Signal Processing First

Lecture 19

Continuous-Time Signals and Systems

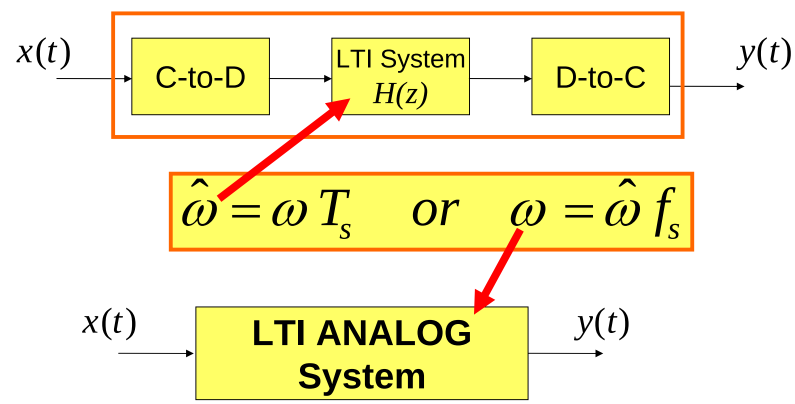
READING ASSIGNMENTS

- This Lecture:
  - Chapter 9, Sects 9-1 to 9-5
- Other Reading:
  - Recitation: Ch. 9, all
  - Next Lecture: Chapter 9, Sects 9-6 to 9-8

LECTURE OBJECTIVES

- Bye bye to D-T Systems for a while
- The UNIT IMPULSE signal
  - Definition
  - Properties
- Continuous-time signals and systems
  - Example systems
  - Review: Linearity and Time-Invariance
  - Convolution integral: impulse response

D-T Filtering of C-T Signals



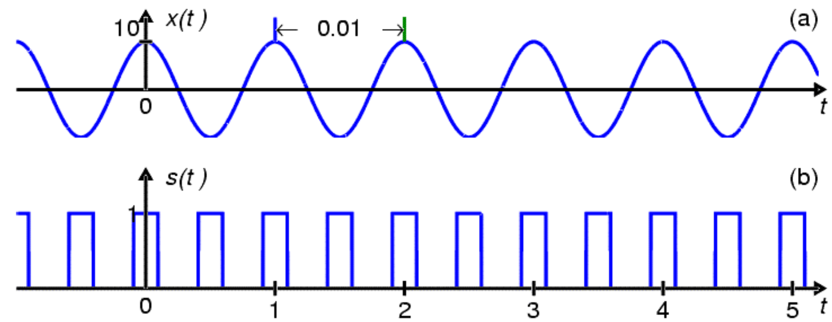
ANALOG SIGNALS x(t)

- INFINITE LENGTH
  - SINUSOIDS: (t = time in secs)
  - PERIODIC SIGNALS
- ONE-SIDED, e.g., for t>0
  - UNIT STEP: u(t)
- FINITE LENGTH
  - SQUARE PULSE
- IMPULSE SIGNAL: $\delta(t)$
- DISCRETE-TIME: x[n] is list of numbers

CT Signals: PERIODIC

$x(t) = 10\cos(200\pi t)$

Sinusoidal signal



INFINITE DURATION

Square Wave

CT Signals: ONE-SIDED

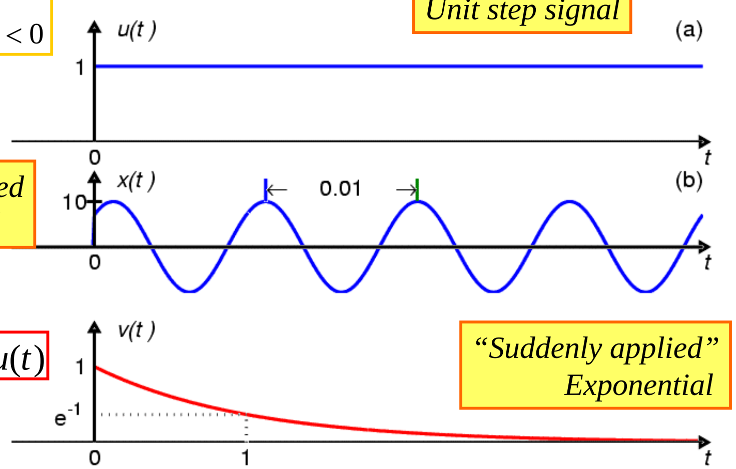
$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

Unit step signal

One-Sided Sinusoid

$v(t) = e^{-t}u(t)$

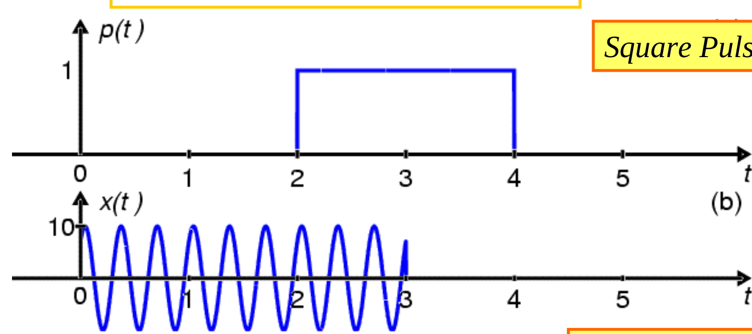
"Suddenly applied" Exponential



CT Signals: FINITE LENGTH

$p(t) = u(t-2) - u(t-4)$

Square Pulse signal



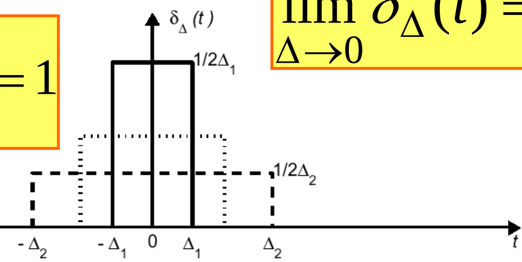
Sinusoid multiplied by a square pulse

What is an Impulse?

- A signal that is “concentrated” at one point.

$$\int_{-\infty}^{\infty} \delta_\Delta(t)dt = 1$$

$$\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t)$$



Defining the Impulse

- Assume the properties apply to the limit:

$$\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t)$$

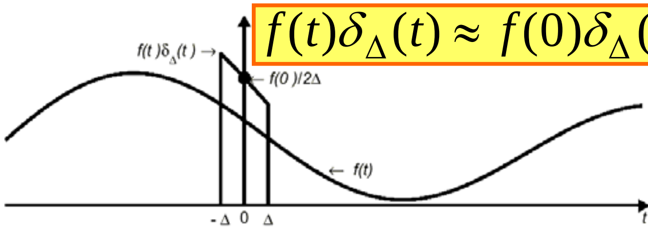
- One **“INTUITIVE”** definition is:

$$\delta(t) = 0, \quad t \neq 0$$ *Concentrated at t=0*

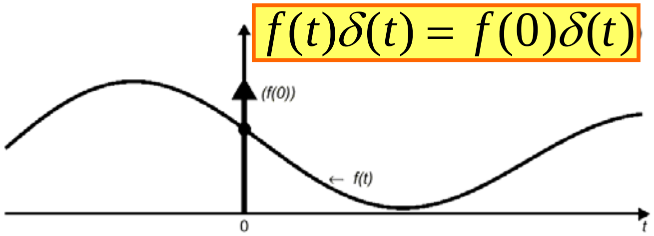
$$\int_{-\infty}^{\infty} \delta(\tau)d\tau = 1$$ *Unit area*

Sampling Property

$$f(t)\delta_\Delta(t) \approx f(0)\delta_\Delta(t)$$

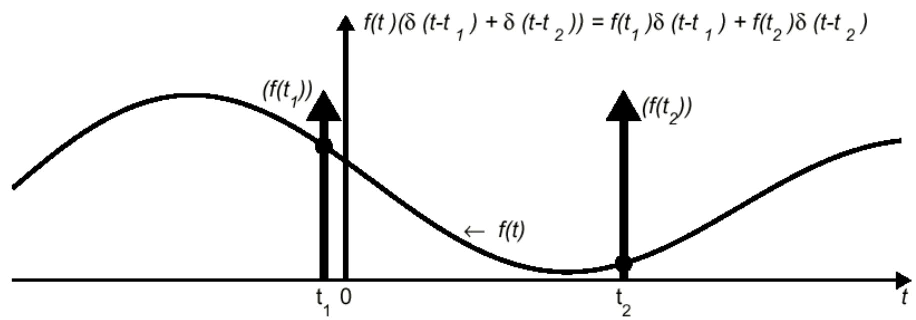


$$f(t)\delta(t) = f(0)\delta(t)$$



General Sampling Property

$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$$



Properties of the Impulse

$\delta(t-t_0)=0, \quad t \neq t_0$ — *Concentrated at one time*

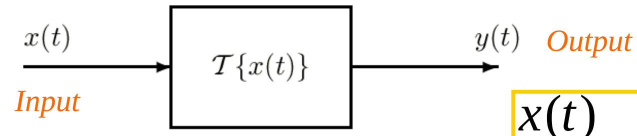
$\int_{-\infty}^{\infty} \delta(t-t_0)dt = 1$ — *Unit area*

$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$ — *Sampling Property*

$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$ — *Extract one value of f(t)*

$\frac{du(t)}{dt} = \delta(t)$ — *Derivative of unit step*

Continuous-Time Systems



$x(t) \mapsto y(t)$

- Examples:
  - Delay $y(t) = x(t-t_d)$
  - Modulator $y(t) = [A + x(t)]\cos\omega_c t$
  - Integrator $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

CT BUILDING BLOCKS

- INTEGRATOR (CIRCUITS)
- DIFFERENTIATOR
- DELAY by t_o
- MODULATOR (e.g., AM Radio)
- MULTIPLIER & ADDER

Ideal Delay:

- Mathematical Definition:

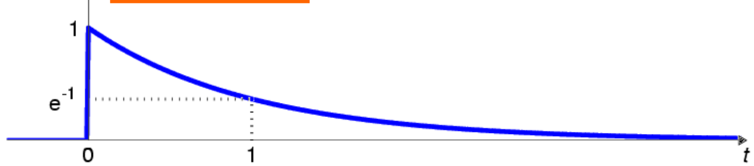
$$y(t) = x(t-t_d)$$

- To find the IMPULSE RESPONSE, *h(t),* let *x(t)* be an impulse, so

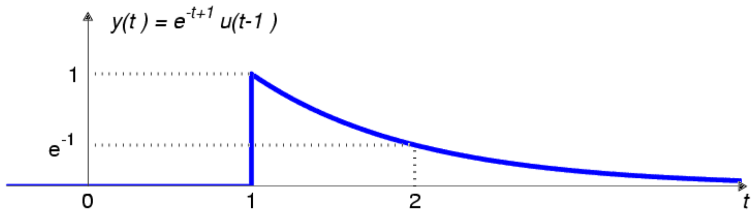
$$h(t) = \delta(t-t_d)$$

Output of Ideal Delay of 1 sec

$x(t) = e^{-t}u(t)$



$$y(t) = x(t-1) = e^{-(t-1)}u(t-1)$$



Integrator:

- Mathematical Definition:

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

Running Integral

- To find the IMPULSE RESPONSE, *h(t)*, let *x(t)* be an impulse, so

$$h(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

Integrator: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

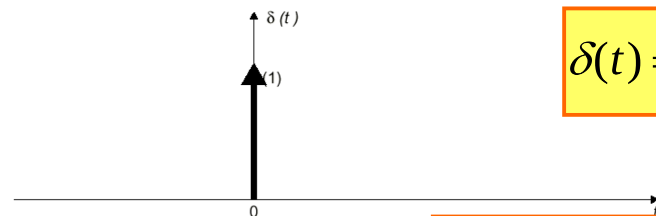
- Integrate the impulse

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

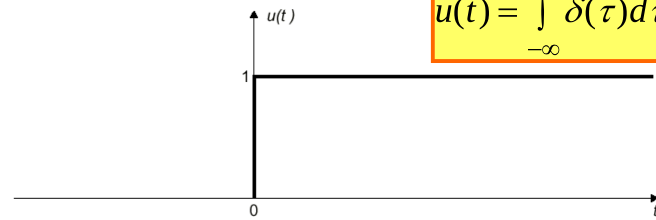
- IF t<0, we get zero
- IF t>0, we get one
  - Thus we have h(t) = u(t) for the integrator

Graphical Representation

$$\delta(t) = \frac{du(t)}{dt}$$



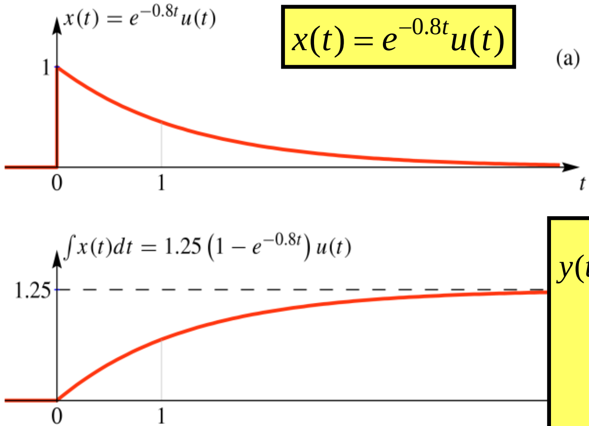
$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



Output of Integrator

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau = x(t) * u(t)$$

$$x(t) = e^{-0.8t}u(t)$$



$$y(t) = \int_{-\infty}^{t} e^{-0.8\tau}u(\tau)d\tau$$

$$= \begin{cases} 0 & t < 0 \\ \int_{0}^{t} e^{-0.8\tau}u(\tau)d\tau & t \geq 0 \end{cases}$$

$$= 1.25(1 - e^{-0.8t})u(t)$$

Differentiator:

- Mathematical Definition:

$$y(t) = \frac{dx(t)}{dt}$$

- To find *h(t)*, let *x(t)* be an impulse, so

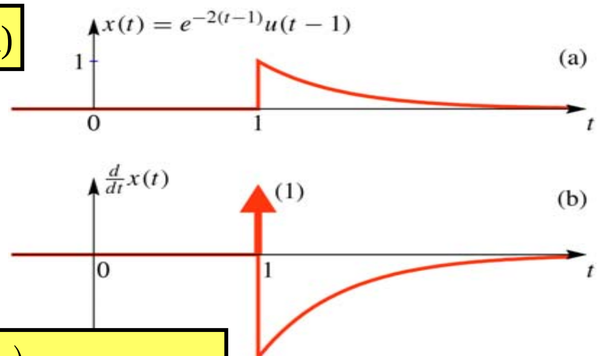
$$h(t) = \frac{d\delta(t)}{dt} = \delta^{(1)}(t)$$

Doublet

Differentiator Output:

$$y(t) = \frac{dx(t)}{dt}$$

$$x(t) = e^{-2(t-1)}u(t-1)$$



$$y(t) = \frac{d}{dt}\left(e^{-2(t-1)}u(t-1)\right)$$

$$= -2e^{-2(t-1)}u(t-1) + e^{-2(t-1)}\delta(t-1)$$

$$= -2e^{-2(t-1)}u(t-1) + 1\delta(t-1)$$

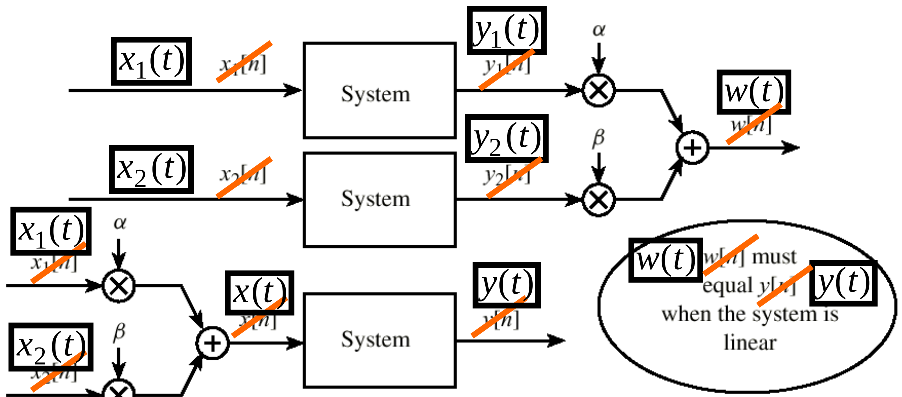
Linear and Time-Invariant (LTI) Systems

- If a continuous-time system is both linear and time-invariant, then the output *y(t)* is related to the input *x(t)* by a ***convolution integral***

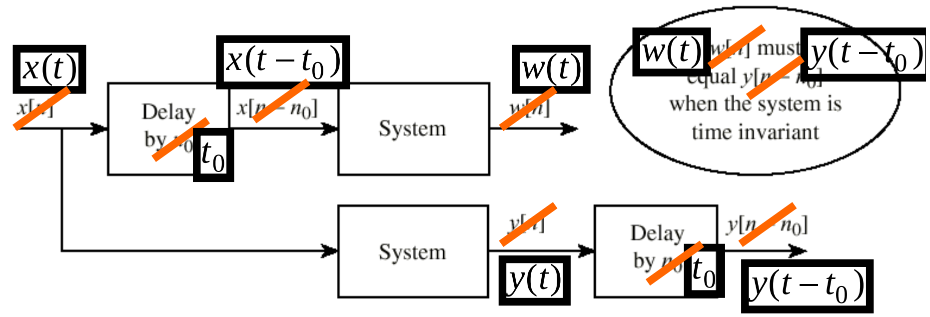
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

where *h(t)* is the ***impulse response*** of the system.

Testing for Linearity



Testing Time-Invariance



Integrator: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

- Linear

$$\int_{-\infty}^{t}[ax_1(\tau)+bx_2(\tau)]d\tau = ay_1(t)+by_2(t)$$

- And Time-Invariant

$$w(t) = \int_{-\infty}^{t} x(\tau - t_0)d\tau \qquad \text{let } \sigma = \tau - t_0$$

$$\Rightarrow \quad w(t) = \int_{-\infty}^{t-t_0} x(\sigma)d\sigma = y(t - t_0)$$

Modulator: $y(t) = [A + x(t)]\cos\omega_c t$

- ***Not*** linear--obvious because

$$[A + ax_1(t) + bx_2(t)] \neq$$
$$[A + ax_1(t)] + [A + bx_2(t)]$$

- ***Not*** time-invariant

$$w(t) = [A + x(t - t_0)]\cos\omega_c t \neq y(t - t_0)$$