GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 3-Mar-00 COURSE: EE 2025

NAME: ______________________________ STUDENT #: ____________

LAST, FIRST

Recitation Section: Circle the date & time when your Recitation Section meets (not Lab):

Mon-3p (L01:Smith)	M-4:30p (L02:Smith)			
Tues-9:30a (L03:Frazier)	T-Noon (L05:Frazier)	T-1:30p (L07:Smith)	T-3p (L09:Yezzi)	T-4:30p (L11:Yezzi)
Wed-3p (L13:Verriest)	W-4:30p (L14:Verriest)			
Thur-9:30a (L04:Smith)	Th-Noon (L06:Bordelon)	Th-1:30p (L08:Bordelon)	Th-3p (L10:Smith)	Th-4:30p (L12:Smith)

- Write your name on the front page ONLY.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY, JUSTIFY your reasoning CLEARLY to receive any partial credit.
  Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

Problem sp-00-Q.2.1:

For each of the following frequency response, pick one of the representations below that defines *exactly* the same LTI system. Write your answer S_1, S_2, S_3, S_4, S_5, or S_6, in the box next to each frequency response. In addition, evaluate the frequency response at $\hat{\omega} = 0$, $\pm\pi$ and $\hat{\omega} = \pm\frac{1}{2}\pi$ as requested for each case; *simplify* the answer to **polar form** and write it in the space provided.

ANS = $\qquad$ $\mathcal{H}(\hat{\omega}) = e^{-j2\hat{\omega}}(2\cos(\hat{\omega}))$

$\mathcal{H}(0) =$

ANS = $\qquad$ $\mathcal{H}(\hat{\omega}) = -e^{-j2\hat{\omega}}$

$\mathcal{H}(0) =$

ANS = $\qquad$ $\mathcal{H}(\hat{\omega}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

$\mathcal{H}(\frac{1}{2}\pi) =$

ANS = $\qquad$ $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j3\hat{\omega}}$

$\mathcal{H}(\pi) =$

POSSIBLE ANSWERS: (impulse response, filter coefficients or difference equation)

S_1 : $h[n] = 3\delta[n] - 3\delta[n-2]$

S_2 : $y[n] = x[n-1] - x[n-3]$

S_3 : $b_k = \{1, 0, 1\}$

S_4 : $h[n] = \delta[n-1] + \delta[n-3]$

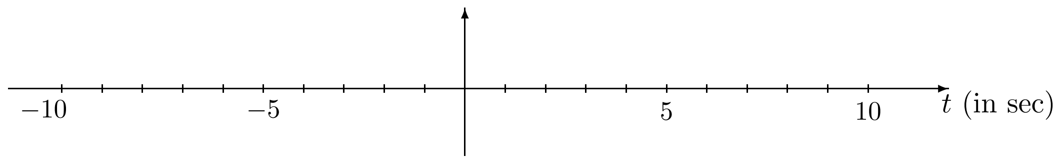
S_5 : $y[n] = -x[n-2]$

S_6 : $b_k = \{1, 1, 1\}$

Problem sp-00-Q.2.2:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} 1 & \text{for } 0 \le t \le 7 \\ 0 & \text{for } 7 < t < 8 \end{cases}$

(a) Assume that the period of $x(t)$ is 8 s. Draw a plot of $x(t)$ over the range $-10 \le t \le 10$ s.

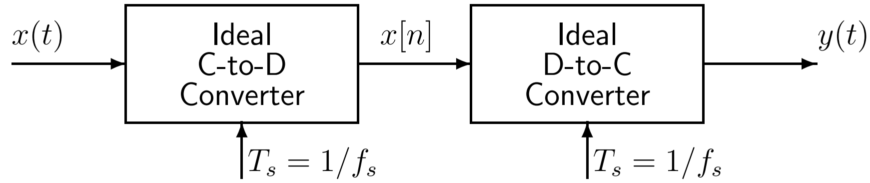


(b) Determine the DC value of $x(t)$.

(c) Write the Fourier integral expression for the coefficient a_3 in terms of the specific signal $x(t)$ defined above. *Set up all the specifics of the integral (e.g., limits of integration), but do not evaluate the integral. All parameters in the integral should have numeric values.*

(d) Evaluate the following integral: $\displaystyle\int_0^9 e^{-j2\pi(15)t/10}dt$ Simplify your answer and express it in **polar form.**

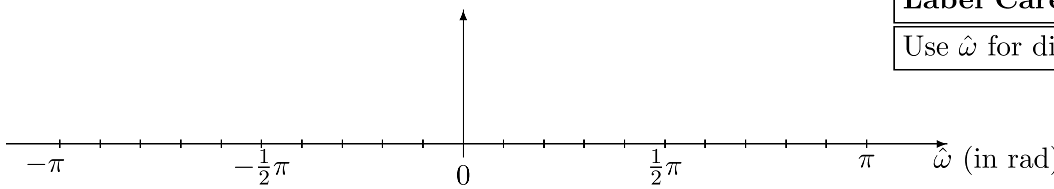
Problem sp-00-Q.2.3:



(a) If the input to the ideal C/D converter is $x(t) = 7\cos(2400\pi t + \pi/3)$, determine the spectrum for $x[n]$ when $f_s = 2000$ samples/sec. Make a $\boxed{\text{plot}}$ for your answer, but label the frequency, amplitude and phase of each spectral component.

Label Carefully

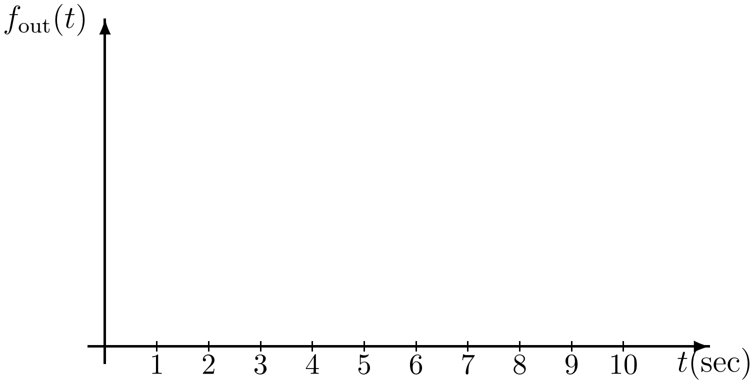
Use $\hat{\omega}$ for digital freq.



(b) Suppose that the input signal is a chirp signal defined as follows:

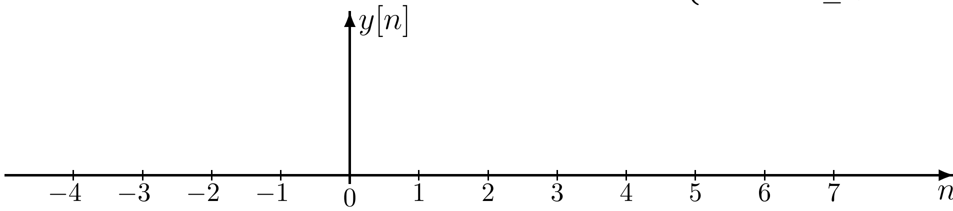
$$x(t) = \cos(2\pi(600)t + 200\pi t^2) \qquad \text{for } 0 \leq t \leq 10 \text{ sec.}$$

If the sampling rate is $f_s = 2000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Recall that this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`, or the DSP-First function `plotspec()`.



Problem sp-00-Q.2.4:

(a) If the filter coefficients of an FIR filter are $\{b_k\} = \{0, 1, 0, -1\}$, make a $\boxed{\text{plot}}$ of the output when the input is the unit step signal: $x[n] = u[n] = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$



Label Carefully

Plot zero values also

(b) Suppose that the frequency response of a different FIR filter is

$$\mathcal{H}(\hat{\omega}) = 2j\sin(\hat{\omega})e^{-j3\hat{\omega}}$$

If the input signal is $x[n] = 5 + 3\cos(\pi n/4)$ for $-\infty < n < \infty$, determine a simple mathematical expression for the output signal $y[n]$.

$\boxed{y[n] = }$

Problem sp-00-Q.2.1:

For each of the following frequency response, pick one of the representations below that defines *exactly* the same LTI system. Write your answer S_1, S_2, S_3, S_4, S_5, or S_6, in the box next to each frequency response. In addition, evaluate the frequency response at $\hat{\omega} = 0, \pi$ and $\hat{\omega} = \pm\frac{1}{2}\pi$ as requested for each case; *simplify* the answer to **polar form** and write it in the space provided.

$\boxed{\text{ANS} = 4}$ $\mathcal{H}(\hat{\omega}) = e^{-j2\hat{\omega}}(2\cos(\hat{\omega}))$

$\boxed{\mathcal{H}(0) = 2}$

$$\mathcal{H}(0) = e^{-j0}(2\cos(0)) = 2$$

$\boxed{\text{ANS} = 5}$ $\mathcal{H}(\hat{\omega}) = -e^{-j2\hat{\omega}}$

$\boxed{\mathcal{H}(0) = e^{j\pi}}$

$$\mathcal{H}(0) = -e^{-j0} = -1 = e^{j\pi}$$

$\boxed{\text{ANS} = 6}$ $\mathcal{H}(\hat{\omega}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

$\boxed{\mathcal{H}(\frac{1}{2}\pi) = e^{-j\pi/2}}$

$$\mathcal{H}(\pi/2) = 1 + e^{-j\pi/2} + e^{-j\pi} = 1 + e^{-j\pi/2} - 1$$
$$= e^{-j\pi/2}$$

$\boxed{\text{ANS} = 2}$ $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j3\hat{\omega}}$

$\boxed{\mathcal{H}(+\pi) = 0}$

$$\mathcal{H}(+\pi) = e^{-j\pi} - e^{-j3\pi} = -1 - (-1) = 0$$

POSSIBLE ANSWERS: (impulse response, filter coefficients or difference equation)

S_1 : $h[n] = 3\delta[n] - 3\delta[n-2]$

$b_k = \{3, 0, -3\}$ $\quad \mathcal{H}(\hat{\omega}) = 3 - 3e^{-j2\hat{\omega}}$

S_2 : $y[n] = x[n-1] - x[n-3]$

$b_k = \{0, 1, 0, -1\}$ $\quad \mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j3\hat{\omega}}$

S_3 : $b_k = \{1, 0, 1\}$

$$\mathcal{H}(\hat{\omega}) = 1 + e^{-j2\hat{\omega}} = e^{-j\hat{\omega}}(e^{j\hat{\omega}} + e^{-j\hat{\omega}}) = e^{-j\hat{\omega}}(2\cos\hat{\omega})$$

S_4 : $h[n] = \delta[n-1] + \delta[n-3]$

$b_k = \{0, 1, 0, 1\}$ $\quad \mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}} + e^{-j3\hat{\omega}} = e^{-j2\hat{\omega}}(e^{j\hat{\omega}} + e^{-j\hat{\omega}}) = e^{-j2\hat{\omega}}(2\cos\hat{\omega})$

S_5 : $y[n] = -x[n-2]$

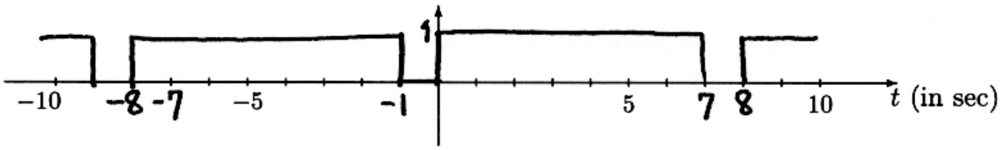
$$\mathcal{H}(\hat{\omega}) = -e^{-j2\hat{\omega}}$$

S_6 : $b_k = \{1, 1, 1\}$ $\quad \mathcal{H}(\hat{\omega}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

Problem sp-00-Q.2.2:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} 1 & \text{for } 0 \le t \le 7 \\ 0 & \text{for } 7 < t < 8 \end{cases}$

(a) Assume that the period of $x(t)$ is 8 s. Draw a plot of $x(t)$ over the range $-10 \le t \le 10$ s.



(b) Determine the DC value of $x(t)$.

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt = \frac{1}{T_0}(\text{area in one period})$$

$$= \frac{1}{8}(1 \times 7) = \frac{7}{8}$$

(c) Write the Fourier integral expression for the coefficient a_3 in terms of the specific signal $x(t)$ defined above. *Set up all the specifics of the integral (e.g., limits of integration), but do not evaluate the integral. All parameters in the integral should have numeric values.*

$k = 3$

$$a_3 = \frac{1}{8}\int_0^7 1\, e^{-j2\pi(3)t/8}\,dt$$

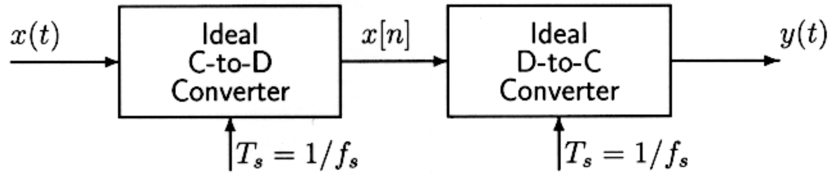
(d) Evaluate the following integral: $\int_0^9 e^{-j2\pi(15)t/10}dt$ Simplify your answer and express it in <u>**polar form.**</u>

$$\int_0^9 e^{-j3\pi t}dt = \left.\frac{e^{-j3\pi t}}{-j3\pi}\right|_0^9 = \frac{e^{-j27\pi} - e^{-j0}}{-j3\pi}$$

$$= \frac{-1-1}{-j3\pi} = \frac{2}{j3\pi} = -j\frac{2}{3\pi} = \frac{2}{3\pi}e^{-j\pi/2}$$

POLAR FORM

Problem sp-00-Q.2.3:



(a) If the input to the ideal C/D converter is $x(t) = 7\cos(2400\pi t + \pi/3)$, determine the spectrum for $x[n]$ when $f_s = 2000$ samples/sec. Make a $\boxed{\text{plot}}$ for your answer, but label the frequency, amplitude and phase of each spectral component.

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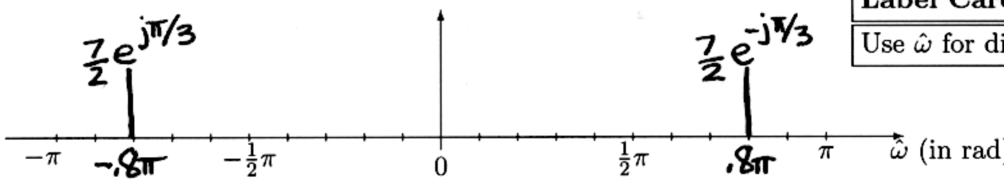
Use $\hat{\omega}$ for digital freq.

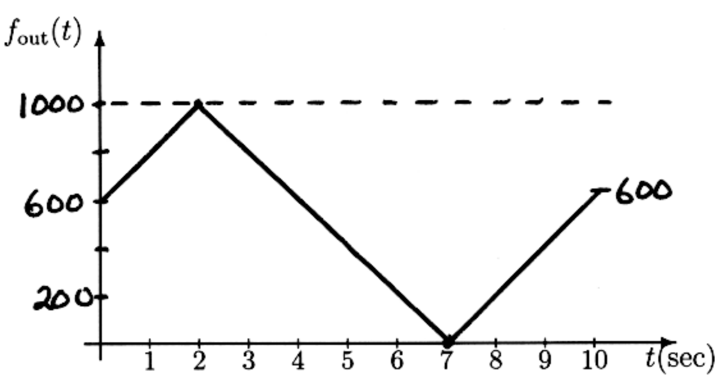


$$x[n] = 7\cos\left(2400\pi \tfrac{n}{2000} + \pi/3\right)$$
$$= 7\cos(1.2\pi n + \pi/3) = 7\cos(-0.8\pi n + \pi/3)$$
$$= 7\cos(0.8\pi n - \pi/3)$$
$$= \frac{7}{2}e^{-j\pi/3}e^{j0.8\pi n} + \frac{7}{2}e^{j\pi/3}e^{-j0.8\pi n}$$

(b) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(2\pi(600)t + 200\pi t^2) \qquad \text{for } 0 \le t \le 10 \text{ sec.}$$

If the sampling rate is $f_s = 2000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Recall that this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`, or the DSP-First function `plotspec()`.



$$\psi(t) = 1200\pi t + 200\pi t^2$$
$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\psi(t)$$
$$= \frac{1}{2\pi}(1200\pi + 400\pi t)$$
$$= 600 + 200t \text{ Hz}$$

$f_i(0) = 600$ Hz

$f_i(2) = 1000$ Hz

$f_i(4) = 1400 \rightarrow 600$

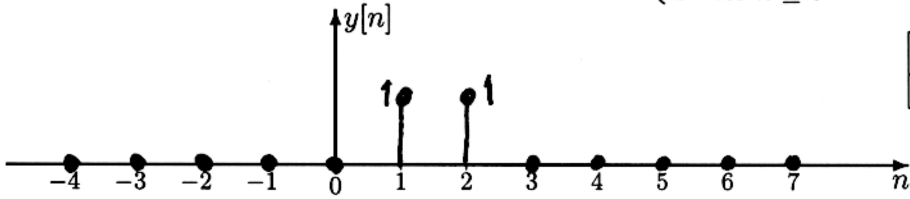
$f_i(7) = 2000 \rightarrow 0$

$f_i(10) = 2600 \rightarrow 600$

D/A will restrict output freqs to the range: $0 \rightarrow \frac{1}{2}f_s = 1000$ Hz

Problem sp-00-Q.2.4:

(a) If the filter coefficients of an FIR filter are $\{b_k\} = \{0, 1, 0, -1\}$, make a $\boxed{\text{plot}}$ of the output when the input is the unit step signal: $x[n] = u[n] = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$



Label Carefully

Plot zero values also

Convolution:

```
1  1  1  1  1  1
0  1  0  -1
------------------------------
0  1  1  1  1  1  1 ...
      0 -1 -1 -1 -1 ...
------------------------------
0  1  1  0  0  0  0 .....
```

(b) Suppose that the frequency response of a different FIR filter is

$$\mathcal{H}(\hat{\omega}) = 2j \sin(\hat{\omega}))e^{-j3\hat{\omega}}$$

If the input signal is $x[n] = 5 + 3\cos(\pi n/4)$ for $-\infty < n < \infty$, determine a simple mathematical expression for the output signal $y[n]$.

$\boxed{y[n] = 0 + 3\sqrt{2}\cos(\pi n/4 - \pi/4)}$

$x[n]$ has two freq. components: $\hat{\omega} = 0$ & $\hat{\omega} = \frac{\pi}{4}$

$\mathcal{H}(\hat{\omega})\big|_{\hat{\omega}=0} = \mathcal{H}(0) = 2j\sin(0)e^{-j0} = 0$

$\mathcal{H}(\frac{\pi}{4}) = 2j\sin(\pi/4)e^{-j3\pi/4} = \sqrt{2}e^{j\pi/2}e^{-j3\pi/4} = \sqrt{2}e^{-j\pi/4}$

$y[n] = 5 \times \mathcal{H}(0) + 3 \times |\mathcal{H}(\frac{\pi}{4})| \cos(\frac{\pi}{4}n + \angle\mathcal{H}(\pi/4))$

$= 5(0) + 3\sqrt{2}\cos(\frac{\pi}{4}n - \frac{\pi}{4})$