 7.3*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

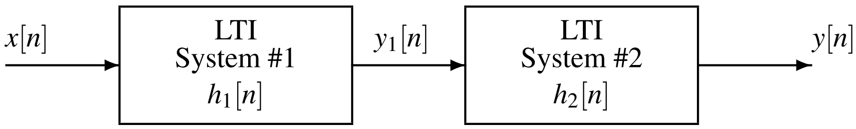

Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is a "blurring" filter described by the difference equation

$$y_1[n] = \sum_{k=1}^{7} \beta^k x[n-k],$$

and System #2 is described by the impulse response

$$h_2[n] = \delta[n-1] - \beta\delta[n-2],$$

where β is a real number. Determine the impulse response sequence, $h_1[n]$, of the first system.

(b) Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(c) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1. Give numerical values of the filter coefficients for the specific case where $\beta = 0.85$.

PROBLEM 7.4*:

Consider a system defined by $\quad y[n] = \sum_{k=2}^{7} (0.6)^{k-3} x[n-k]$

(a) Determine the filter order (M), the filter length (L), and the filter coefficients $\{b_k\}$.
Note: the first few filter coefficients, b_0 and b_1, are zero.

(b) Suppose that the input $x[n]$ is nonzero only for $19 \le n \le 33$. Where will the output start, i.e., what is the index of the first nonzero $y[n]$?

(c) Where does $y[n]$ end, i.e., determine the index of the last non-zero value in the output sequence $y[n]$?

(d) If we define the length of the input signal in part (b) to be the number of nonzero values starting at $n = 19$ and ending at $n = 33$, then its length is 15 samples over its nonzero region. Determine the total length of the **output** signal (in samples) over its nonzero region. Explain by relating the nonzero output length to the nonzero input length and the length of the nonzero part of $h[n]$ which is 6.

Hint: Draw a sketch similar to Fig. 5.5 (on p. 105) to illustrate the regions where the output signal is zero and nonzero.