GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2009
Problem Set #1

Assigned: 17-Aug-09

Due Date: Week of 24-Aug-09

Reading: In *SP First*: App. A on *Complex Numbers,* pp. 430–451; and Ch. 2 on *Sinusoids,* pp. 8–43.

The web site for the course uses **`t-square`**: `https://t-square.gatech.edu`
The login for **`t-square`** is your GT login.

$\Longrightarrow$ Please check **`t-square`** daily. All official course announcements will be posted there.

Turn in all **STARRED** problems. Some subset of these problems will be randomly selected for grading.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM. After this assignment is handed in by everyone, solutions will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

Two-Part Format for HW Solutions: For each homework problem, two distinct pieces of information are required for a complete solution:

(a) *Approach:* Write a clear explanation of ***how*** you are going to solve the problem. Write in complete sentences. This explanation should be written with little or no mathematical formulas, and it should also be written so that it is independent of the specific numerical values in the problem.

(b) *Details:* Carry out the solution of the particular problem. Details mean getting the algebra correct, making precise plots, and doing the numerical calculations are the key.

Complex Numbers: A complex number is just an ordered pair of real numbers. Several different mathematical notations can be used to represent complex numbers. In *rectangular form* we will use all of the following notations:

$$\begin{aligned} z &= (x,y) \\ &= x + jy \qquad\qquad \text{where } j = \sqrt{-1} \\ &= \Re e\{z\} + j\Im m\{z\} \end{aligned}$$

Note that $i = \sqrt{-1}$ is typical notation in most math courses. The pair (x,y) can be drawn as a vector, such that x is the horizontal coordinate and y the vertical coordinate in a two-dimensional space. Addition of complex numbers is the same as vector addition; i.e., add the real parts and add the imaginary parts.

In *polar form* we will use the *complex exponential* notation:

$$\begin{aligned} z &= |z|e^{j\arg z} \\ &= re^{j\theta} \end{aligned}$$

where $|z| = r = \sqrt{x^2+y^2}$ and $\theta = \arctan(y/x) = \arg z$. In a vector drawing, r is the length and θ the direction of the vector measured from the positive x-axis. The angle is often called the *argument* of the complex number. Here is another notation that is much less common:

$$z = r\angle\theta$$

Euler's Formula:

$$re^{j\theta} = r\cos\theta + jr\sin\theta$$

can be used to convert between Cartesian and polar forms.

Some of these problems should be a review of complex numbers learned in high school. In these problems, a calculator will be useful for doing the complex arithmetic, especially if it is one that accepts both polar and cartesian formats. It is essential to learn how to use the polar format feature. However, it is also worthwhile to be able to do the calculations by hand, and *visualize* the calculation to *understand* what your calculator is doing.

PROBLEM 1.1*:

Convert the following to polar form:

(a) $z = -2\pi$
(b) $z = (-33 - j33)^2$
(c) $z = 1 - j\sqrt{3}$
(d) $z = (-j8)^3$
(e) $z = (3, -4)$
(f) $z = (-3 + j4)^7$

Give numerical values for the magnitude, and the angle (or phase) in radians.

PROBLEM 1.2*:

Convert the following to rectangular form (by using Euler's formula):

(a) $z = 8e^{j(-5\pi/6)}$
(b) $z = e^{\pi - j(5\pi/4)}$
(c) $z = \pi\angle(71\pi/2)$
(d) $z = \pi^e\angle(-41\pi)$

Give numerical values for the real and imaginary parts.

PROBLEM 1.3*:

Evaluate the following and give the answer in both rectangular and polar form. In all cases, assume that the complex numbers are $z_1 = -1 - j$ and $z_2 = \sqrt{3}e^{-j(17\pi/6)}$.

(a) Conjugate: z_1^*
(b) jz_2
(c) z_2/z_1
(d) z_2^2
(e) $z_1^{-1} = 1/z_1$
(f) $z_1 z_2$
(g) $z_1 + z_2^*$
(h) $|z_2|^2 = z_2 z_2^*$
(i) $z_2 + z_2^*$

Note: z^* means the "conjugate" of z. Part (h) is the *magnitude-squared,* which can also be written as the product of the complex number and its complex conjugate.

PROBLEM 1.4*:

Plot two periods of the following sinusoids over the time-interval $0 \le t \le 2T$, where T is the period:

(a) A cosine wave with a period of 50π secs, an amplitude of five, and a phase of -1.0472 radians.

(b) $x(t) = \ln(e^7)\cos(\frac{7\pi}{30}t - \frac{\pi}{2})$

(c) $x(t) = \sqrt{e}\cos(20\pi(t - 0.05))$

PROBLEM 1.5*:

The waveform in the following figure (generated from the MATLAB GUI `sindrill`) can be expressed as

$$x(t) = A\cos[\omega_0(t - t_d)] = A\cos(\omega_0 t + \phi) = A\cos(2\pi f_0 t + \phi)$$

From the waveform, determine A, ω_0, f_0, t_d, and ϕ. Choose the value of ϕ such that $-\pi < \phi \leq \pi$. Make a careful note that the time axis is in units of milliseconds (ms).