GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 20-Nov-06 COURSE: ECE-2025

NAME: ______________________________ GT #: ______________
LAST, FIRST (ex: gtz123a)

3 points 3 points 3 points

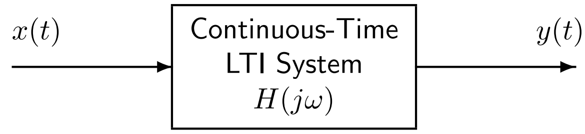
Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-9:30am (Juang)		
	L07:Tues-Noon (Juang)		L08:Thur-Noon (Altunbasak)
	L09:Tues-1:30pm (Taylor)		L10:Thurs-1:30pm (Altunbasak)
L01:M-3pm (Madisetti)	L11:Tues-3pm (Taylor)	L02:W-3pm (Clements)	L12:Thur-3pm (Romberg)
L03:M-4:30pm (Madisetti)		L04:W-4:30pm (Clements)	L12:Thur-4:30pm (Romberg)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	-3	

PROBLEM fa-06-Q.3.1:

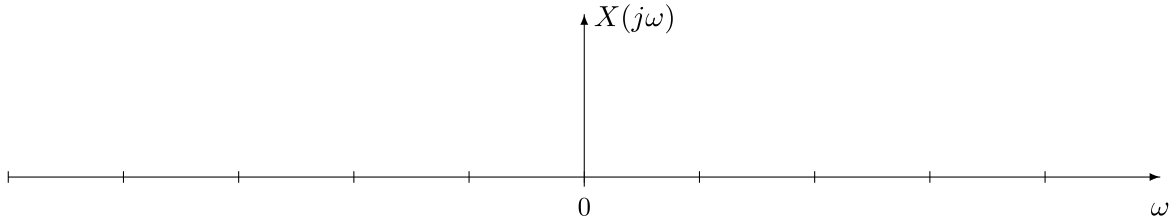


The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^{2} a_k e^{j10kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1/\pi}{1+k^2} & k \neq 0 \\ 0.1 & k = 0 \end{cases}$$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$X(j\omega) =$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{j\omega}{20 + j\omega}$$

Evaluate the frequency response at $\omega = 10$, giving your answer in polar form (with numerical values):

at $\omega = 10$, $|H(j\omega)| =$

at $\omega = 10$, $\angle H(j\omega) =$

(c) For $x(t)$ given above, the output signal can be written as $y(t) = \sum_{k=-2}^{2} b_k e^{jk\omega_0 t}$

Determine the numerical values of the parameters ω_0, b_0 and b_1, i.e., b_k for $k = 0, 1$.

$\omega_0 =$

$b_0 =$

$b_1 =$

PROBLEM fa-06-Q.3.2:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = -3e^{-3t/4}u(t) + 4\delta(t)$

$\boxed{}$

(b) $x(t) = 4e^{(-3+j4)t}u(t)$

$\boxed{}$

(c) $x(t) = \delta(t-6)\,\sin(\pi t)$

$\boxed{}$

(d) $x(t) = u(t-2) - u(t-4)$

$\boxed{}$

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = \dfrac{16j\omega}{3+j4\omega}$

[1] $X(j\omega) = 2e^{-j3\omega}\,\dfrac{\sin(3\omega)}{\omega}$

[2] $X(j\omega) = \dfrac{-12}{3+j4\omega}$

[3] $X(j\omega) = j2e^{-j3\omega}\,\sin(3\omega)$

[4] $X(j\omega) = 0$

[5] $X(j\omega) = 2e^{-j3\omega}\,\dfrac{\sin(\omega)}{\omega}$

[6] $X(j\omega) = \dfrac{\sin(\omega)}{\omega/2}$

[7] $X(j\omega) = \pi\delta(\omega-\pi) + \pi\delta(\omega+\pi)$

[8] $X(j\omega) = \frac{1}{2\pi}e^{-j4\omega} * [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)]$

[9] $X(j\omega) = \dfrac{4}{3+j(\omega-4)}$

PROBLEM fa-06-Q.3.3:

A continuous-time LTI system is defined by the input/output relation $y(t) = \int_{t-1}^{t+3} 5x(\tau - 2)d\tau$

(a) Determine the impulse response, $h(t)$, of this system.

(b) Is this a stable system? **Explain** with a proof (if true) or counter-example (if false).

(c) Use convolution to determine $r(t) = p(t) * p(t)$ when the signal $p(t)$ is the pulse

$$p(t) = u(t - \tfrac{1}{2}) - u(t + \tfrac{1}{2})$$

Note: this part is independent of part (a).

PROBLEM fa-06-Q.3.4:

For each of the following expressions, select the correct match from the second list below.
Write each answer in the box provided. (The operator $*$ denotes convolution.)

(a) $x(t) = \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\delta(t-\tau)d\tau$

$\boxed{}$

(b) $x(t) = [\delta(t-2) + \delta(t-1)] * [\delta(t) - \delta(t-1)]$

$\boxed{}$

(c) $x(t) = \frac{d}{dt}\left\{e^{-t}u(t-2)\right\}$

$\boxed{}$

(d) $x(t) = e^{-2t}\delta(t-1) + u(t)\delta(t+2)$

$\boxed{}$

(e) $x(t) = \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\delta(\tau-2)d\tau$

$\boxed{}$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[0] $x(t) = e^{-t}u(t)$

[1] $x(t) = u(t-4)$

[2] $x(t) = e^{-2}$

[3] $x(t) = e^{-2}\delta(t-1) + \delta(t-2)$

[4] $x(t) = -e^{-t}u(t-2) + e^{-2}\delta(t-2)$

[5] $x(t) = e^{-2}\delta(t-1)$

[6] $x(t) = e^{-2}\delta(t-2)$

[7] $x(t) = \delta(t-1) - \delta(t-3)$

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 20-Nov-06 COURSE: ECE-2025

NAME: Answer Key
LAST, FIRST

GT #: Version-1
(ex: gtz123a)

3 points 3 points 3 points

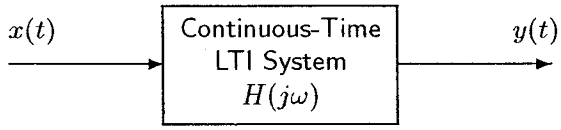
Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-9:30am (Juang)		
	L07:Tues-Noon (Juang)		L08:Thur-Noon (Altunbasak)
	L09:Tues-1:30pm (Taylor)		L10:Thurs-1:30pm (Altunbasak)
L01:M-3pm (Madisetti)	L11:Tues-3pm (Taylor)	L02:W-3pm (Clements)	L12:Thur-3pm (Romberg)
L03:M-4:30pm (Madisetti)		L04:W-4:30pm (Clements)	L12:Thur-4:30pm (Romberg)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	−3	

PROBLEM fa-06-Q.3.1:



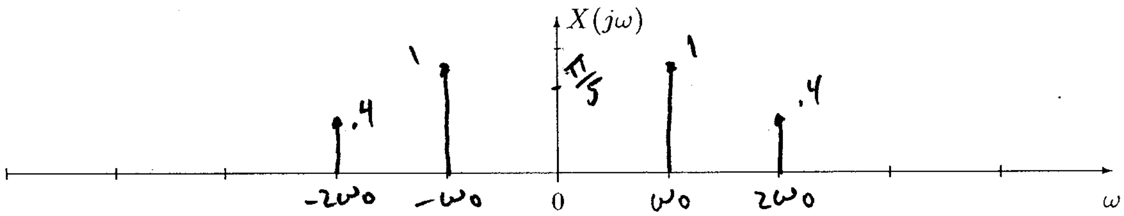
The periodic input to the above system is defined by the equation:

$a_1 = \frac{1}{2\pi}$, $a_2 = \frac{1}{5\pi}$

$$x(t) = \sum_{k=-2}^{2} a_k e^{j10kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1/\pi}{1+k^2} & k \neq 0 \\ 0.1 & k = 0 \end{cases}$$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$$\boxed{X(j\omega) = \sum_{k=-2}^{2} 2\pi a_k \delta(\omega - k\omega_0) \quad \text{where } a_k = \begin{cases} \frac{1/\pi}{1+k^2} & k \neq 0 \\ 0.1 & k = 0 \end{cases}}$$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{j\omega}{20 + j\omega}$$

Evaluate the frequency response at $\omega = 10$, giving your answer in polar form (with numerical values):

$$\frac{j10}{20+j10} = \frac{j}{2+j} \qquad \left(\frac{j}{2+j}\right)\left(\frac{-j}{2-j}\right) = \frac{1}{4+1} = 1/5$$

at $\omega = 10$, $\lvert H(j\omega)\rvert =$ $\frac{1}{\sqrt{5}}$	$= 0.4472$
at $\omega = 10$, $\angle H(j\omega) =$ 1.107 rad	$= 0.352\pi$, or 63.43°

(c) For $x(t)$ given above, the output signal can be written as $y(t) = \sum_{k=-2}^{2} b_k e^{jk\omega_0 t}$

Determine the numerical values of the parameters ω_0, b_0 and b_1.

$$b_k = a_k H(j\omega_0 k)$$
$$b_0 = a_0 H(j0) = 0$$
$$b_1 = a_1 H(j10) = \left(\frac{1}{2\pi}\right)\left(\frac{j}{2+j}\right)$$

$\boxed{\omega_0 = 10}$

$\boxed{b_0 = 0}$

$\boxed{b_1 = \left(\frac{j}{2+j}\right)\frac{1}{2\pi}} = 0.07118\, e^{j1.107}$

0.352π
or
63.43°

PROBLEM fa-06-Q.3.2:
For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = -3e^{-3t/4}u(t) + 4\delta(t)$

$\boxed{0}$

$$\frac{-3}{\frac{3}{4}+j\omega} + 4 = \frac{-12}{3+j4\omega} + \frac{12+j16\omega}{3+j4\omega} = \frac{j16\omega}{3+j4\omega}$$

(b) $x(t) = 4e^{(-3+j4)t}u(t)$

$\boxed{9}$

$$\left(4e^{-3t}u(t)\right)e^{j4t} \Leftrightarrow \left(\frac{4}{3+j\omega} * 2\pi\delta(\omega-4)\right)\frac{1}{2\pi}$$

$$\frac{4}{3+j(\omega-4)}$$

(c) $x(t) = \delta(t-6)\sin(\pi t)$

$\boxed{4}$

(d) $x(t) = u(t-2) - u(t-4)$

$\boxed{5}$

$T=2,\ t_d = 3$

$$\frac{\sin(\omega)}{\omega/2} \cdot e^{-j3\omega}$$

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = \dfrac{16j\omega}{3+j4\omega}$

[1] $X(j\omega) = 2e^{-j3\omega}\,\dfrac{\sin(3\omega)}{\omega}$

[2] $X(j\omega) = \dfrac{-12}{3+j4\omega}$

[3] $X(j\omega) = j2e^{-j3\omega}\sin(3\omega)$

[4] $X(j\omega) = 0$

[5] $X(j\omega) = 2e^{-j3\omega}\,\dfrac{\sin(\omega)}{\omega}$

[6] $X(j\omega) = \dfrac{\sin(\omega)}{\omega/2}$

[7] $X(j\omega) = \pi\delta(\omega-\pi) + \pi\delta(\omega+\pi)$

[8] $X(j\omega) = \frac{1}{2\pi}e^{-j4\omega} * [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)]$

[9] $X(j\omega) = \dfrac{4}{3+j(\omega-4)}$

PROBLEM fa-06-Q.3.3:

A continuous-time LTI system is defined by the input/output relation $y(t) = \int_{t-1}^{t+3} 5x(\tau - 2)d\tau$

(a) Determine the impulse response, $h(t)$, of this system.

$$h(t) = \int_{t-1}^{t+3} 5\delta(\tau-2)\,d\tau = \boxed{\begin{cases} 5 & -1 \le t \le 3 \\ 0 & \text{else} \end{cases}}$$

$t+3-2 > 0 \qquad t > -1$

$t-1-2 < 0 \qquad t < 3$

(b) Is this a stable system? **Explain** with a proof (if true) or counter-example (if false).

yes, $\int_{-\infty}^{\infty} |h(t)|\,dt = 20$ which is finite

(c) Use the convolution integral to determine $r(t) = p(t) * p(t)$ when the signal $x(t)$ is the pulse

$$p(t) = u(t - \tfrac{1}{2}) - u(t + \tfrac{1}{2})$$

$$\int_{-\infty}^{\infty} [u(t-\tfrac{1}{2}) - u(t+\tfrac{1}{2})][u(t-\tau-\tfrac{1}{2}) - u(t-\tau+\tfrac{1}{2})]\,d\tau$$

regions

$t-\tau-\frac{1}{2} < 0 \qquad 0$

$t-\tau-\frac{1}{2} > 0, \; t-\tau+\frac{1}{2} < 0$

$$= \int_{-1/2}^{1/2} u(t-\tau-\tfrac{1}{2}) - u(t-\tau+\tfrac{1}{2})\,d\tau$$

$$\int_{-1/2}^{1/2} u(t-\tau-\tfrac{1}{2})\,d\tau - \underbrace{\int_{-1/2}^{1/2} u(t-\tau+\tfrac{1}{2})\,d\tau}$$

0 for $t-\tau+\frac{1}{2} < 0$

$t < \tau - \frac{1}{2}$ $\quad (\tau_{max} = \frac{1}{2}$

0 for $t < 0$

1 for t

$$r(t) = \begin{cases} 0 & t < -1 \\ t-1 & -1 \le t < 0 \\ 1-t & 0 \le t < 1 \\ 0 & t > 1 \end{cases}$$

PROBLEM fa-06-Q.3.4:

For each of the following expressions, select the correct match from the second list below. (The operator $*$ denotes convolution.) ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\delta(t-\tau)d\tau$ $= e^{-t}u(t) * \delta(t) = e^{-t}u(t)$

$\boxed{0}$

(b) $x(t) = [\delta(t-2) + \delta(t-1)] * [\delta(t) - \delta(t-1)]$ $= \cancel{\delta(t-2)} + \delta(t-1) - \delta(t-3) - \cancel{\delta(t-2)}$

$\boxed{7}$

(c) $x(t) = \frac{d}{dt}\{e^{-t}u(t-2)\}$ $e^{-\cancel{t}\,2}\delta(t-2) - e^{-t}u(t-2)$

$\boxed{4}$

(d) $x(t) = e^{-2t}\delta(t-1) + u(t)\delta(t+2)$ $e^{-2}\delta(t-1)$

$\boxed{5}$

(e) $x(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\delta(\tau-2)d\tau$ e^{-2}

$\boxed{2}$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[0] $x(t) = e^{-t}u(t)$

[1] $x(t) = u(t-4)$

[2] $x(t) = e^{-2}$

[3] $x(t) = e^{-2}\delta(t-1) + \delta(t-2)$

[4] $x(t) = -e^{-t}u(t-2) + e^{-2}\delta(t-2)$

[5] $x(t) = e^{-2}\delta(t-1)$

[6] $x(t) = e^{-2}\delta(t-2)$

[7] $x(t) = \delta(t-1) - \delta(t-3)$