

GEORGIA INSTITUTE OF TECHNOLOGY  
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING  
QUIZ #1

COURSE: ECE 2025

DATE: 19-Sept-03

Key

LAST, FIRST

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Recitation Section: Circle the day & time when your Recitation Section meets:

L02:Thur-9:30 (G-K. Chang)

L04:Thur-12:00 (G-K. Chang)

L06:Thur-1:30 (T. Zhou)

L08:Thur-3:00 (T. Zhou)

L10:Thur-4:30 (G. Casinovi)

L13:Mon-3:00 (J. McClellan)

L16:Wed-4:30 (R. Butera)

Savannah (G. AlRegib)

- Write your name on the front page **ONLY**. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ( $8\frac{1}{2}'' \times 11''$ ) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, **JUSTIFY** your reasoning clearly to receive any partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

| <i>Problem</i> | <i>Value</i> | <i>Score</i> |
|----------------|--------------|--------------|
| 1              | 20           |              |
| 2              | 20           |              |
| 3              | 20           |              |
| 4              | 20           |              |
| 5              | 20           |              |

Problem Q1.1:

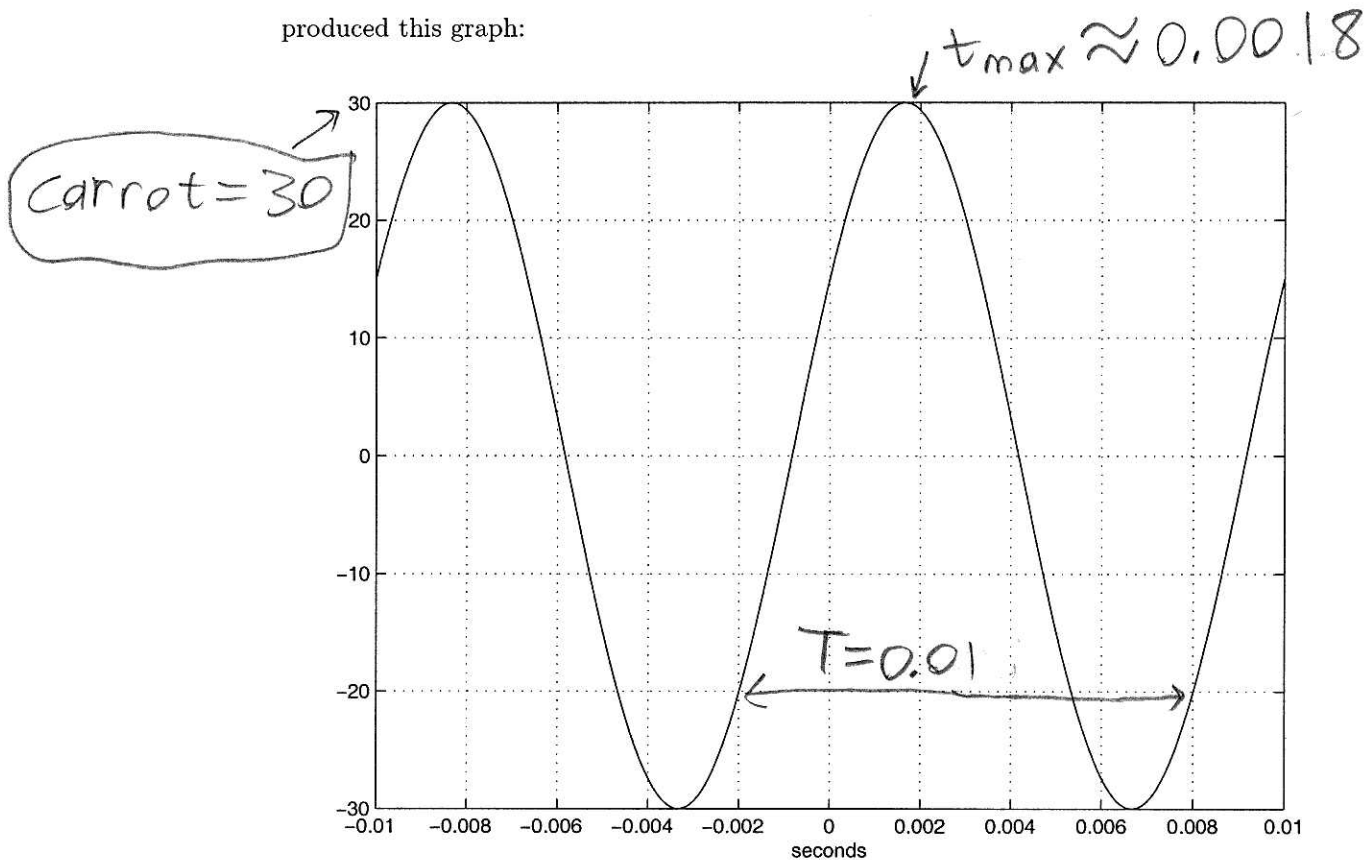
(a) The following lines, typed at the MATLAB prompt,

```
t = -0.01:0.0001:0.01;  
x = carrot*cos(2*pi*celery*t+pi/cucumber);  
plot(t,x);
```

```
xlabel('seconds')
```

```
grid on
```

produced this graph:



The variable cucumber is an *integer*. Find the values of the variables carrot, celery, and cucumber.

$$\text{celery} = f = \frac{1}{T} = \frac{1}{0.01} = \boxed{100}$$

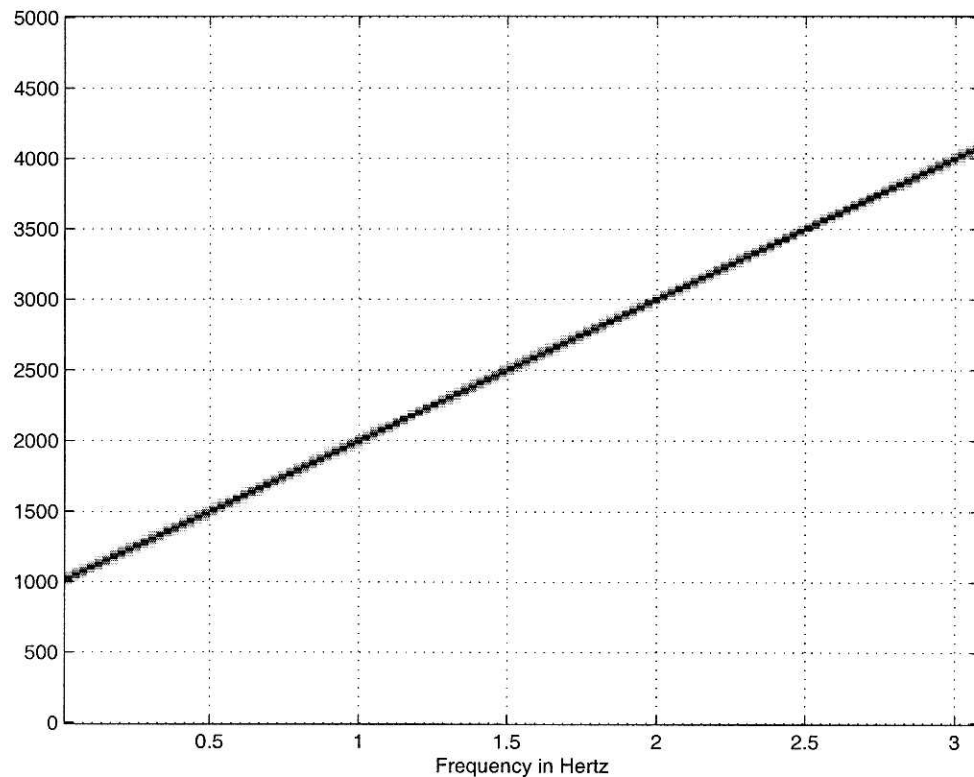
$$\phi = -\omega t_{\max} \approx -2\pi \times 100 \times 0.0018 \\ \approx -0.36\pi$$

$$\text{cucumber} \approx \frac{-1}{-0.36} \approx \boxed{-3} \quad (\text{rounding to nearest integer})$$

(b) The following lines, typed at the MATLAB prompt,

```
fs = 10000;  
t = 0:(1/fs):3.1;  
x = cos(2*pi*pizza*t.^2 + 2*pi*soda*t);  
  
plotspec(x,fs,512); grid on  
xlabel('Time in seconds'); xlabel('Frequency in Hertz');
```

produced this graph:



Find the values of the variables pizza and soda.

$$f_i(t) = 2 \cdot \text{pizza} \cdot t + \text{soda}$$

$$f_i(0) = \boxed{\text{soda} = 1000}$$

$$f_i(3) = 4000 = 2 \cdot \text{pizza} \cdot 3 + 1000$$

$$3000 = 6 \text{pizza}$$

$$\text{pizza} = \boxed{500}$$

**Problem Q1.2:**

(a) Find  $|(9 + 9j)(4e^{j0.7})|$ . (Be sure to notice those vertical bars.)

$$\begin{aligned} &= |9 + 9j| \times 4 = (9\sqrt{2})4 \\ &= \boxed{36\sqrt{2}} \end{aligned}$$

(b) Let  $\Im m$  denote taking the imaginary part of a complex variable. Find  $\Im m\{x(t)x^*(t - 0.075)\}$ , where  $x(t) = e^{j10\pi t}$ . Reduce your answer to a real number.

$$\begin{aligned} &\Im m\{x(t)x^*(t - 0.075)\} \\ &= \Im m\{e^{j10\pi t} e^{-j10\pi(t - 0.075)}\} = \Im m\{e^{j0.75\pi}\} \\ &= \sin\left(\frac{3}{4}\pi\right) = \boxed{\frac{\sqrt{2}}{2} \text{ or } \frac{1}{\sqrt{2}}} \end{aligned}$$

Problem Q1.3:

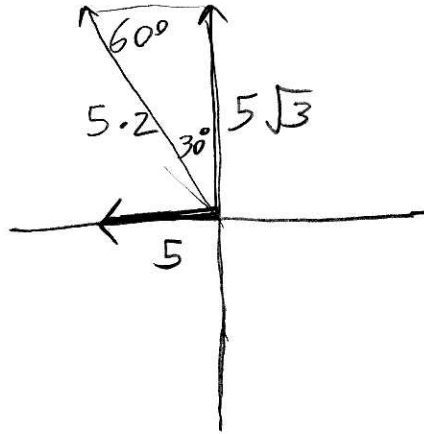
Consider the equation

$$A \cos(\omega t + \phi) = 5\sqrt{3} \cos(870\pi t + \pi/2) - 5 \cos(870\pi t)$$

(a) Find  $A$  and  $\omega$ .

$$A = 5 \cdot 2 = 10$$

$$\omega = 870\pi$$



(b) Circle the correct  $\phi$ :

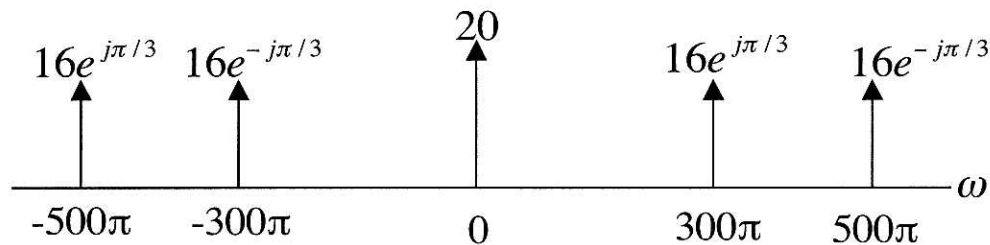
- |                                       |                      |
|---------------------------------------|----------------------|
| (a) $\phi = \pi/6$                    | (h) $\phi = -\pi/6$  |
| (b) $\phi = \pi/3$                    | (i) $\phi = -\pi/3$  |
| (c) $\phi = \pi/4$                    | (j) $\phi = -\pi/4$  |
| (d) $\phi = \pi/2$                    | (k) $\phi = -\pi/2$  |
| <u>(e) <math>\phi = 2\pi/3</math></u> | (l) $\phi = -2\pi/3$ |
| (f) $\phi = 3\pi/4$                   | (m) $\phi = -3\pi/4$ |
| (g) $\phi = 5\pi/6$                   | (n) $\phi = -5\pi/6$ |

$$\phi = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Notice you could pull the 5 out in front

**Problem Q1.4:**

The signal  $x(t)$  has the two-sided spectrum:



(a) Express  $x(t)$  as a constant plus a **sum** of two cosines, i.e. express it in a form like:

$$?? \cos(?t + ?) + ? \cos(?t + ?)$$

$$20 + 32 \cos(300\pi t + \frac{\pi}{3}) + 32 \cos(500\pi t - \frac{\pi}{3})$$

(apply Euler's, so multiply by 2)

(b) Express  $x(t)$  as a constant plus a **product** of two cosines, i.e. express it in a form like:

$$?? \cos(?t + ?) \cos(?t + ?)$$

(apply Euler's twice, so multiply by 4)

$$20 + 64 \cos\left(\underbrace{400\pi t}_{=\frac{300\pi + 500\pi}{2}}\right) \cos\left(\underbrace{100\pi t - \frac{\pi}{3}}_{\frac{500\pi - 300\pi}{2}}\right)$$

**Problem Q1.5:**

For part (a) and (b), suppose  $x(t) = 15 + 20 \cos(700\pi t + 6\pi/11) + 14 \sin(1100\pi t)$ .

$$j = \cos(1100\pi t - \frac{\pi}{2})$$

(a) What is the fundamental period of  $x(t)$  in seconds?

$$\omega_0 = 100\pi, \quad T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{100\pi} = \boxed{\frac{1}{50} \text{ s}}$$

$$\text{or } \boxed{0.02 \text{ s}}$$

(b) Find the Fourier coefficients  $a_k$  of  $x(t)$  for  $k = 0, 7, -7, 11$ , and  $-11$ .

$$a_0 = 15$$

$$a_7 = \frac{20}{2} e^{j\frac{6\pi}{11}} = \boxed{10 e^{j\frac{6\pi}{11}}}$$

$$a_{-7} = \boxed{10 e^{-j\frac{6\pi}{11}}}$$

$$a_{11} = \frac{14}{2} e^{j\frac{\pi}{2}} = \boxed{7 e^{-j\frac{\pi}{2}}}$$

$$a_{-11} = \boxed{7 e^{j\frac{\pi}{2}}}$$

(c) This part (c) is independent of parts (a) and (b). The first classes at Georgia Tech were held in 1888. Suppose  $x(t)$  is a periodic signal with period  $T = 1$ , defined over  $0 \leq t < 1$  by

$$x(t) = t^{1888}$$

Find the Fourier series coefficient  $a_0$  of  $x(t)$ . Note we only want the coefficient for  $k = 0$ .

$$a_0 = \int_0^1 t^{1888} dt = \frac{1}{1889} t^{1889} \bigg|_{t=0}^{t=1}$$

$$= \boxed{\frac{1}{1889}}$$