Signal Processing First

Lecture 15
Zeros of H(z) and the Frequency Domain

READING ASSIGNMENTS

- This Lecture:
  - Chapter 7, Section 7-6 to end
- Other Reading:
  - Recitation & Lab: Chapter 7
    - ZEROS (and POLES)
  - Next Lecture:Chapter 8

LECTURE OBJECTIVES

- ZEROS and POLES
- Relate H(z) to FREQUENCY RESPONSE

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- THREE DOMAINS:
  - Show Relationship for FIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

DESIGN PROBLEM

- Example:
  - Design a Lowpass FIR filter (Find b_k)
  - Reject completely 0.7π, 0.8π, and 0.9π
    - This is NULLING
  - Estimate the filter length needed to accomplish this task. How many b_k ?
- Z POLYNOMIALS provide the TOOLS

Z-Transform DEFINITION

- POLYNOMIAL Representation of LTI SYSTEM: $H(z) = \sum_n h[n] z^{-n}$

APPLIES to Any SIGNAL

- EXAMPLE:

$$\{h[n]\} = \{2, 0, -3, 0, 2\}$$

$$H(z) = 2z^{-0} + 0z^{-1} - 3z^{-2} + 0z^{-3} + 2z^{-4}$$
$$= 2 - 3z^{-2} + 2z^{-4}$$
$$= 2 - 3(z^{-1})^2 + 2(z^{-1})^4$$

POLYNOMIAL in z^{-1}

CONVOLUTION PROPERTY

- Convolution in the n-domain
  - SAME AS
- Multiplication in the z-domain

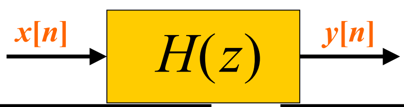
$$y[n] = h[n] * x[n] \quad \leftrightarrow \quad Y(z) = H(z)X(z)$$

MULTIPLY z-TRANSFORMS

$$y[n] = x[n] * h[n]$$
$$= \sum_{k=0}^{M} h[k]x[n-k]$$

FIR Filter

CONVOLUTION EXAMPLE



$$x[n] = \delta[n-1] + 2\delta[n-2] \qquad h[n] = \delta[n] - \delta[n-1]$$

$$y[n] = x[n] * h[n]$$

$$X(z) = z^{-1} + 2z^{-2} \qquad H(z) = 1 - z^{-1}$$

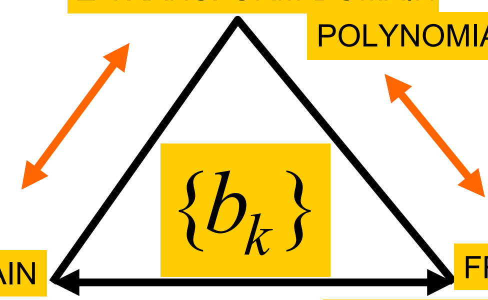
$$Y(z) = (z^{-1} + 2z^{-2})(1 - z^{-1}) = z^{-1} + z^{-2} - 2z^{-3}$$

$$y[n] = \delta[n-1] + \delta[n-2] - 2\delta[n-3]$$

THREE DOMAINS

Z-TRANSFORM-DOMAIN

POLYNOMIALS: H(z)



TIME-DOMAIN

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

FREQ-DOMAIN

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

FREQUENCY RESPONSE ?

- Same Form:

$\hat{\omega}-\text{Domain}$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k (e^{j\hat{\omega}})^{-k}$$

$$z = e^{j\hat{\omega}}$$

$z-\text{Domain}$

$$H(z) = \sum_{k=0}^{M} b_k z^{-k}$$

SAME COEFFICIENTS

ANOTHER ANALYSIS TOOL

- z-Transform POLYNOMIALS are EASY !
  - ROOTS, FACTORS, etc.
- **ZEROS and POLES: where is H(z) = 0 ?**
- The z-domain is COMPLEX
  - H(z) is a COMPLEX-VALUED function of a COMPLEX VARIABLE z.

ZEROS of H(z)

- Find z, where H(z)=0

$$H(z) = 1 - \tfrac{1}{2} z^{-1}$$

$$1 - \tfrac{1}{2} z^{-1} = 0 \ ?$$

$$z - \tfrac{1}{2} = 0$$

$$\text{Zero at}: z = \tfrac{1}{2}$$

ZEROS of H(z)

- Find z, where H(z)=0
  - Interesting when z is ON the unit circle.

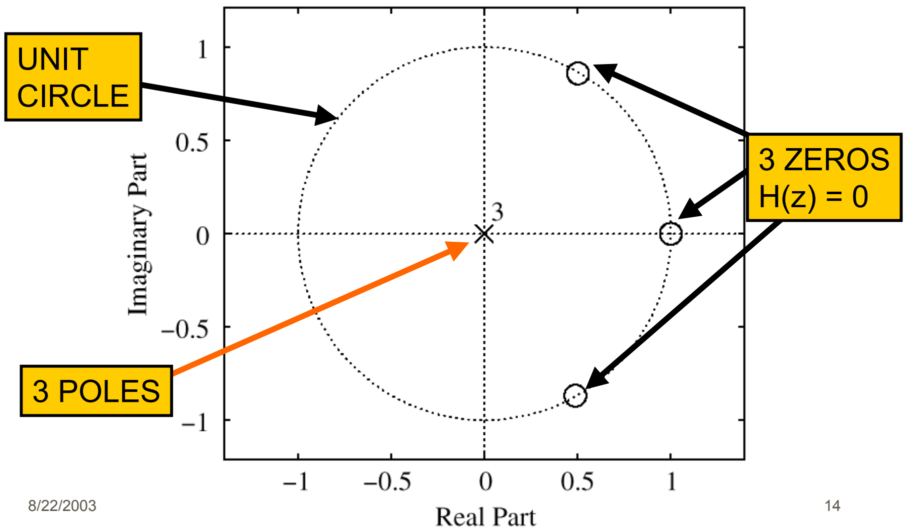
$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(z) = (1 - z^{-1})(1 - z^{-1} + z^{-2})$$

$$\text{Roots}: z = 1, \tfrac{1}{2} \pm j\tfrac{\sqrt{3}}{2}$$

$$e^{\pm j\pi/3}$$

PLOT ZEROS in z-DOMAIN



POLES of H(z)

- Find z, where $H(z) \to \infty$
  - Not very interesting for the FIR case

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(z) = \frac{z^3 - 2z^2 + 2z \;\; -1}{z^3}$$

$$\text{Three Poles at}: z = 0$$

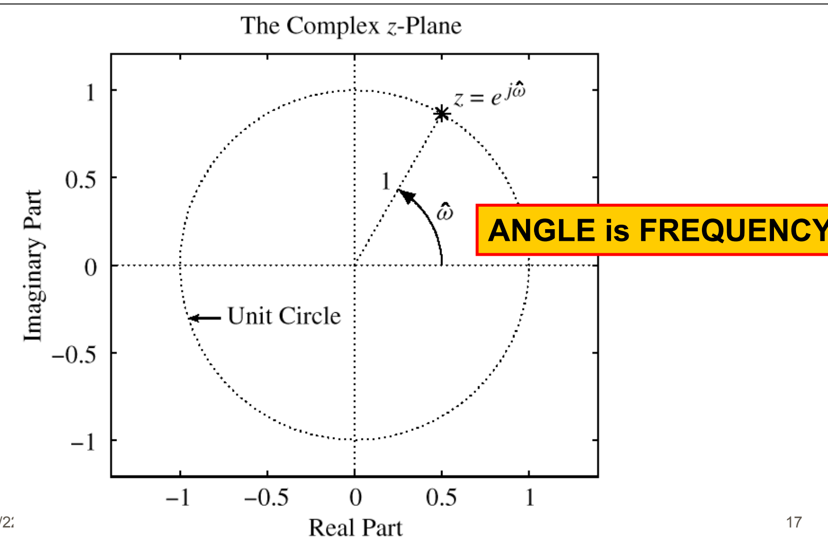
FREQ. RESPONSE from ZEROS

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

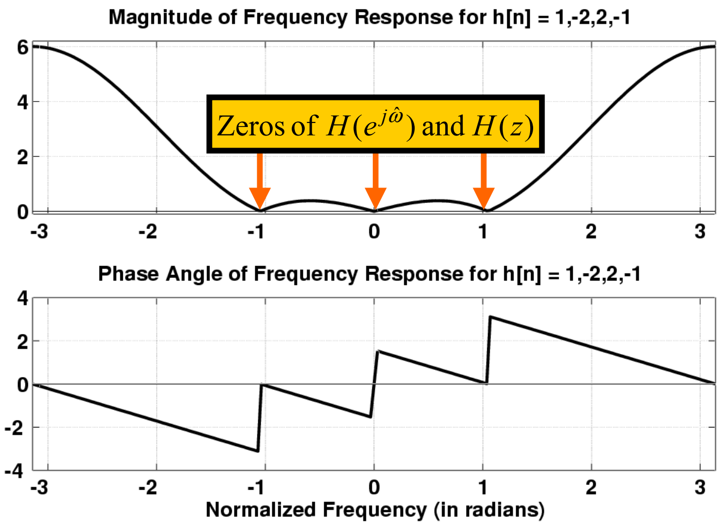
- Relate H(z) to FREQUENCY RESPONSE
- EVALUATE H(z) on the **UNIT CIRCLE**
  - ANGLE is same as FREQUENCY

$$z = e^{j\hat{\omega}} \quad (\text{as } \hat{\omega} \text{ varies})$$
$$\text{defines a CIRCLE, radius} = 1$$

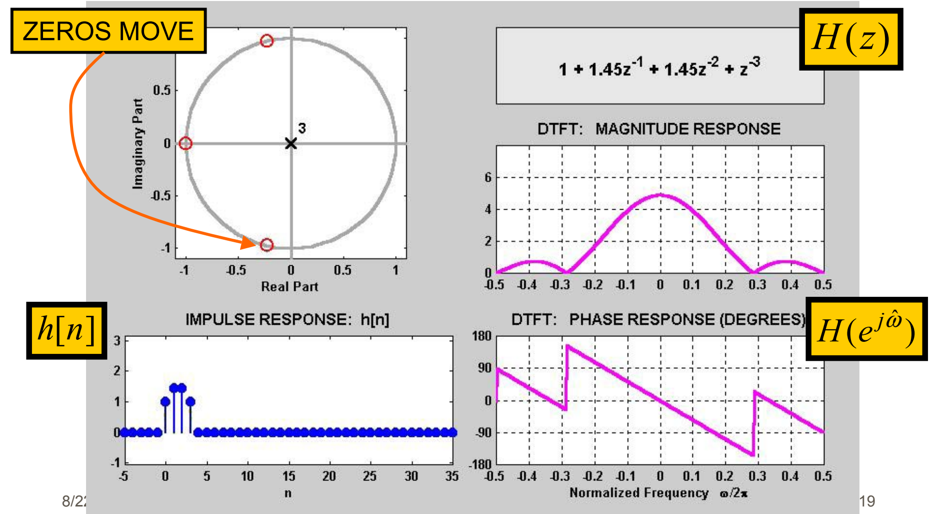
$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$



FIR Frequency Response



3 DOMAINS MOVIE: FIR



NULLING PROPERTY of H(z)

- When H(z)=0 on the unit circle.
  - Find inputs x[n] that give zero output

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

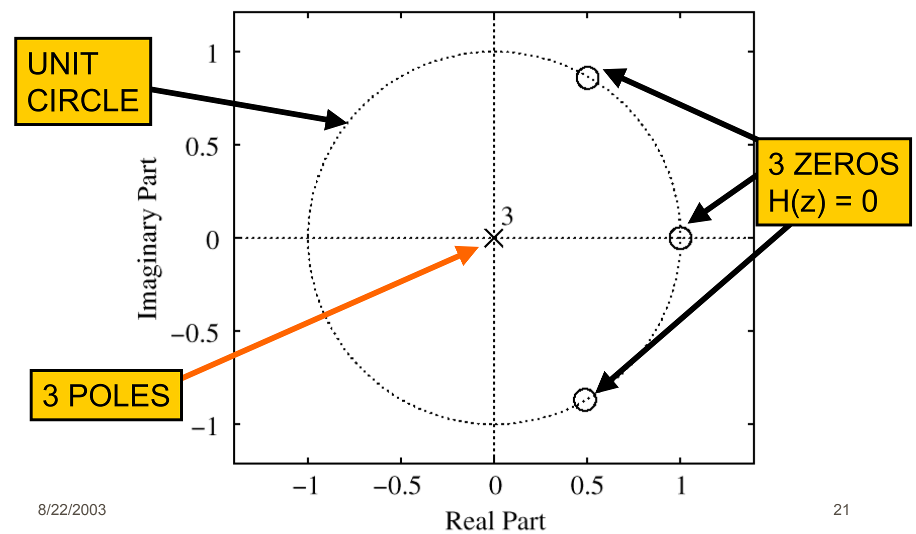
$x[n]$ → $H(z)$ → $y[n]$

$$H(e^{j\pi/3}) = ?$$

$$x[n] = e^{j(\pi/3)n}$$

$$y[n] = H(e^{j(\pi/3)}) \cdot e^{j(\pi/3)n}$$

PLOT ZEROS in z-DOMAIN



NULLING PROPERTY of H(z)

- Evaluate H(z) at the input "frequency"

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

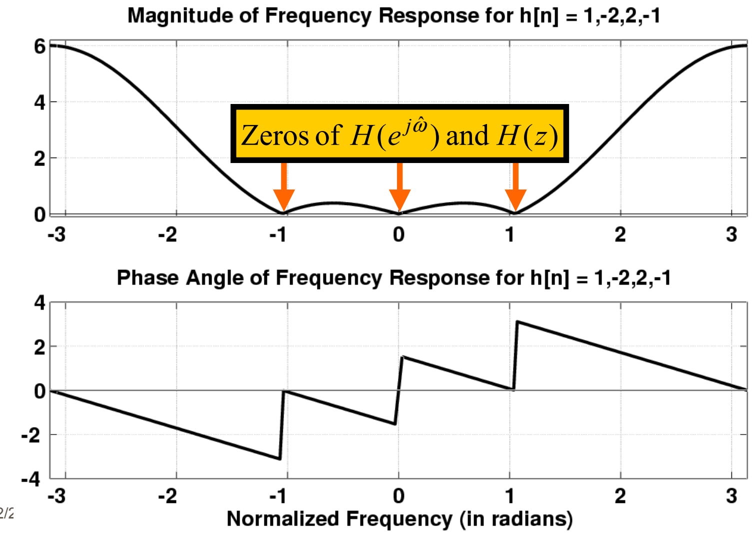
$$y[n] = H(e^{j\pi/3}) \cdot e^{j(\pi/3)n}$$

$$y[n] = (1 - 2e^{-j\pi/3} + 2e^{-j2\pi/3} - e^{-j3\pi/3}) \cdot e^{j(\pi/3)n}$$

$$(1 - 2(\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}) + 2(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}) - (-1))$$

$$y[n] = (1 - 1 + j\sqrt{3} - 1 - j\sqrt{3} + 1) \cdot e^{j(\pi/3)n} = 0$$

FIR Frequency Response



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PeZ Demo: Zero Placing