12.1

a) $$x(t) = \sum_{n=-\infty}^{\infty} \pi \, \delta\left(t - \frac{n}{10}\right)$$

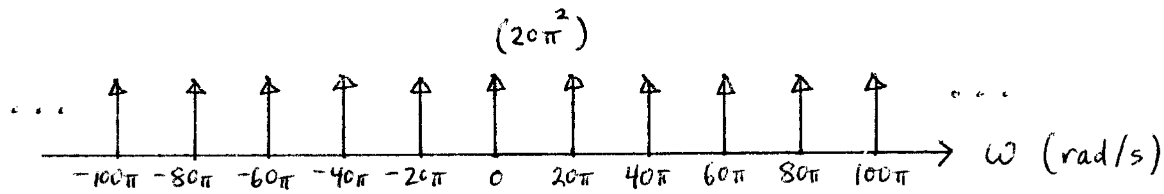
$$X(j\omega) = \mathcal{F}\left\{ \sum_{n=-\infty}^{\infty} \pi \, \delta\left(t - \frac{n}{10}\right) \right\}$$

$$= \pi \, \mathcal{F}\left\{ \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{n}{10}\right) \right\}$$

$$= \pi \, \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi}{T} k\right), \quad T = \frac{1}{10}$$

$$= 20\pi^2 \sum_{k=-\infty}^{\infty} \delta(\omega - k20\pi)$$

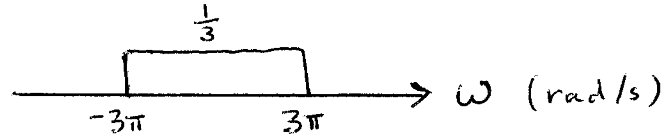
$X(j\omega)$:



b) $$H(j\omega) = \mathcal{F}\left\{ \frac{\sin(3\pi t)\cos(80\pi t)}{3\pi t} \right\}$$

in order to use modulation property, first compute

$$\mathcal{F}\left\{ \frac{\sin(3\pi t)}{3\pi t} \right\} = \frac{1}{3} \mathcal{F}\left\{ \frac{\sin(3\pi t)}{\pi t} \right\} = \frac{1}{3}\left\{ u(\omega + 3\pi) - u(\omega - 3\pi) \right\}$$

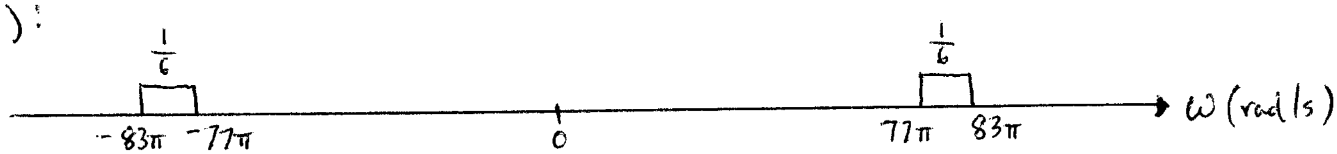


$$\Rightarrow H(j\omega) = \frac{1}{2} \cdot \frac{1}{3} \left\{ u(\omega - 80\pi + 3\pi) - u(\omega - 80\pi - 3\pi) \right\}$$

$$+ \frac{1}{2} \cdot \frac{1}{3} \left\{ u(\omega + 80\pi + 3\pi) - u(\omega + 80\pi - 3\pi) \right\}$$

$$= \frac{1}{6} \left\{ u(\omega - 77\pi) - u(\omega - 83\pi) \right\} + \frac{1}{6} \left\{ u(\omega + 83\pi) - u(\omega + 77\pi) \right\}$$

$H(j\omega)$:



c) $Y(j\omega) = H(j\omega)X(j\omega)$

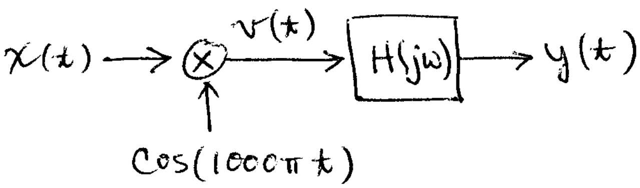
$Y(j\omega)$:

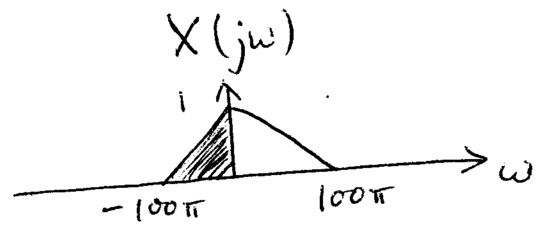
$$Y(j\omega) = \frac{10\pi^2}{3}\left(\delta(\omega+80\pi) + \delta(\omega-80\pi)\right)$$

$$y(t) = \mathcal{F}^{-1}\left\{\frac{10\pi}{3}\left(\pi\,\delta(\omega+80\pi) + \pi\,\delta(\omega-80\pi)\right)\right\}$$

$$= \frac{10\pi}{3}\cos(80\pi t), \qquad -\infty < t < \infty$$

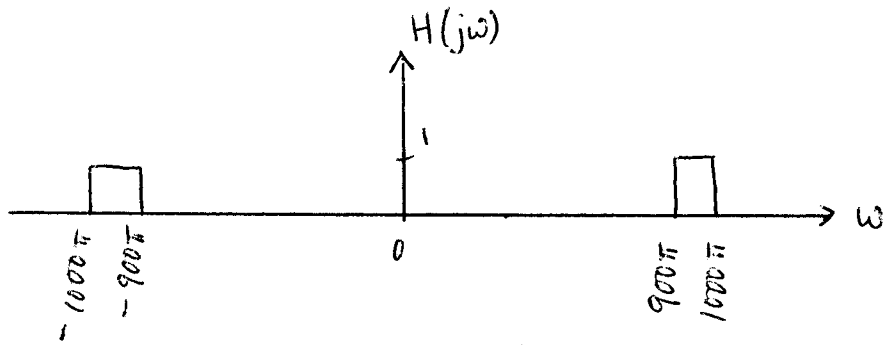
12.2





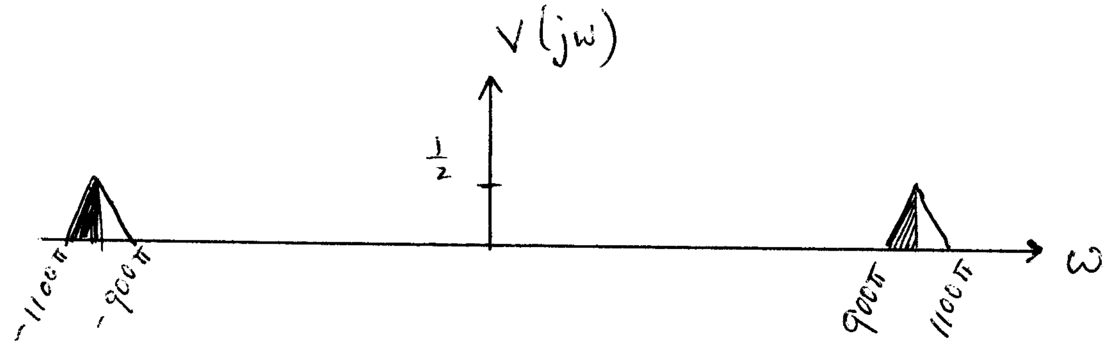
(a) $H(j\omega) = \begin{cases} 1, & 900\pi < |\omega| < 1000\pi \\ 0, & \text{otherwise} \end{cases}$

$|\omega| = \begin{cases} \omega, & \omega \geq 0 \\ -\omega, & \omega < 0 \end{cases} \Rightarrow H(j\omega) = 1$ if $\begin{cases} 900\pi < \omega < 1000\pi \\ \text{and} \\ -1000\pi < \omega < -900\pi \end{cases}$

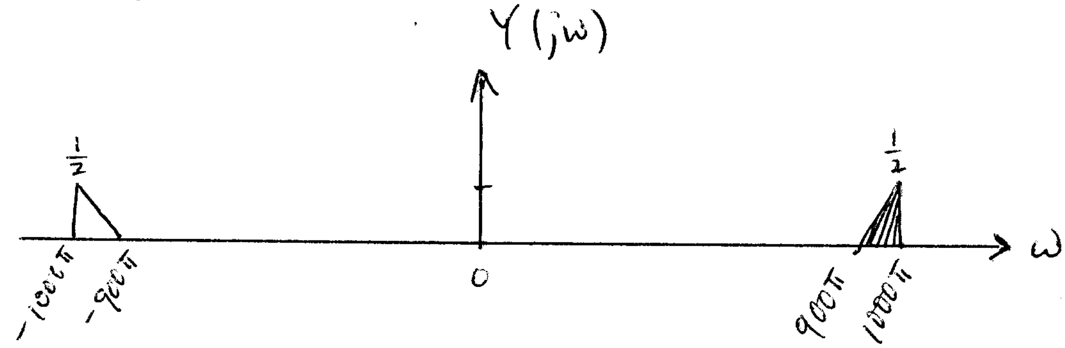


(b) $V(j\omega) = \mathcal{F}\{x(t)\cos(1000\pi t)\}$

$$= \tfrac{1}{2}X\left(j(\omega-1000\pi)\right) + \tfrac{1}{2}X\left(j(\omega+1000\pi)\right)$$



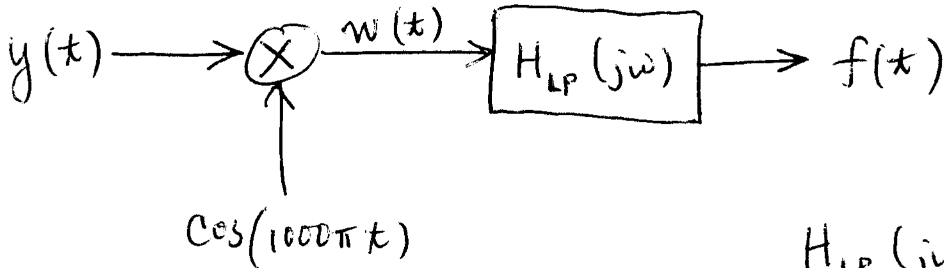
(c) $Y(j\omega) = H(j\omega)V(j\omega)$

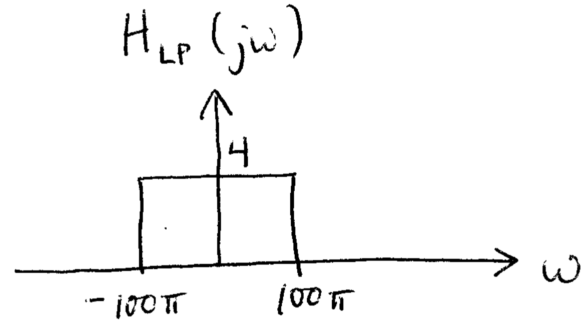


(d) The baseband spectrum $X(j\omega)$ centered at $\omega = 0$ has a negative-frequency sideband (shaded) and a positive-frequency sideband (unshaded). The modulator created a double sideband representation, but the bandpass filter removed the two superfluous sidebands. Hence, the spectral width occupied by $Y(j\omega)$ matches that occupied by $X(j\omega)$. We call $V(j\omega)$ a double-sideband ~~single~~ signal and $Y(j\omega)$ a single-sideband signal.

12.3

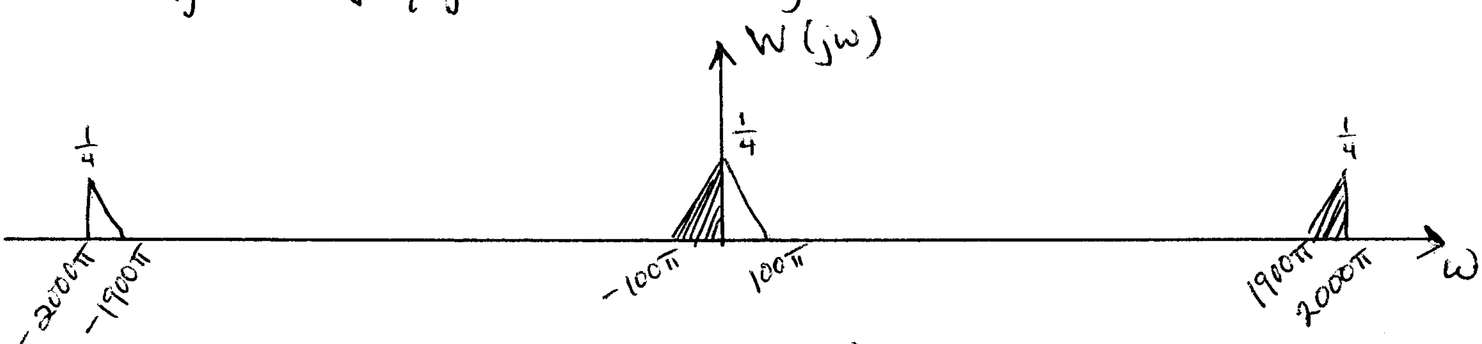
The purpose of this problem is to extend the previous problem by showing how $x(t)$ can be recovered from $y(t)$ by a demodulation process.

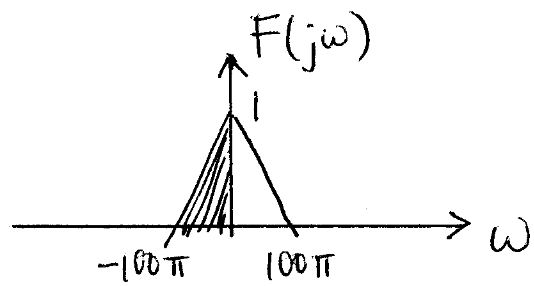




The above de-modulation process results in $f(t) = x(t)$. We prove this by establishing that $F(j\omega) = X(j\omega)$.

$$W(j\omega) = \mathcal{F}\{y(t)\cos(1000\pi t)\}$$





$$
\begin{aligned}
f(t) &= \mathcal{F}^{-1}\{F(j\omega)\} \\
&= \mathcal{F}^{-1}\{X(j\omega)\} \\
&= x(t)
\end{aligned}
$$

12.4

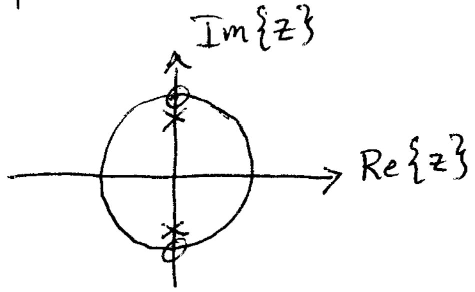
a) $y[n] = -0.9\,y[n-2] + x[n] + x[n-2]$

$$Y(z)(1+0.9z^{-2}) = X(z)(1+z^{-2})$$

$$H(z) = \frac{1+z^{-2}}{1+0.9z^{-2}} = \frac{z^2+1}{z^2+0.9}$$

zeros @ roots of $z^2+1 \Rightarrow z^2 = -1 = j^2 \Rightarrow z = \pm j$

poles @ roots of $z^2+0.9 \Rightarrow z^2 = -0.9 = 0.9j^2 \Rightarrow z = \pm j\sqrt{0.9}$



This IIR filter is stable, and it provides nulling at $\hat{\omega} = \pm\frac{\pi}{2}$ or, since $f_s = 1000$ samples/s, at $\omega = \hat{\omega} f_s = \pm 500\pi$ rad/s.

$$H_{eff}(j\omega) = H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\omega/f_s}$$

$$= \frac{1+e^{-j2\hat{\omega}}}{1+0.9e^{-j2\hat{\omega}}}\Bigg|_{\hat{\omega}=\omega/f_s}$$

$$= \frac{1+e^{-j\omega/500}}{1+0.9e^{-j\omega/500}}$$

If $\omega = \pm 500\pi$ rad/s, then $1+e^{-j\omega/500} = 1+e^{\mp j\pi} = 1-1 = 0$ so that $H_{eff}(j\omega) = 0$ as well.

b) The only requirement for $Y(j\omega) = H_{eff}(j\omega)X(j\omega)$ to be a valid expression is that aliasing must not occur. Since ω_{max} for the given $X(j\omega)$ is $\omega_{max} = 500\pi$ rad/s, aliasing will not occur provided that $\omega_s \geq 2\omega_{max}$,

$2\pi f_s \geq 2(500\pi) \Rightarrow f_s \geq 500$ samples/s.

c) In this case, we want $y(t) = x(t)$, so this requires

i) no aliasing
ii) an ideal low-pass digital filter
iii) a sufficiently narrow spectrum for $x(t)$

From the given $X(j\omega)$, we see that $\omega_{max} = 500\pi$ rad/s and aliasing will be avoided if $f_s \geq 500$ samples/s. From the given $H(e^{j\hat{\omega}})$, we see that the digital filter has an ideal low-pass characteristic with passband $-\frac{\pi}{2} < \hat{\omega} < \frac{\pi}{2}$ or, using $\hat{\omega} = \omega/f_s$, $-\frac{\pi}{2} f_s < \omega < \frac{\pi}{2} f_s$. If the sampled input signal is to pass through the digital filter unaltered, then this requires $\frac{\pi}{2} f_s \geq \omega_{max}$ or $\frac{\pi}{2} f_s \geq 500\pi$ which implies $f_s \geq 1000$ samples/s. So condition iii) is the limiting condition, and thus $f_s \geq 1000$ samples/s.

<u>12.5</u>

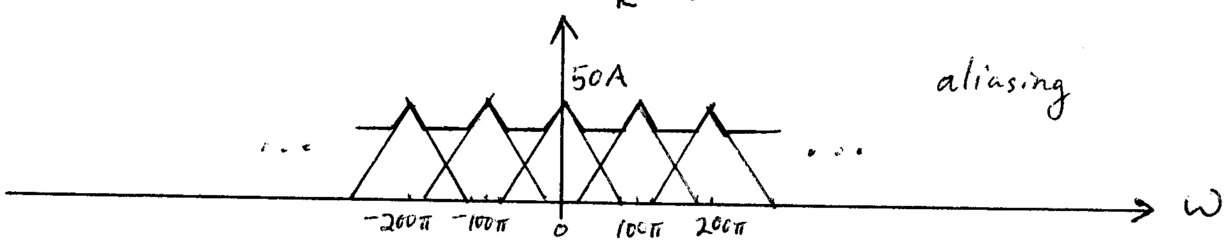
a) $$x_s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t - nT_s)$$

$$\begin{aligned}
X_s(j\omega) &= \mathcal{F}\left\{ \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t - nT_s) \right\} \\
&= \sum_{n=-\infty}^{\infty} x(nT_s)\, \mathcal{F}\{ \delta(t - nT_s) \} \\
&= \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j\omega n T_s} \\
&= \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\hat{\omega} n}, \qquad \hat{\omega} = \omega T_s \\
&= X(e^{j\hat{\omega}})
\end{aligned}$$

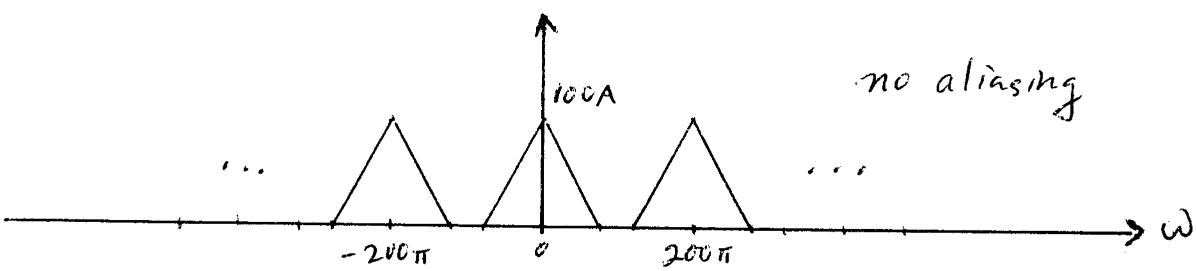
So the CTFT of $x_s(t)$ is equivalent to the DTFT of $x[n]$. Interesting, but not relevant to the problem at hand, since $x(t)$ has been characterized by a given $X(jw)$. For this problem, it's preferable to proceed as follows. According to (12.43) in the text,

$$X_s(jw) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(w - kw_s))$$

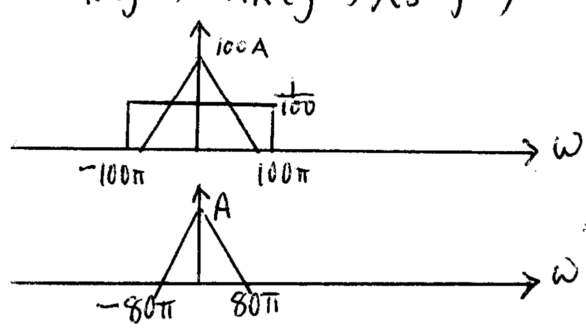
(a) $w_s = 100\pi$, $T_s = \frac{1}{50}$ $\Rightarrow$ $X_s(jw) = 50 \sum_{k=-\infty}^{\infty} X(j(w - k100\pi))$



(b) $w_s = 200\pi$, $T_s = \frac{1}{100}$ $\Rightarrow$ $X_s(jw) = 100 \sum_{k=-\infty}^{\infty} X(j(w - k200\pi))$



(c) using sampling frequency from part (b) $\Rightarrow$ $X_R(jw) = H_R(jw) X_s(jw)$



$\Rightarrow$ $X_R(jw) = X(jw)$

$x_r(t) = x(t)$

12.6

$$
\begin{aligned}
x(t) &= -100t\, e^{-25t^2} \\
&= -100t\, e^{-(5t)^2} \\
&= 2 \frac{d}{dt}\left\{ e^{-(5t)^2} \right\}
\end{aligned}
$$

$$
\begin{aligned}
X(j\omega) &= \mathcal{F}\left\{ 2 \frac{d}{dt}\left\{ e^{-(5t)^2} \right\} \right\} \\
&= 2\, \mathcal{F}\left\{ \frac{d}{dt}\left\{ e^{-(5t)^2} \right\} \right\} \quad \text{, linearity} \\
&= 2j\omega\, \mathcal{F}\left\{ e^{-(5t)^2} \right\}, \quad \text{differentiation} \\
&= 2j\omega \left\{ \frac{1}{5} \mathcal{F}\left\{ e^{-t^2} \right\} \Big|_{\omega \leftarrow \frac{\omega}{5}} \right\}, \quad \text{scaling} \\
&= 2j\omega \frac{1}{5} \left\{ \sqrt{\pi}\, e^{-\omega^2/4} \Big|_{\omega \leftarrow \frac{\omega}{5}} \right\}, \quad \text{Gaussian} \\
&= \frac{2\sqrt{\pi}}{5} j\omega\, e^{-\left(\frac{\omega}{5}\right)^2/4} \\
&= \frac{2\sqrt{\pi}}{5} j\omega\, e^{-\frac{\omega^2}{100}}
\end{aligned}
$$