

PROBLEM s-05-Q.4.1:

In each of the following cases, simplify the expression *as much as possible*. Provide some explanation or intermediate steps for each answer. *Note:* Star * is the convolution operator.

(a) $\left. \frac{\sin(9\omega/2)}{4\omega} \right|_{\omega=0}$ (i.e., evaluate at $\omega = 0$)

(b) $(\delta(t) + \delta(t - 3)) * (\delta(t - 4) - \delta(t - 1))$

(c) $\left\{ e^{-(t+5)^2} u(t+5) \right\} * h(t) = 5e^{-t^2} u(t)$ (find $h(t)$ that satisfies this equation)

(d) $\int_{-\infty}^{\infty} u(t+1)u(4-t)dt$

(e) $\int_{-\infty}^{-4} \delta(\tau+3)d\tau$

PROBLEM s-05-Q.4.2:

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a simple formula that is *real-valued*.

Explain each answer (briefly) by stating which property and/or transform pair you used.

(a) Find $x(t)$ when $X(j\omega) = \frac{12}{6 + j4\omega}$.

(b) Find $s(t)$ when $S(j\omega) = j50 \sin(200\omega)$.

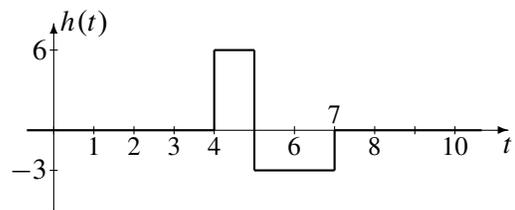
(c) Find $r(t)$ when $R(j\omega) = \pi^2 \delta(\omega) + 13\pi \delta(\omega + 100\pi) + 13\pi \delta(\omega - 100\pi)$.

(d) Find $H(j\omega)$ when $h(t) = \delta(t) - \frac{\sin(6\pi t)}{\pi t}$.

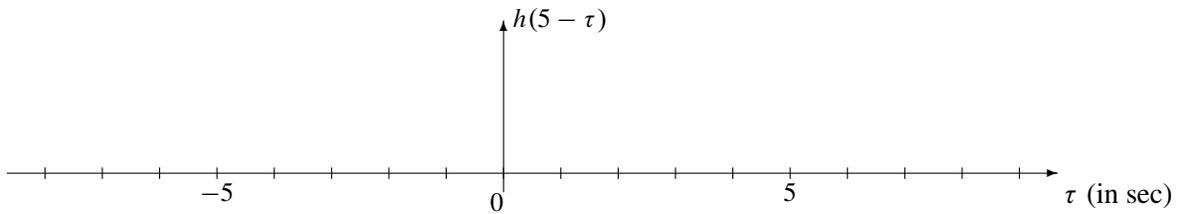
(e) Sketch the magnitude of $H(j\omega)$ obtained in the previous part, i.e., $|H(j\omega)|$ versus ω .

PROBLEM s-05-Q.4.3:

A linear time-invariant system has this impulse response:



- (a) Plot $h(t - \tau)$ versus τ , for $t = 5$. Label your plot carefully.



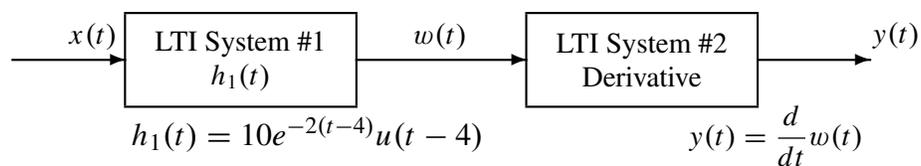
- (b) If the input signal is the unit-step signal, $x(t) = u(t)$, use the convolution integral to find $y(5)$; i.e., the value of the output signal, $y(t) = x(t) * h(t)$, when $t = 5$.

- (c) When the input signal is $x(t) = u(t)$ and the impulse response is $h(t)$ given above, it turns out that the output is **zero** for $t < T_1$ and for $t > T_2$. Find the values of T_1 and T_2 . **Explain** your answers. You may “flip and shift” either $x(t)$ or $h(t)$, whichever leads to the easiest explanation.

$T_1 =$ $T_2 =$

PROBLEM s-05-Q.4.4:

A cascade of linear time-invariant systems is depicted by the following block diagram:



- (a) If the input to the first system is a sinusoid:

$$x(t) = 200 \cos(2t)$$

Determine the output of the *first system*, $w(t)$. Give your answer in the *simplest possible form*.

- (b) If the input to the first system is a shifted unit-step signal, $x(t) = u(t + 4)$, determine the overall output signal, $y(t)$.