GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2009
Problem Set #3

Assigned: 31-Aug-09
Due Date: Week of 7-Sep-09

Quiz #1 will be held in lecture on Monday 14-Sep-09. It will cover material from Chapters 2 and 3 (sections 3-1 and 3-2), as represented in Problem Sets #1, #2, and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides)

Reading: In *SP First*, Chapter 3: *Spectrum Representation,* Sections 3-1, 3-2 and 3-3.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
For the week of September 7 only, students in the **Monday recitations** should turn in HW #3 on Wednesday (9-Sep-09) at their lab time.

Turn in all **STARRED** problems. Some subset of these problems will be randomly selected for grading.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM. After this assignment is handed in by everyone, solutions will be posted to the web.

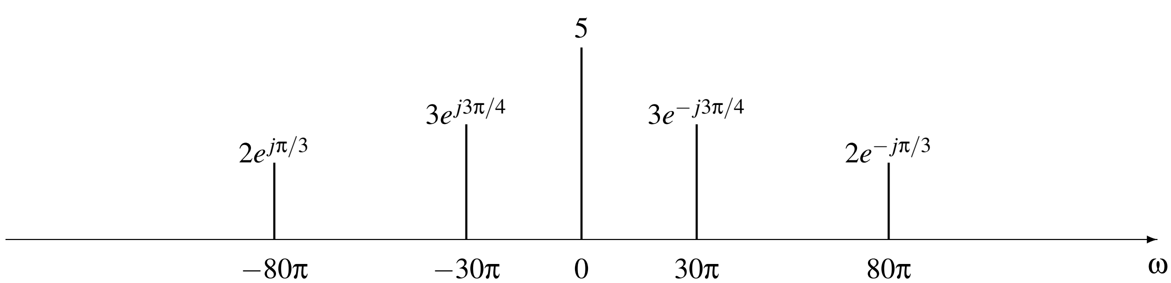
Two-Part Format for HW Solutions: For each homework problem, two distinct pieces of information are required for a complete solution:

(a) *Approach:* Write a clear explanation of ***how*** you are going to solve the problem. Write in complete sentences. This explanation should be written with little or no mathematical formulas, and it should also be written so that it is independent of the specific numerical values in the problem.

(b) *Details:* Carry out the solution of the particular problem. Details mean getting the algebra correct, making precise plots, and doing the numerical calculations are the key.

PROBLEM 3.1*:

A real signal $x(t)$ has the following two-sided spectrum:



(a) Write an equation for $x(t)$ as a sum of cosines.

(b) Plot the spectrum of the signal $y(t) = x(t - 0.01)$.

(c) Plot the spectrum of the signal $z(t) = x(t) + 2\cos(30\pi t + 0.25\pi)$.

PROBLEM 3.2*:
Determine the spectrum for each of the following signals. Give your answer as a plot.

(a) $x(t) = \sin^3(5\pi t)$

(b) $y(t) = [\cos(2\pi t)\cos(14\pi t)]^2$

PROBLEM 3.3*:
Signal Processing First, Chapter **3**, Problem **19**, page 69–70.

PROBLEM 3.4*:
The two-sided spectrum of a signal $x(t)$ is given in the following table:

Frequency (rad/sec)	Complex Amplitude
$-\omega_2$	$2e^{j2\pi/3}$
-40π	X_{-1}
0	B
ω_1	$\sqrt{3} - j\sqrt{3}$
60π	X_2

(a) If $x(t)$ is a *real* signal, determine the numerical values of the parameters: X_{-1}, X_2, ω_1 and ω_2.

(b) Write an expression for $x(t)$ involving only real numbers and cosine functions.

(c) Determine a reasonably small value for B that guarantees that the signal is nonnegative (i.e., $x(t) \geq 0$), and justify your answer. By "reasonably small", we mean that you don't have to find the absolute smallest value of B, but your answer should be justifiable for this specific signal (i.e., you shouldn't just be guessing a large value such as 10^{10}).

PROBLEM 3.5*:
In AM radio, the transmitted signal is voice (or music) mixed with a *carrier signal.* The carrier is a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta has a *carrier frequency* of 750 kHz. If we use the notation $v(t)$ to denote the voice/music signal, then the actual transmitted signal for WSB might be:

$$x(t) = (v(t) + A)\cos(2\pi(750 \times 10^3)t)$$

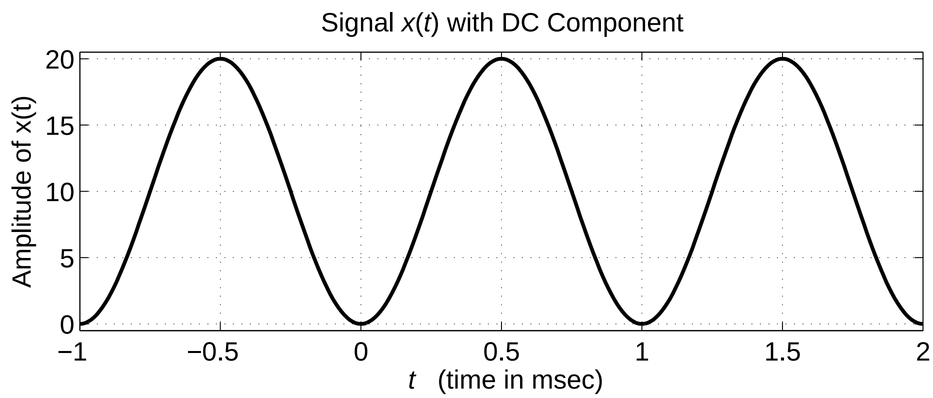
where A is a constant.
Note: The constant A is introduced to make the AM receiver design easier, in which case A would be chosen so that $(A + v(t))$ is positive.

(a) Voice-band signals tend to contain frequencies less than 4000 Hz (4 kHz). Suppose that $v(t)$ is a 2500 Hz sinusoid, $v(t) = \cos(2\pi(2500)t - 0.4\pi)$. Draw the spectrum for $v(t)$.

(b) Now draw the spectrum for $x(t)$, assuming a carrier at 750 kHz (with zero phase). Use $v(t)$ from part (a) and assume that $A = 2.3$.
Hint: Substitute for $v(t)$ and expand $x(t)$ into a sum of cosine terms with different frequencies.

Note that the following problem is not "starred" and does not have to be turned in. It is only suggested as extra practice.

PROBLEM 3.6:



The above signal $x(t)$ consists of a DC component plus a cosine signal. The terminology *DC component* means a component that is constant versus time.

(a) What is the frequency of the DC component? What is the frequency of the cosine component?

(b) Write an equation for the signal $x(t)$. You should be able to determine numerical values for all the amplitudes, frequencies, and phases in your equation by inspection of the above graph.

(c) Plot the two-sided spectrum of the signal $x(t)$. Show the complex amplitudes for each positive and negative frequency component contained in $x(t)$.