Problem fall-99-Q.1.1:

For each of the following signals, pick one of the representations below that defines *exactly* the same signal. Write your answer ((a), (b), (c), (d) or (e)) in the box next to each signal.

$\text{ANS} = \boxed{}$ $\qquad e^{j\pi/3}e^{j100\pi t} + e^{-j\pi/3}e^{-j100\pi t}$

$\text{ANS} = \boxed{}$ $\qquad e^{j2\pi/3}e^{j100\pi t} + e^{-j2\pi/3}e^{-j100\pi t}$

$\text{ANS} = \boxed{}$ $\qquad -2\cos(100\pi t + 2\pi/3)$

$\text{ANS} = \boxed{}$ $\qquad 2\cos(100\pi t + 2\pi/3)$

$\text{ANS} = \boxed{}$ $\qquad 2\cos(100\pi t + 4\pi/3)$

POSSIBLE ANSWERS:

Your answer will be one of the following choices. **Please note that you may not need to use all of the following signals. If that is the case, then one or more of the following signals will be used more than one time to match the above signals.**

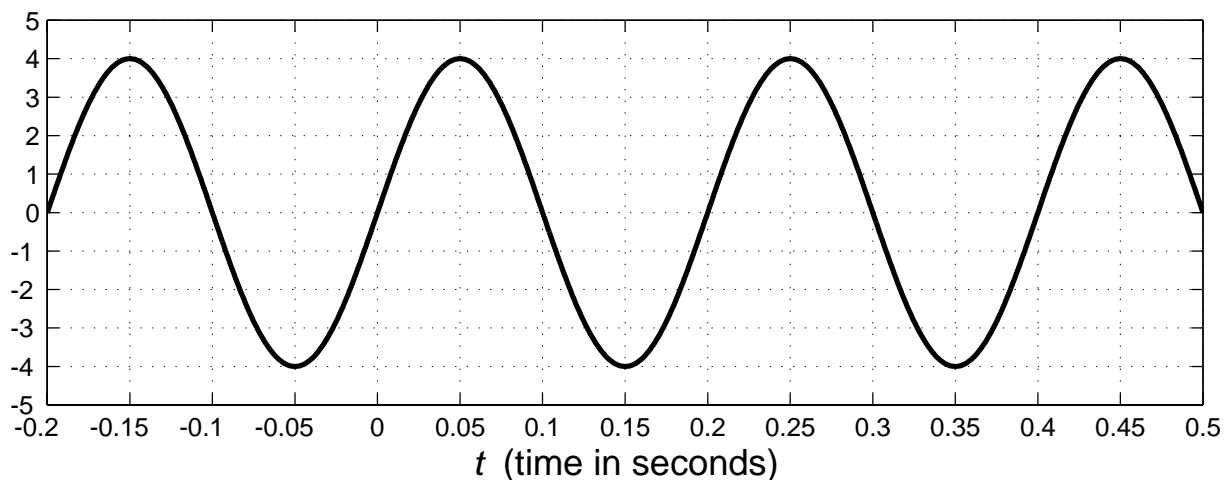(a) $x_a(t) = \cos(100\pi t - \pi/3)$

(b) $x_b(t) = 2\cos(100\pi t + \pi/3)$

(c) $x_c(t) = \Re\left\{2e^{-j2\pi/3}e^{j100\pi t}\right\}$

(d) $x_d(t) = \Re\left\{(-1 + j\sqrt{3})e^{j100\pi t}\right\}$

(e) $x_e(t) = 2\cos(100\pi t - \pi/3)$

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Problem fall-99-Q.1.2:

Sinusoidal Signal $x(t) = A\cos(\omega_0 t + \phi)$

The above graph is a plot of a sinusoidal signal $x(t) = A\cos(\omega_0 t + \phi)$.

(a) Determine numerical values for A, ω_0 and ϕ with $-\pi < \phi \leq \pi$.

(b) By a suitable choice of delay t_d, we can shift $x(t)$ to obtain the new signal

$$y(t) = x(t - t_d) = A\cos(\omega_0 t) \tag{1}$$

There are an infinite number of values of t_d that satisfy Equation (1). Give an equation for these values. *If you cannot write the general expression, give at least* **two** *different values of* t_d.

Problem fall-99-Q.1.3:

Define $x(t)$ as
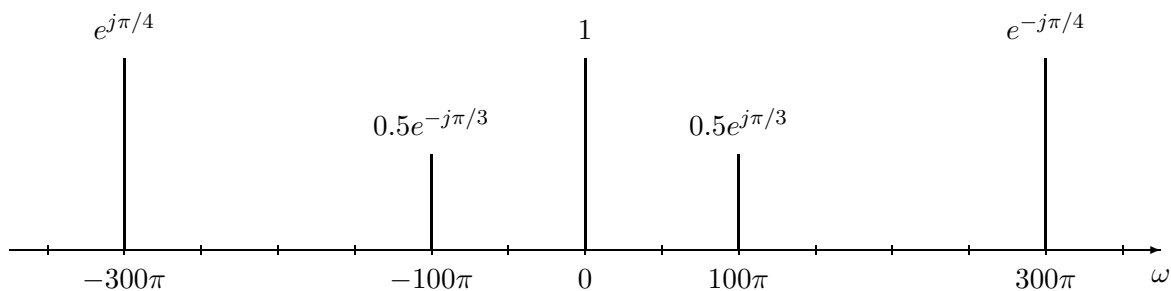
$$x(t) = 10\cos(20\pi t - \pi/6) + 10\sqrt{3}\cos(20\pi(t + 1/30))$$

(a) Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ, as well as ω_0.

(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).

Problem fall-99-Q.1.4:

The spectrum of a signal $x(t)$ is shown in the following figure:

$$e^{j\pi/4} \qquad\qquad 1 \qquad\qquad e^{-j\pi/4}$$

$$0.5e^{-j\pi/3} \qquad\qquad 0.5e^{j\pi/3}$$

$$-300\pi \qquad\qquad -100\pi \qquad 0 \qquad 100\pi \qquad\qquad 300\pi \qquad \omega$$

Note carefully that the frequency axis is radian frequency (ω) *not* cyclic frequency (f).

(a) Write an equation for $x(t)$ in terms of cosine functions.

(b) Is $x(t)$ periodic? **You must explain this answer. Why or why not?**
If it is periodic, what is the fundamental frequency and corresponding period of $x(t)$?

(c) A new signal is defined as $y(t) = \cos(\beta t + \pi) + x(t)$. It is known that $y(t)$ is periodic with period $T_0 = 0.04$ sec. Determine **two** positive values for the frequency β that will satisfy this condition.

(d) Using either of the frequencies β found in (c), modify the spectrum plot above so that it becomes the spectrum of $y(t)$.

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Quiz #1B   - September 20, 1999 ver. 2

Q.1   (See Quiz 1A).

1. . . . . .   **b**
2 . . . . . .   d
3. . . . . .   e
4. . . . . .   d
5   c

Q.2. a) $A = 4$

$T_0 = 0.2 \text{sec} \Rightarrow f_0 = 5 Hz \Rightarrow \omega_0 = 10\pi \text{ Rad/sec}.$

$\phi = -2\pi f \, t_{peak} = -10\pi (0.05) = -\frac{\pi}{2}.$

b)
$$y(t) = x(t - t_d) = A \cos\left(\omega_0 (t - t_d) + \phi\right)$$
$$= A \cos\left(\omega_0 t + \phi - \omega_0 t_d\right)$$
$$= A \cos(\omega_0 t)$$

$\Rightarrow \quad \phi - \omega_0 t_d = (2\pi) \cdot k \qquad k \in I.$

$$t_d = \frac{\phi - 2\pi k}{\omega_0} = \frac{-\frac{\pi}{2} - 2\pi k}{10\pi}$$
$$= \frac{-\frac{1}{2} - 2k}{10}, \quad k \in I.$$

8.3. $x(t) = 10\cos\left(20\pi t - \pi/6\right) + 10\sqrt{3}\,\cos\left(20\pi(t + 1/30)\right)$

a)

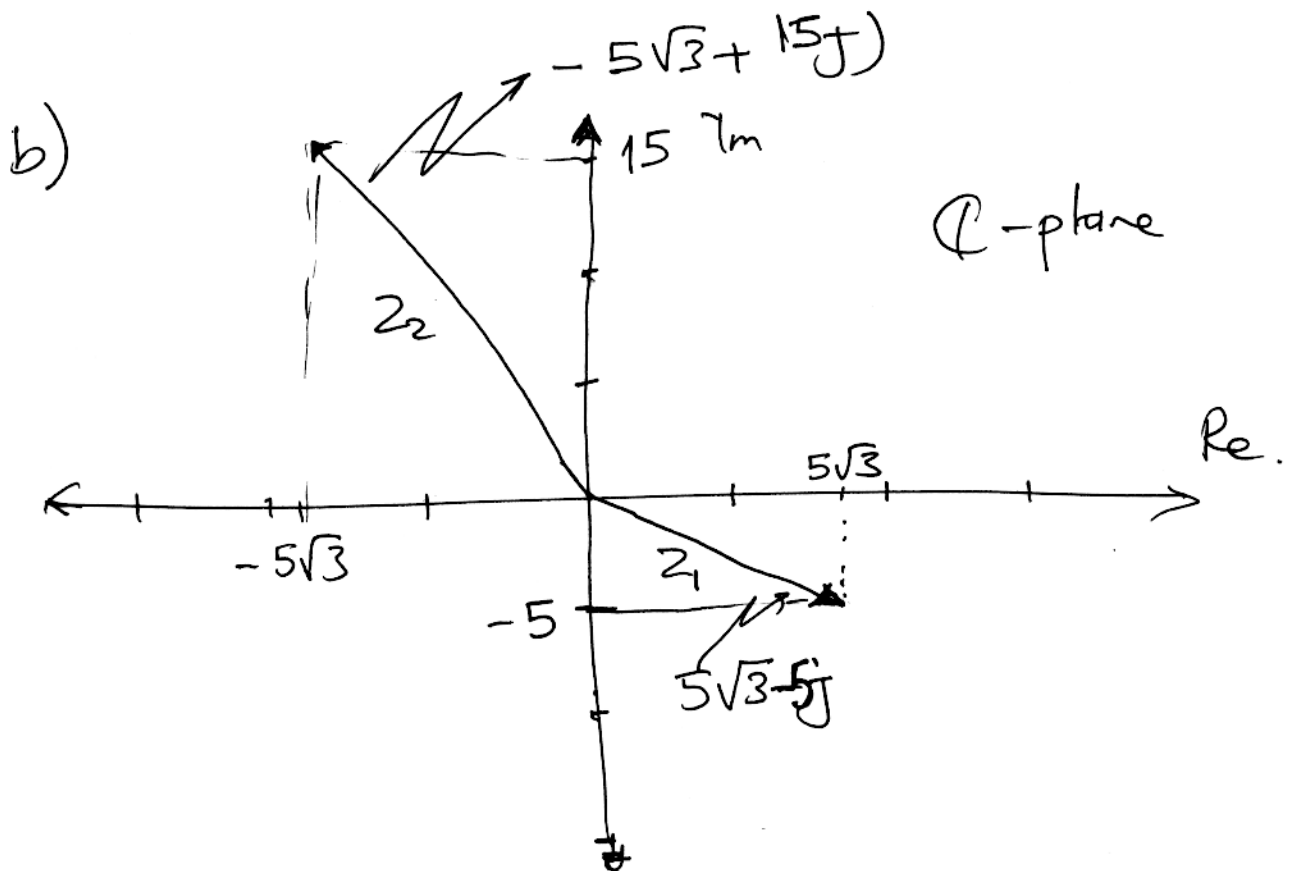$$= 10\cos\left(20\pi t - \pi/6\right) + 10\sqrt{3}\,\cos\left(20\pi t + \tfrac{2\pi}{3}\right)$$

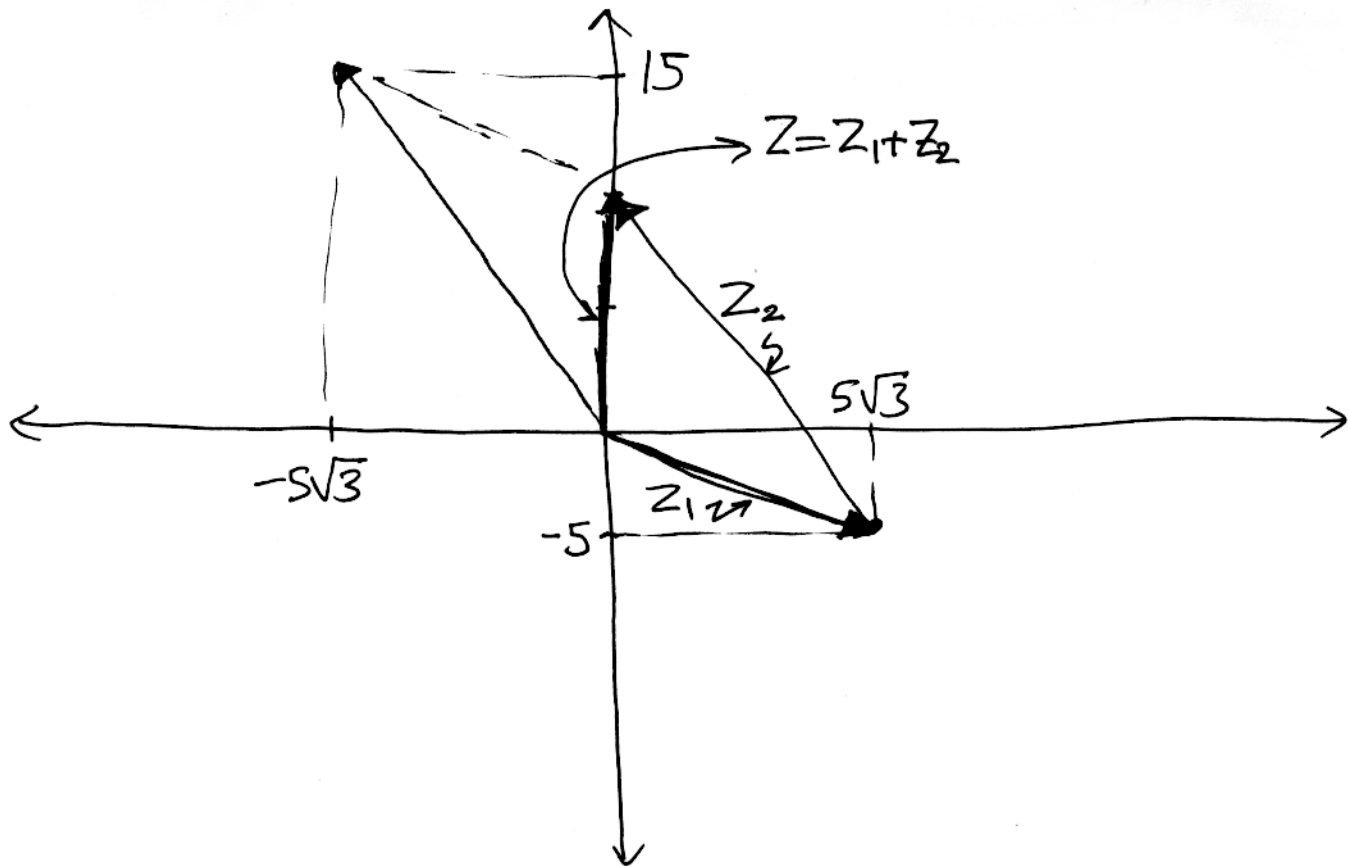$$Z_1 = 10.e^{-j\pi/6}, \qquad Z_2 = 10\sqrt{3}.\,e^{j\,2\pi/3}$$

$$Z_1 = 10.\left(\tfrac{\sqrt{3}}{2} - \tfrac{1}{2}j\right); \qquad Z_2 = 10\sqrt{3}.\left(-\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}j\right)$$

$$= 5\sqrt{3} - 5j, \qquad\qquad Z_2 = -5\sqrt{3} + 15j$$

$$Z = Z_1 + Z_2 = 10j = 10.e^{j\pi/2}$$

$$x(t) = 10\cos\left(20\pi t + \pi/2\right)$$

b)

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$Z = Z_1 + Z_2$

Z_2

$5\sqrt{3}$

$-5\sqrt{3}$

Z_1

-5

8.4. a)

$$x(t) = 1 + \cos(100\pi t + \pi/3) + 2\cos(300\pi t + (-\pi/4))$$

b.) Periodic. Harmonically related freq.

$$f_1 = \frac{100\pi}{2\pi} = 50$$

$$f_2 = \frac{300\pi}{2\pi} = 150$$

$$f_0 = \gcd(f_1, f_2) = 50. \Rightarrow T_0 = \frac{1}{f_0} = \frac{1}{50} \text{ sec.}$$
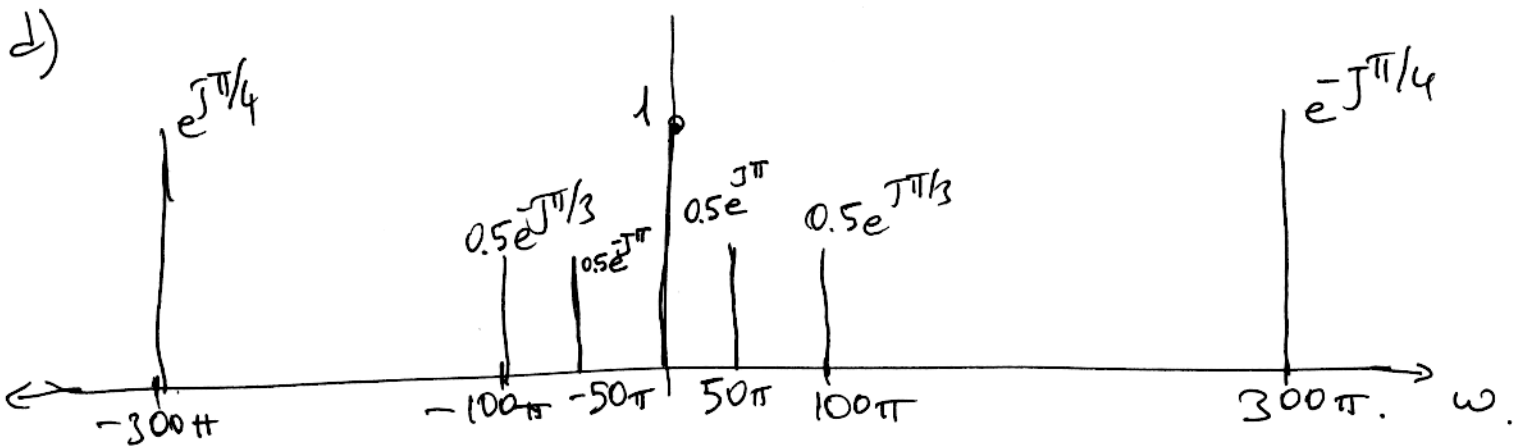
$$= 0.02 \text{ sec.}$$

c. $y(t) = \cos(\beta t + \pi) + x(t)$

$T_0 = 0.04 \Rightarrow f_0 = \dfrac{1}{0.04} = 25.$

$f_1 = 50$
$f_2 = 150$ $\Rightarrow$ $25 = \gcd\left(50, 150, \dfrac{\beta}{2\pi}\right)$
$f_3 = \beta/2\pi$

$\Rightarrow$ $\dfrac{\beta_1}{2\pi} = 25 \Rightarrow \beta_1 = 50\pi.$

$\dfrac{\beta_2}{2\pi} = 75 \Rightarrow \beta_2 = 150\pi.$

d)

Note: β can be any odd multiple of 50π.