GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 19-Sept-03 COURSE: ECE 2025

NAME: Key STUDENT #: _____

LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-9:30 (G. Li)	L02:Thur-9:30 (G-K. Chang)
L03:Tues-12:00 (G. Li)	L04:Thur-12:00 (G-K. Chang)
L05:Tues-1:30 (M. Richards)	L06:Thur-1:30 (T. Zhou)
L07:Tues-3:00 (M. Richards)	L08:Thur-3:00 (T. Zhou)
L09:Tues-4:30 (Y. Altunbasak)	L10:Thur-4:30 (G. Casinovi)
L11:Tues-6:00 (Y. Altunbasak)	L13:Mon-3:00 (J. McClellan)
L14:Wed-3:00 (R. Butera)	L16:Wed-4:30 (R. Butera)
Savannah (G. AlRegib)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, justify your reasoning clearly to receive any partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

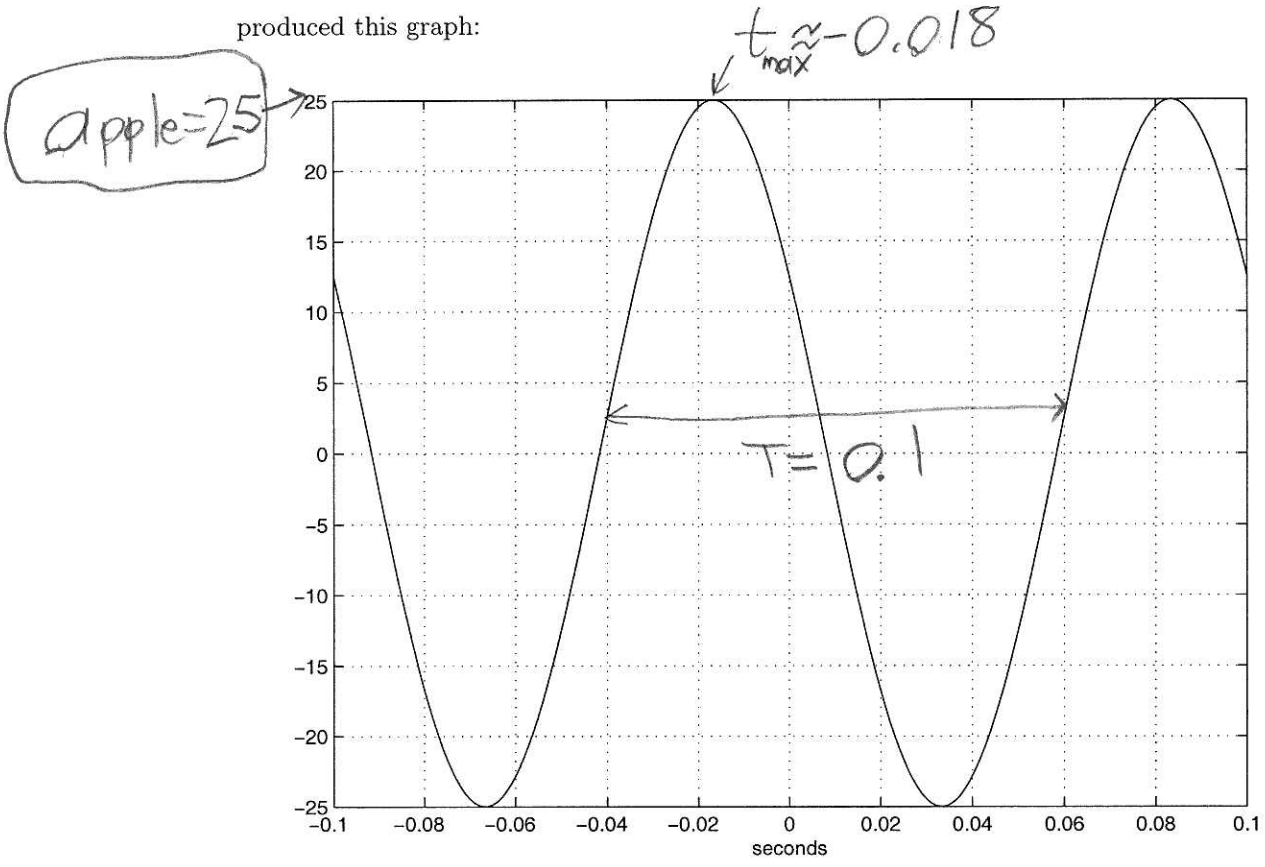
Problem Q1.1:

(a) The following lines, typed at the MATLAB prompt,

```
t = -0.1:0.001:0.1;
x = apple*cos(2*pi*banana*t+pi/grape);
plot(t,x);

xlabel('seconds')
grid on
```

produced this graph:



The variable grape is an *integer*. Find the values of the variables apple, banana, and grape.

$$banana = f = \frac{1}{T} = \frac{1}{0.1} = 10$$

$$\phi = -\omega t_{max} \approx -2\pi \times 10 \times (-0.018)$$
$$= 0.36\pi$$

$$grape \approx \frac{1}{0.36} = \boxed{3}$$ (rounding to nearest integer)

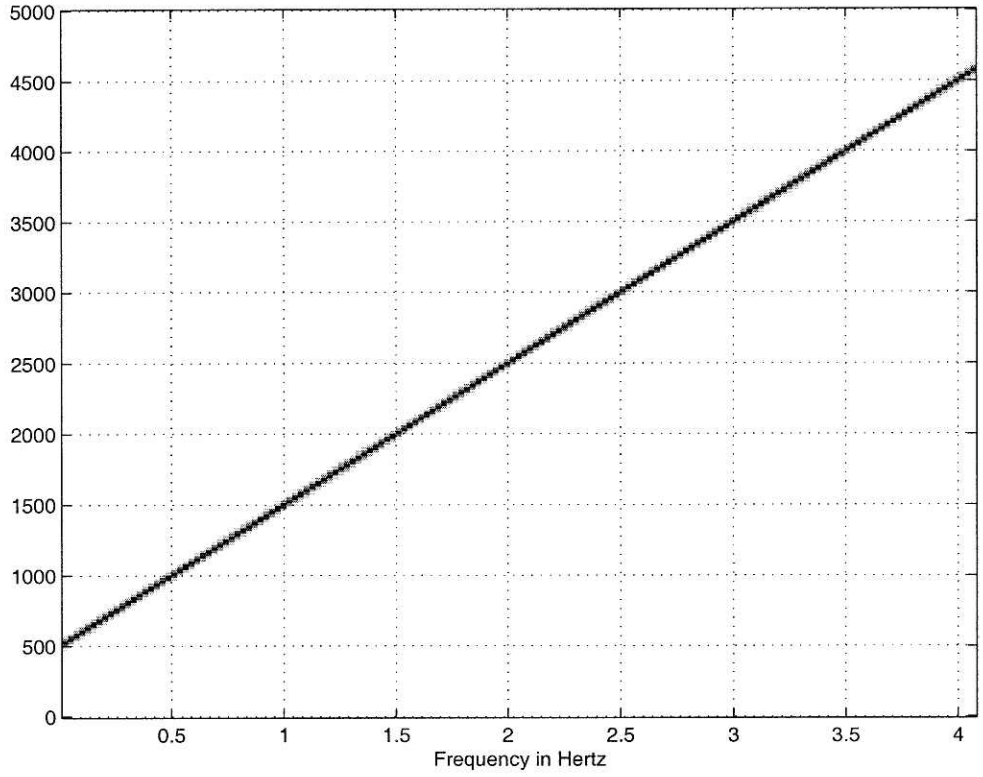
Apple = 25

(b) The following lines, typed at the MATLAB prompt,

```
fs = 10000;
t = 0:(1/fs):4.1;
x = cos(2*pi*ham*t .^ 2 + 2*pi*cheese*t);

plotspec(x,fs,512); grid on
xlabel('Time in seconds'); xlabel('Frequency in Hertz');
```

produced this graph:



Find the values of the variables ham and cheese.

$F_i(t) = 2\,ham \cdot t + cheese$

$F_i(0) = \boxed{500 = cheese}$

$F_i(4) = 2\,ham \cdot 4 + 500 = 4500$

$8\,ham = 4000$

$\boxed{ham = 500}$

Problem Q1.2:

(a) Let $\Im m$ denote taking the imaginary part of a complex variable. Find $\Im m\{x(t)x^*(t-0.002)\}$, where $x(t) = e^{j250\pi t}$. Reduce your answer to a real number.

$$Im\{e^{j250\pi t} e^{-j250\pi(t-0.002)}\}$$
$$= Im\{e^{j0.5\pi}\} = \sin\left(\frac{\pi}{2}\right) = \boxed{1}$$

(b) Find $|(3+3j)(5e^{j0.2})|$. (Be sure to notice those vertical bars.)

$$= |3+3j| \times 5 = (3\sqrt{2})\,5 = 15\sqrt{2}$$

Problem Q1.3:

Notice you could pull the 8 out in front

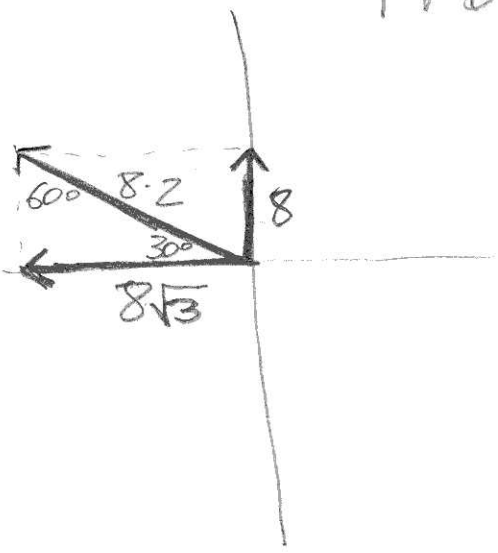
Consider the equation

$$A\cos(\omega t + \phi) = 8\cos(425\pi t + \pi/2) - 8\sqrt{3}\cos(425\pi t)$$

(a) Find A and ω.

$A = 8 \cdot 2 = 16$

$\omega = 425\pi$



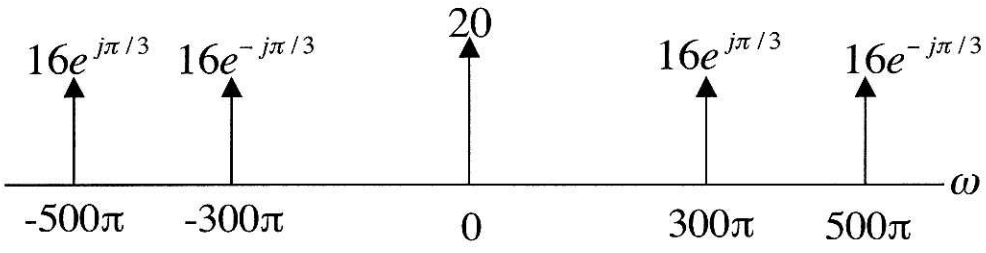
(b) Circle the correct ϕ:

(a) $\phi = \pi/6$	(h) $\phi = -\pi/6$
(b) $\phi = \pi/3$	(i) $\phi = -\pi/3$
(c) $\phi = \pi/4$	(j) $\phi = -\pi/4$
(d) $\phi = \pi/2$	(k) $\phi = -\pi/2$
(e) $\phi = 2\pi/3$	(l) $\phi = -2\pi/3$
(f) $\phi = 3\pi/4$	(m) $\phi = -3\pi/4$
(g) $\phi = 5\pi/6$	(n) $\phi = -5\pi/6$

$\phi = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Problem Q1.4:

The signal $x(t)$ has the two-sided spectrum:



(a) Express $x(t)$ as a constant plus a **sum** of two cosines, i.e. express it in a form like:

$$?+?\cos(?t+?)+?\cos(?t+?)$$

$$20+32\cos\left(300\pi t+\frac{\pi}{3}\right)+32\cos\left(500\pi t-\frac{\pi}{3}\right)$$

(apply Euler's, so multiply by 2)

(b) Express $x(t)$ as a constant plus a **product** of two cosines, i.e. express it in a form like:

$$?+?\cos(?t+?)\cos(?t+?)$$

(apply Euler's twice, so multiply by 4)

$$20+64\cos(400\pi t)\cos\left(100\pi t-\frac{\pi}{3}\right)$$

$\frac{-300\pi+500\pi}{2}$ $\qquad$ $\frac{500\pi-300\pi}{2}$

Problem Q1.5:

$\cos(2500\pi t - \frac{\pi}{2})$

For part (a) and (b), suppose $x(t) = 7 + 6\cos(2400\pi t + 3\pi/4) + 10\sin(2500\pi t)$.

(a) What is the fundamental period of $x(t)$ in seconds?

$\omega_0 = 100\pi$

$T_0 = \frac{2\pi}{100\pi} = \frac{1}{50}$ s or 0.02 s

(b) Find the ~~Fourier~~ coefficients a_k of $x(t)$ for $k = 0, 24, -24, 25$, and -25.

$a_0 = 7$

$a_{24} = \frac{6}{2}e^{j3\pi/4} = 3e^{j\frac{3\pi}{4}}$ $\quad a_{-24} = 3e^{-j\frac{3\pi}{4}}$

$a_{25} = \frac{10}{2}e^{j\frac{\pi}{2}} = 5e^{j\frac{\pi}{2}}$ $\quad a_{25} = 5e^{j\frac{\pi}{2}}$

(c) This part (c) is independent of parts (a) and (b). Georgia Tech played its first football game against the University of Georgia in 1893. Suppose $x(t)$ is a periodic signal with period $T = 1$, defined over $0 \le t < 1$ by

$$x(t) = t^{1893}$$

Find the Fourier series coefficient a_0 of $x(t)$. Note we only want the coefficient for $k = 0$.

$$a_0 = \int_0^1 t^{1893}\,dt = \frac{1}{1894}t^{1894}\Big|_{t=0}^{t=1}$$

$$= \frac{1}{1894}$$