GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 24-Oct-03 COURSE: ECE-2025

NAME: ______________________________ GT #: ____________________

LAST, FIRST

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L01: Tues-9:30 (G. Li)
L02: Thur-9:30 (G-K. Chang)
L03: Tues-12:00 (G. Li)
L04: Thur-12:00 (G-K. Chang)
L05: Tues-1:30 (M. Richards)
L06: Thur-1:30 (T. Zhou)
L07: Tues-3:00 (M. Richards)
L08: Thur-3:00 (T. Zhou)
L09: Tues-4:30 (Y. Altunbasak)
L10: Thur-4:30 (G. Casinovi)
L11: Tues-6:00 (Y. Altunbasak)
L13: Mon-3:00 (J. McClellan)
L14: Wed-3:00 (R. Butera)
L16: Wed-4:30 (R. Butera)
Savannah (G. AlRegib)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answers in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

PROBLEM f-03-Q.2.1:
Questions about discrete-time signals and systems:

(a) Make a stem plot of $y[n] = \delta[n-3] * (u[n+1] - u[n-4])$, where $*$ represents convolution.

(b) Make a stem plot of $y[n] = \delta[n-3]\,(u[n+1] - u[n-4])$ versus n.

(c) Suppose a system is specified by the input/output relationship $y[n] = (x[n^a - b])^c$ where b is any integer, but a and c are *positive* integers.

In answering each of the following questions, give the *minimum* set of numerical constraints, i.e., don't give any more constraints than necessary to achieve the desired property for each question.
For example, your answer might be $b = -7$, $a = 3$, *and* c *is unconstrained.*
On this subproblem, you do not need to give any explanations. Feel free to use your intuition!

(i) What numerical constraints must we put on a, b, and c to ensure the system is *linear*?

(ii) What numerical constraints must we put on a, b, and c to ensure the system is *time-invariant?*

(iii) What numerical constraints must we put on a, b, and c to ensure the system is *causal*?

PROBLEM f-03-Q.2.2:

Suppose an LTI system has a frequency response given by $\boxed{H(e^{j\hat{\omega}}) = 3j\,\sin(5\hat{\omega})e^{-j6\hat{\omega}}}$

(a) Find $h[n]$, the system's impulse response, in terms of a sum of delta functions.

(b) Plot the magnitude of the frequency response versus $\hat{\omega}$.

(c) Evaluate the phase of the frequency response at $\hat{\omega} = 0.1\pi$. *The answer should be a number.*

PROBLEM f-03-Q.2.3:

Consider the signal $x[n] = 3\delta[n] - 2\delta[n-1] + \delta[n-2]$

(a) Suppose $x[n]$ is input to an LTI system described by the difference equation

$$y_a[n] = x[n] + 2x[n-2] - x[n-3]$$

Find the output $y_a[n]$. Express your answer as a sum of delta functions.

(b) Now suppose we have another LTI system with impulse response $h_b[n]$ which is unknown; but if we input the $x[n]$ given above, we get the output

$$y_b[n] = 6\delta[n] - 4\delta[n-1] + 2\delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - \delta[n-5]$$

The impulse response $h_b[n]$ can be expressed in the form: $h_b[n] = A\delta[n-p] + B\delta[n-q]$.
Find numerical values for A, B, p, and q.

PROBLEM f-03-Q.2.4:

A few questions about sampling:

(a) Suppose the sinusoid $x(t) = \cos(3000\pi t + \pi/3)$ is input to an ideal continuous-to-discrete (C-D) converter with sampling frequency f_s.

(i) Find an f_s which would result in a discrete-time signal $x[n] = \cos(0.5\pi n + \pi/3)$.

(ii) Find an f_s which would result in a discrete-time signal $x[n] = \cos(0.5\pi n - \pi/3)$.

(b) Suppose a disc with a spot painted at one point along the edge is rotating ***clockwise*** at a certain speed, and a movie camera operating at **24 frames per second** is filming the rotating disc. Give ***two*** different ***nonzero*** disc rotation speeds, in terms of revolutions per second, which would make it look like the spot is standing still.

PROBLEM f-03-Q.2.5:

Suppose a discrete-time LTI system has frequency response $\boxed{H(e^{j\hat{\omega}}) = \frac{1}{7}\frac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)}\exp\left(-j\frac{9}{2}\hat{\omega}\right)}$

(a) If the input to this system is

$$x[n] = 3 + \cos\left(\frac{\pi}{5}n\right) + \frac{1}{4}\cos\left(\frac{2\pi}{5}n\right) + \frac{1}{9}\cos\left(\frac{3\pi}{5}n\right) + \frac{1}{16}\cos\left(\frac{4\pi}{5}n\right),$$

Find the output $y[n]$ as a very simple formula. ***Explain your reasoning.*** Be clever.

(b) Suppose we want to implement the system with the $H(e^{j\hat{\omega}})$ given above with the following fragment of MATLAB code, where the variable `yy` contains the output and the variable `xx` contains the input:

```
hh = ????;
yy = conv(hh,xx);
```

Specify the row vector `hh`, i.e., `hh = [stuff goes here]`.

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QUIZ #2

DATE: ~~24~~-Oct-03 COURSE: ECE-2025

NAME: Answer Key
LAST, FIRST

GT #: version A

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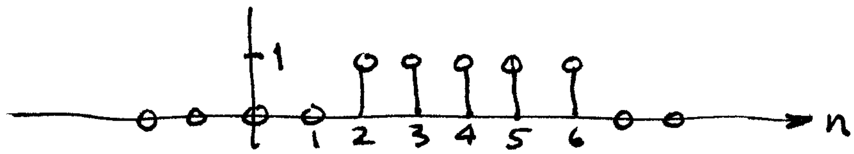
PROBLEM f-03-Q.2.1:
Questions about discrete-time signals and systems:

(a) Make a stem plot of $y[n] = \delta[n-3] * (u[n+1] - u[n-4])$, where $*$ represents convolution.

$y[n] = u[n-2] - u[n-7]$ because $\delta[n-3] * x[n] = x[n-3]$

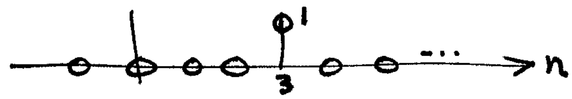


(b) Make a stem plot of $y[n] = \delta[n-3]\,(u[n+1] - u[n-4])$ versus n.

Multiplication. $\delta[n-3]$ is nonzero only when $n=3$

At $n=3$, $u[n+1] = u[4] = 1$ and $u[n-4] = u[3-4] = u[-1] = 0$

$\Rightarrow y[n] = \delta[n-3]$



(c) Suppose a system is specified by the input/output relationship $y[n] = (x[n^a - b])^c$ where b is any integer, but a and c are *positive* integers.

In answering each of the following questions, give the *minimum* set of numerical constraints, i.e., don't give any more constraints than necessary to achieve the desired property for each question. *For example, your answer might be $b = -7$, $a = 3$, and c is unconstrained.*
On this subproblem, you do not need to give any explanations. Feel free to use your intuition!

(i) What numerical constraints must we put on a, b, and c to ensure the system is *linear*?

$c = 1$; a & b would be unconstrained.

Then $y[n] = x[n^a - b]$

$\lambda_1 y_1[n] + \lambda_2 y_2[n] = \lambda_1 x_1[n^a - b] + \lambda_2 x_2[n^a - b]$

(ii) What numerical constraints must we put on a, b, and c to ensure the system is *time-invariant?*

$a = 1$; b & c unconstrained

Then $y[n] = (x[n-b])^c$

$y[n-n_0] = (x[(n-n_0)-b])^c$

(iii) What numerical constraints must we put on a, b, and c to ensure the system is *causal*?

$a = 1$ & $b \geq 0$; c unconstrained

Then $y[n] = (x[n-b])^c$

If $a > 1$, e.g. $a = 2$, then $y[n] = (x[n^2 - b])^c$

Thus when $n = b$, $y[b+1]$ depends on $x\underbrace{[b^2+2b+1-b]}_{x[b+b^2+1]}$

which is in the future.

PROBLEM f-03-Q.2.2:

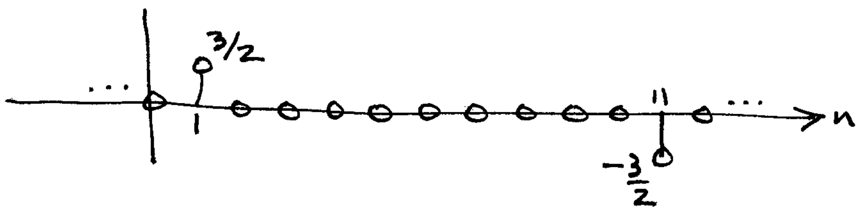
Suppose an LTI system has a frequency response given by $\boxed{H(e^{j\hat{\omega}}) = 3j\sin(5\hat{\omega})e^{-j6\hat{\omega}}}$

(a) Find $h[n]$, the system's impulse response, in terms of a sum of delta functions.

$$H(e^{j\hat{\omega}}) = 3j\, e^{-j6\hat{\omega}}\left(\frac{e^{j5\hat{\omega}} - e^{-j5\hat{\omega}}}{2j}\right)$$

$$= \frac{3}{2}\left(e^{-j\hat{\omega}} - e^{-j11\hat{\omega}}\right)$$

$$\Rightarrow h[n] = \frac{3}{2}\delta[n-1] - \frac{3}{2}\delta[n-11]$$



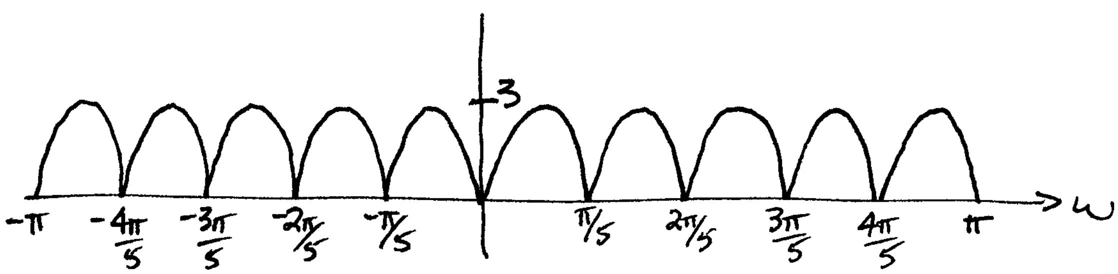
(b) Plot the magnitude of the frequency response versus $\hat{\omega}$.

$$|H(e^{j\hat{\omega}})| = |3\sin(5\hat{\omega})|$$

$\sin(5\hat{\omega}) = 0$ when $5\hat{\omega} = k\pi$

$\Rightarrow \hat{\omega} = k\pi/5$

Plot from $-\pi < \hat{\omega} \leq \pi$



(c) Evaluate the phase of the frequency response at $\hat{\omega} = 0.1\pi$. *The answer should be a number.*

$\angle H(e^{j\hat{\omega}}) = -6\hat{\omega} + \pi/2$ because $j = e^{j\pi/2}$

At $\hat{\omega} = 0.1\pi$

$$\angle H(e^{j\hat{\omega}}) = -0.6\pi + 0.5\pi = -0.1\pi \text{ rads.}$$

PROBLEM f-03-Q.2.3:

Consider the signal $x[n] = 3\delta[n] - 2\delta[n-1] + \delta[n-2]$

(a) Suppose $x[n]$ is input to an LTI system described by the difference equation

$$y_a[n] = x[n] + 2x[n-2] - x[n-3]$$

Find the output $y_a[n]$. Express your answer as a sum of delta functions.

$b_k = \{1, 0, 2, -1\}$

$$\begin{array}{cccccc}
3 & -2 & 1 & & & \\
1 & 0 & 2 & -1 & & \\
\hline
3 & -2 & 1 & & & \\
 & 0 & 0 & 0 & & \\
 & & 6 & -4 & 2 & \\
 & & & -3 & 2 & -1 \\
\hline
3 & -2 & 7 & -7 & 4 & -1
\end{array}$$

$$y_a[n] = 3\delta[n] - 2\delta[n-1] + 7\delta[n-2] - 7\delta[n-3] + 4\delta[n-4] - \delta[n-5]$$

(b) Now suppose we have another LTI system with impulse response $h_b[n]$ which is unknown; but if we input the $x[n]$ given above, we get the output

$$y_b[n] = 6\delta[n] - 4\delta[n-1] + 2\delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - \delta[n-5]$$

The impulse response $h_b[n]$ can be expressed in the form: $h_b[n] = A\delta[n-p] + B\delta[n-q]$. Find numerical values for A, B, p, and q.

$$h_b[n] * x[n] = (A\delta[n-p] + B\delta[n-q]) * x[n]$$
$$= A\,x[n-p] + B\,x[n-q]$$

So, look for shifted, scaled copies of $x[n]$. For $n = 0, 1, 2$, the values of $y_b[n]$ are twice those of $x[n]$. For $n = 3, 4, 5$ the values of $y_b[n]$ are negative w.r.t. $x[n]$. Thus

$$y_b[n] = 2x[n] - x[n-3]$$

$\Rightarrow A = 2 \qquad p = 0$

$B = -1 \qquad q = 3$

PROBLEM f-03-Q.2.4:

A few questions about sampling:

(a) Suppose the sinusoid $x(t) = \cos(3000\pi t + \pi/3)$ is input to an ideal continuous-to-discrete (C-D) converter with sampling frequency f_s.

(i) Find an f_s which would result in a discrete-time signal $x[n] = \cos(0.5\pi n + \pi/3)$.

$$\hat{\omega} = 0.5\pi$$

$$2\pi f = 3000\pi \text{ rad/s}$$

$$\hat{\omega} = \frac{2\pi f}{f_s} \Rightarrow f_s = \frac{2\pi f}{\hat{\omega}} = \frac{3000\pi}{0.5\pi} = 6000 \text{ Hz}$$

(ii) Find an f_s which would result in a discrete-time signal $x[n] = \cos(0.5\pi n - \pi/3)$.

To make the sign of the phase flip we need a folded alias. A negative frequency line from the spectrum of $x(t)$ must end up at $\hat{\omega} = 0.5\pi$

$$\Rightarrow \hat{\omega} = \frac{-2\pi f}{f_s} + 2\pi \ell \quad \text{where } \ell = \text{integer}.$$

$$f_s = \frac{-2\pi f}{\hat{\omega} - 2\pi \ell} .$$

Pick $\ell = 1$ (one choice) $f_s = \frac{-3000\pi}{0.5\pi - 2\pi} = 2000 \text{ Hz}$

(b) Suppose a disc with a spot painted at one point along the edge is rotating ***clockwise*** at a certain speed, and a movie camera operating at **24 frames per second** is filming the rotating disc. Give ***two*** different ***nonzero*** disc rotation speeds, in terms of revolutions per second, which would make it look like the spot is standing still.

If the rotation speed equals the "sampling rate" of the camera, the spot will always be in the same location. Thus, 24 rev/sec works.

If the spot makes two complete revolutions between flashes, the spot will appear to stand still. Thus 48 rev/sec works.

In general, 24n rev/sec works where n = integer. n > 0.

PROBLEM f-03-Q.2.5:

Suppose a discrete-time LTI system has frequency response $\boxed{H(e^{j\hat{\omega}}) = \frac{1}{7}\frac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)}\exp\left(-j\frac{\cancel{8}\,9}{2}\hat{\omega}\right)}$

(a) If the input to this system is

$$x[n] = 3 + \cos\left(\frac{\pi}{5}n\right) + \frac{1}{4}\cos\left(\frac{2\pi}{5}n\right) + \frac{1}{9}\cos\left(\frac{3\pi}{5}n\right) + \frac{1}{16}\cos\left(\frac{4\pi}{5}n\right),$$

Find the output $y[n]$ as a very simple formula. ***Explain your reasoning.*** Be clever.

Each sinusoidal component in $x[n]$ experiences a magnitude & phase change given by $H(e^{j\hat{\omega}})$ at $\hat{\omega} = 0, \pi/5, 2\pi/5, 3\pi/5$ and $4\pi/5$

At $\hat{\omega}=0$, $H(e^{j0}) = \lim_{\hat{\omega}\to 0} \frac{1}{7}\frac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)} = \frac{10}{7}$

At $\hat{\omega} = \pi k/5$, the numerator, $\sin(5\hat{\omega})$, is zero so $H(e^{j\hat{\omega}}) = 0$, when $\hat{\omega} = \pi k/5$

Thus, $y[n] = 3\left(\frac{10}{7}\right) + 0 + 0 + 0 + 0 = 30/7$

(b) Suppose we want to implement the system with the $H(e^{j\hat{\omega}})$ given above with the following fragment of MATLAB code, where the variable yy contains the output and the variable xx contains the input:

```
hh = ????;
yy = conv(hh,xx);
```

Specify the row vector hh, i.e., hh = `[stuff goes here]`.

$\frac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)} e^{-j\frac{9}{2}\hat{\omega}}$ is the freq. resp. of a 10-pt running sum filter.

The $\frac{1}{7}$ multiplier means that all the $h[n]$'s have to be scaled by $1/7$. Thus

hh = ones(1,10)/7;

or

hh = [1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7];