GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2009
Problem Set #5

Assigned: 21-Sep-09
Due Date: Week of 28-Sep-09

Reading: In *SP First*, Chapter 3: *Spectrum Representation,* Sections 3-4 through 3-8 and Chapter 4: *Sampling and Aliasing,* Sections 4-1 and 4-2.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Turn in all **STARRED** problems. Some subset of these problems will be randomly selected for grading.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM. After this assignment is handed in by everyone, solutions will be posted to the web.

Two-Part Format for HW Solutions: For each homework problem, two distinct pieces of information are required for a complete solution:

(a) *Approach:* Write a clear explanation of ***how*** you are going to solve the problem. Write in complete sentences. This explanation should be written with little or no mathematical formulas, and it should also be written so that it is independent of the specific numerical values in the problem.

(b) *Details:* Carry out the solution of the particular problem. Details mean getting the algebra correct, making precise plots, and doing the numerical calculations are the key.

PROBLEM 5.1*:
Suppose that a periodic signal has a period of 4 s, and is defined *over one period* as:

$$x(t) = 5|t| \quad \text{for } -2 \le t \le 2$$

Then this signal is actually a scaled version of the triangle wave in Section 3-6.4 (page 55) of *SP-First*.

(a) Draw a plot of $x(t)$ over the range $-6 \le t \le 6$ s.

(b) Determine a general expression for all the Fourier series coefficients, a_k. No integrals should be required if you use the Fourier series information given in equation (3.39) on p. 56.
Hint: exploit the *amplitude scaling property* which says that the Fourier coefficients of $Ax(t)$ are Aa_k, where $\{a_k\}$ are the Fourier coefficients of $x(t)$.

(c) Determine the DC value by integrating $x(t)$ over one period, and dividing by the period.

(d) Make a spectrum plot of the triangle-wave signal defined above, showing a frequency range in hertz that includes the first three harmonics, i.e., $k = 0, \pm 1, \pm 2, \pm 3$.

(e) Define $x_3(t)$ as the signal synthesized from the first three harmonics (plus DC). Write a mathematical formula for $x_3(t)$ as a constant plus sinusoidal terms.
Note: you can check your work on this problem and the previous problem by using MATLAB to plot $x_3(t)$ for two or three periods—you should get a close approximation to a triangle wave.

PROBLEM 5.2*:
A periodic signal $x(t)$ is represented as a Fourier series of the form

$$x(t) = -2 + \sum_{k=-\infty}^{\infty} k^2 e^{j150\pi kt}$$

(a) Determine the fundamental **period** of the signal $x(t)$, i.e., the minimum period. Explain.

(b) Define a new signal by adding a sinusoid to $x(t)$

$$y(t) = 3\sin(300\pi t) + x(t)$$

The new signal, $y(t)$ can be expressed in the following Fourier Series with new coefficients $\{b_k\}$:

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j150\pi kt}$$

Fill in the following tables, giving *numerical values* for each $\{a_k\}$ and $\{b_k\}$ in polar form.
Note: A magnitude value must be nonnegative.
Hint: Find a simple relationship between $\{b_k\}$ and $\{a_k\}$, where $\{a_k\}$ denotes the Fourier coefficients of the original signal $x(t)$.

Signal: $x(t)$

a_k	Mag	Phase
a_3		
a_2		
a_1		
a_0		
a_{-1}		
a_{-2}		
a_{-3}		

Signal: $y(t)$

b_k	Mag	Phase
b_3		
b_2		
b_1		
b_0	***SAME***	
b_{-1}		
b_{-2}		
b_{-3}		

Note: Whenever a b_k coefficient is equal the corresponding a_k coefficient, just write ***SAME*** in the b_k table.

PROBLEM 5.3*:
Consider the cosine wave

$$x(t) = 10\cos(880\pi t + \phi)$$

Suppose that we obtain a sequence of numbers by sampling the waveform at equally spaced time instants nT_s. In this case, the resulting sequence would have values

$$x[n] = x(nT_s) = 10\cos(880\pi nT_s + \phi)$$

for $-\infty < n < \infty$. Suppose that $T_s = 0.0001$ sec.

(a) How many samples will be taken in one period of the cosine wave?

(b) Now consider another waveform $y(t)$ such that

$$y(t) = 10\cos(\omega_0 t + \phi)$$

Find a frequency $\omega_0 > 880\pi$ such that $y(nT_s) = x(nT_s)$ for all integers n.
Hint: Use the fact that $\cos(\theta + 2\pi n) = \cos(\theta)$ if n is an integer.

(c) For the frequency found in (b), what is the total number of samples taken in one period of $x(t)$?

PROBLEM 5.4*:
Suppose that a discrete-time signal $x[n]$ is given by the formula

$$x[n] = 2.2\cos(0.3\pi n - \pi/3)$$

and that it was obtained by sampling a continuous-time signal $x(t) = A\cos(2\pi f_0 t + \phi)$ at a sampling rate of $f_s = 6000$ samples/sec. Determine three different continuous-time signals that could have produced $x[n]$. All these continuous-time signals should have a frequency less than 8 kHz. Write the mathematical formula for all three.

PROBLEM 5.5*:
When watching old TV movies, all of us have seen the phenomenon where a wagon wheel appears to move backwards. The same illusion can also be seen in automobile commercials, when the hubcaps of a car or truck have a spoked pattern. Both of these are due to the fact that the video consists of a sequence of samples (frames) of the scene. For this problem, assume that the frame sampling rate is 25 frames per second (the PAL standard used in Europe).

In the figure to the right, a seven-spoked wheel is shown. Assume that the diameter of this wheel is 0.5 meters. In addition, assume that the wheel is rotating clockwise, so that if attached to a car, the car would be traveling to the right *at a constant speed.* However, when seen on TV the spoke pattern of the car wheel appears to stand still. How fast is the car traveling (in kilometers per hour)? Derive a general equation that will make it easy to give all possible answers.