

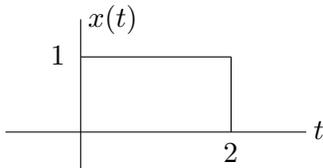


**Problem FALL-01-F.1:**

- (a) A continuous-time linear, time-invariant system has the impulse response

$$h(t) = \delta(t) + A\delta(t - \Delta).$$

Find the output of the system,  $y(t)$ , when the input is the signal  $x(t) = u(t) - u(t - 2)$  sketched below. Let  $A = -1$  and  $\Delta = 2$ . Express your answer as a plot.



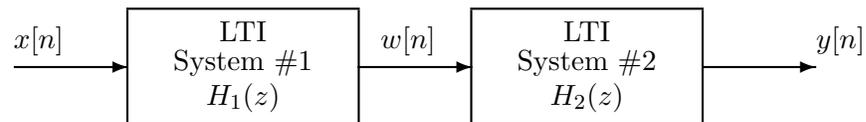
- (b) Assuming that the input signal is the same as in part (a), find values for  $A$  and  $\Delta$  that will create the output signal  $y(t) = u(t + 2) - u(t - 2)$ .

$A =$

$\Delta =$

**Problem FALL-01-F.2:**

A cascade of two FIR discrete-time systems is depicted by the following block diagram:



The systems are defined by the following:

$$H_1(z) = (1 - z^{-3}) \quad \text{and} \quad h_2[n] = (-0.9)^{n-1}u[n-1].$$

- (a) If the input to the first system is

$$x[n] = u[n],$$

determine the output,  $w[n]$ , of the **first** system.

$w[n] =$

- (b) Determine the system function  $H(z)$  of the overall system.

$H(z) =$

- (c) Determine the impulse response of the the overall system.

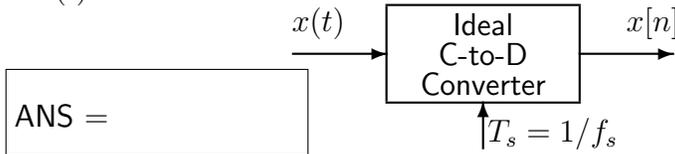
$h[n] =$

**Problem FALL-01-F.3:**

For each short question, pick a correct frequency<sup>1</sup> and enter its letter in the answer box<sup>2</sup>:

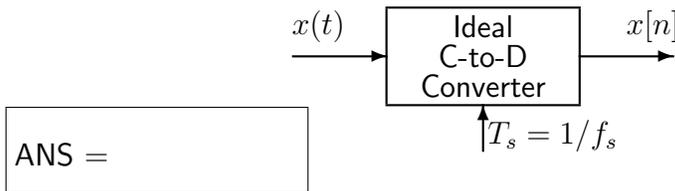
**Frequency**

- (a) If the output from an ideal C/D converter is  $x[n] = A \cos(\pi n)$ , and the sampling rate is 10000 samples/sec, then determine one possible value of the input frequency of  $x(t)$ :



- (a) 8000 Hz
- (b) 5000 Hz
- (c) 4000 Hz
- (d) 1600 Hz
- (e) 1200 Hz
- (f) 1000 Hz
- (g) 800 Hz
- (h) 500 Hz
- (i) 400 Hz

- (b) If the output from an ideal C/D converter is  $x[n] = A \cos(\pi n)$ , and the input signal  $x(t)$  defined by:  $x(t) = A \cos(5000\pi t)$  then determine one possible value of the sampling frequency of the C-to-D converter:



- (c) Determine the Nyquist rate for sampling the signal  $x(t)$  defined by:  $x(t) = \Re\{e^{j4000\pi t} + e^{j3000\pi t}\}$ .



<sup>1</sup>Some questions have more than one answer, but you only need to pick one correct answer from the list.

<sup>2</sup>It is possible to use an answer more than once.

**Problem FALL-01-F.4:**

For each of the following problems, **SIMPLIFY** your answer as much as possible.

(a) Evaluate  $\Re\{x[n+1]x^*[n-1]\}$  when  $x[n] = j5e^{-j(0.2)\pi n}$ .

(b) Evaluate the following expression,  $|1 + e^{j\pi/3}|^2 =$

(c) Evaluate the following integral,  $\int_{-\infty}^{\infty} \delta(t - 3) \cos(2\pi t) e^{-j\pi t/2} dt$

(d) Evaluate the following integral,  $\int_{-\infty}^{\infty} e^{-\pi t/2} u(t - 3) e^{-j\pi t/2} dt$ .

**Problem FALL-01-F.5:**

In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or plot. ( The symbol \* denotes convolution.)

(a) Find  $Y(j\omega)$  when  $y(t) = h(t) * x(t) = \cos(t) * \frac{\sin(\pi t/2)}{t/2}$ .

(b) Find  $h(t)$  when  $H(j\omega) = j\delta(\omega - 1) * [u(\omega) - u(\omega + 2)] - 1$ .

(c) Find  $v(t)$  when  $V(j\omega) = 3(u(\omega + 1) - u(\omega - 5))$ .

(d) Find  $H(j\omega)$  when  $h(t) = e^{-2t}u(t - 1)$ .

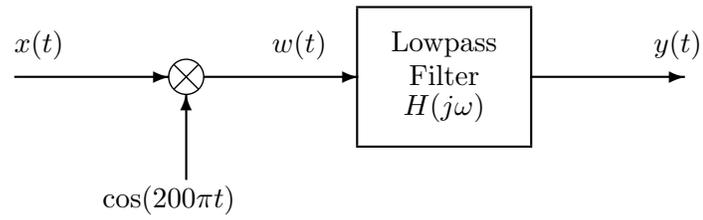
**Problem FALL-01-F.6:**

- (a) The Fourier coefficients for the Fourier Series of a periodic signal  $x(t)$  are defined using the following MATLAB code:

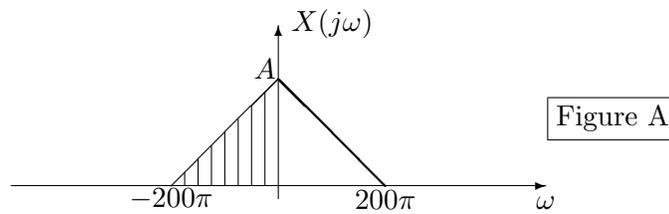
```
N = 2;
for k = -N:N
    if k == 0
        ak(k+N+1) = 1.5; % DC term
    else
        ak(k+N+1) = (j*pi*k)*(1 + exp(j*k*pi));
    end
end
```

If the fundamental frequency is 50 Hz, sketch the Fourier transform of the signal,  $X(j\omega)$ .

(b)



In the above modulation/filtering system, assume that the input signal  $x(t)$  has a bandlimited Fourier transform,  $X(j\omega)$ , as depicted in Figure A below.

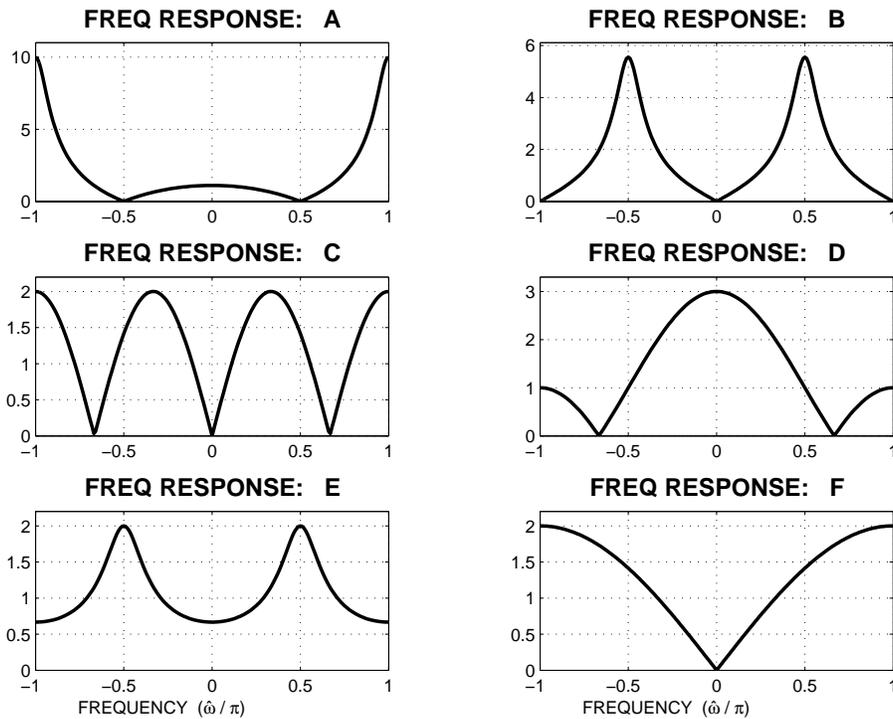


The frequency response of the lowpass filter is

$$H(j\omega) = \begin{cases} 2 & |\omega| \leq 200\pi \\ 0 & |\omega| > 200\pi \end{cases}$$

Draw  $Y(j\omega)$ .

**Problem FALL-01-F.7:**



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an  $H(z)$  or a difference equation) matches the frequency response (magnitude only). NOTE: the frequency axis is **normalized**; it is  $\hat{\omega}/\pi$ .

$$\mathcal{S}_1 : y[n] = x[n] - x[n - 1]$$

$$\mathcal{S}_2 : y[n] = -0.8y[n - 1] + x[n] + x[n - 2]$$

$$\mathcal{S}_3 : y[n] = -0.5y[n - 2] + x[n - 1]$$

$$\mathcal{S}_4 : y[n] = 0.8y[n - 1] + 0.5x[n]$$

$$\mathcal{S}_5 : H(z) = \frac{1 - z^{-2}}{1 + 0.64z^{-2}}$$

$$\mathcal{S}_6 : H(z) = 1 + z^{-1} + z^{-2}$$

$$\mathcal{S}_7 : H(z) = \frac{1 + z^{-1}}{1 - 0.9z^{-1}}$$

$$\mathcal{S}_8 : H(z) = z^{-1} - z^{-4}$$

Mark your answers in the following table:

| FREQUENCY RESPONSE | SYSTEM ( $\mathcal{S}_\#$ ) | FREQUENCY RESPONSE | SYSTEM ( $\mathcal{S}_\#$ ) |
|--------------------|-----------------------------|--------------------|-----------------------------|
| A                  |                             | B                  |                             |
| C                  |                             | D                  |                             |
| E                  |                             | F                  |                             |