ECE 2025 Spring 2006 Hw 8

8.1

(a) $y[n] = x[n] - 3x[n-1] + x[n-2]$

$h[n] = y[n]\big|_{x[n]=\delta[n]} = \delta[n] - 3\delta[n-1] + \delta[n-2]$

$H(z) = \mathcal{Z}\{h[n]\} = 1 - 3z^{-1} + z^{-2}$

$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} = 1 - 3e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

(b) $h[n] = \frac{1}{7}\left(\delta[n-3] - \delta[n-10]\right)$

$y[n] = \frac{1}{7}\left(x[n-3] - x[n-10]\right)$

$H(z) = \frac{1}{7}\left(z^{-3} - z^{-10}\right)$

$H(e^{j\hat{\omega}}) = \frac{1}{7}\left(e^{-j3\hat{\omega}} - e^{-j10\hat{\omega}}\right)$

(c) $H(e^{j\hat{\omega}}) = j\sin(2.5\hat{\omega})\,e^{-j2.5\hat{\omega}}$

$= j\left(\dfrac{e^{j2.5\hat{\omega}} - e^{-j2.5\hat{\omega}}}{2j}\right)e^{-j2.5\hat{\omega}}$

$= \frac{1}{2}\left(1 - e^{-j5\hat{\omega}}\right)$

$H(z) = \frac{1}{2}\left(1 - z^{-5}\right)$

$h[n] = \frac{1}{2}\left(\delta[n] - \delta[n-5]\right)$

$y[n] = \frac{1}{2}\left(x[n] - x[n-5]\right)$

8.2

(a) $H(z) = \frac{4}{z^3} = 4z^{-3}$

$$H(e^{j\hat{\omega}}) = 4e^{-j3\hat{\omega}}$$

$$h[n] = 4\delta[n-3]$$

$$y[n] = 4x[n-3]$$

(b) $H(z) = (1+z^{-1})(1-e^{j\frac{\pi}{4}}z^{-1})(1-e^{-j\frac{\pi}{4}}z^{-1})$

$$= (1+z^{-1})(1-(e^{j\frac{\pi}{4}}+e^{-j\frac{\pi}{4}})z^{-1}+z^{-2})$$

$$= (1+z^{-1})(1-2\cos\tfrac{\pi}{4}z^{-1}+z^{-2})$$

$$= 1-\frac{2}{\sqrt{2}}z^{-1}+z^{-2}+z^{-1}-\frac{2}{\sqrt{2}}z^{-2}+z^{-3}$$

$$= 1+(1-\sqrt{2})z^{-1}+(1-\sqrt{2})z^{-2}+z^{-3}$$

$$H(e^{j\hat{\omega}}) = 1+(1-\sqrt{2})e^{-j\hat{\omega}}+(1-\sqrt{2})e^{-j2\hat{\omega}}+e^{-j3\hat{\omega}}$$

$$h[n] = \delta[n]+(1-\sqrt{2})\delta[n-1]+(1-\sqrt{2})\delta[n-2]+\delta[n-3]$$

$$y[n] = x[n]+(1-\sqrt{2})x[n-1]+(1-\sqrt{2})x[n-2]+x[n-3]$$

(c) $H(z) = \frac{1+z^{-11}}{1+z^{-1}} = \frac{z^{11}+1}{z^{11}+z^{10}} = \frac{z^{11}+1}{z^{10}(z+1)}$

$$z^{11}+1=0 \Rightarrow z^{11}=-1$$

$$(re^{j\theta})^{11} = e^{j\pi}e^{j2\pi\ell}$$

$$r^{11}=1 \Rightarrow r=1$$

$$11\theta = \pi+2\pi\ell \Rightarrow \theta = \frac{\pi+2\pi\ell}{11}, \quad \ell=0,\dots,10$$

$$z^{11}+1=\left(z-e^{j\frac{\pi}{11}}\right)\left(z-e^{j\frac{3\pi}{11}}\right)\left(z-e^{j\frac{5\pi}{11}}\right)\cdots\left(z-e^{j\frac{21\pi}{11}}\right)$$

Note: the factor $\left(z-e^{j\frac{11\pi}{11}}\right)=\left(z-e^{j\pi}\right)=(z+1)$ cancels out!

$$\Rightarrow H(z)=\frac{\left(z-e^{j\frac{\pi}{11}}\right)\cdots\left(z-e^{j\frac{9\pi}{11}}\right)\left(z-e^{j\frac{13\pi}{11}}\right)\cdots\left(z-e^{j\frac{21\pi}{11}}\right)}{z^{10}}$$

$$=\left(1-e^{j\frac{\pi}{11}}z^{-1}\right)\cdots\left(1-e^{j\frac{9\pi}{11}}z^{-1}\right)\left(1-e^{j\frac{13\pi}{11}}z^{-1}\right)\cdots\left(1-e^{j\frac{21\pi}{11}}z^{-1}\right)$$

another perspective:

$$
\begin{array}{r|l}
 & 1-z^{-1}+z^{-2}-z^{-3}\quad\cdots\quad+z^{-10} \\
\hline
1+z^{-1} & 1+0z^{-1}+0z^{-2}+0z^{-3}+0z^{-4}+0z^{-5}+0z^{-6}+0z^{-7}+0z^{-8}+0z^{-9}+0z^{-10}+z^{-11} \\
 & \underline{1+z^{-1}} \\
 & \quad -z^{-1} \\
 & \quad \underline{-z^{-1}-z^{-2}} \\
 & \qquad z^{-2} \\
 & \qquad \underline{z^{-2}+z^{-3}} \\
 & \qquad\quad -z^{-3} \\
 & \qquad\quad \underline{-z^{-3}-z^{-4}} \\
 & \qquad\qquad z^{-4} \\
 & \qquad\qquad \vdots
\end{array}
$$

$$\Rightarrow \underline{H(z)=1-z^{-1}+z^{-2}-z^{-3}+\cdots+z^{-10}}$$

double check: $(1+z^{-1})H(z)\stackrel{?}{=}1+z^{-11}$

$$
\begin{array}{l}
1-z^{-1}+z^{-2}-z^{-3}+\cdots-z^{-9}+z^{-10} \\
\quad+z^{-1}-z^{-2}+z^{-3}-\cdots+z^{-9}-z^{-10}+z^{-11}=1+z^{-11}\ \checkmark
\end{array}
$$

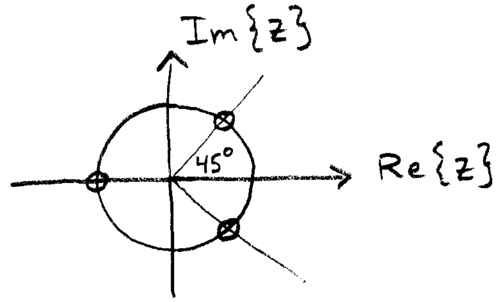
$$\therefore\left.\begin{array}{l}
H(e^{j\hat{\omega}})=1-e^{-j\hat{\omega}}+e^{-j2\hat{\omega}}-e^{-j3\hat{\omega}}+\cdots+e^{-j10\hat{\omega}} \\
h[n]=\delta[n]-\delta[n-1]+\delta[n-2]-\delta[n-3]+\cdots+\delta[n-10] \\
y[n]=x[n]-x[n-1]+x[n-2]-x[n-3]+\cdots+x[n-10]
\end{array}\right\}*$$

8.3 $\quad H(z) = (1+z^{-1})(1-e^{j\frac{\pi}{4}}z^{-1})(1-e^{-j\frac{\pi}{4}}z^{-1})$ Same as in 8.2 (b)

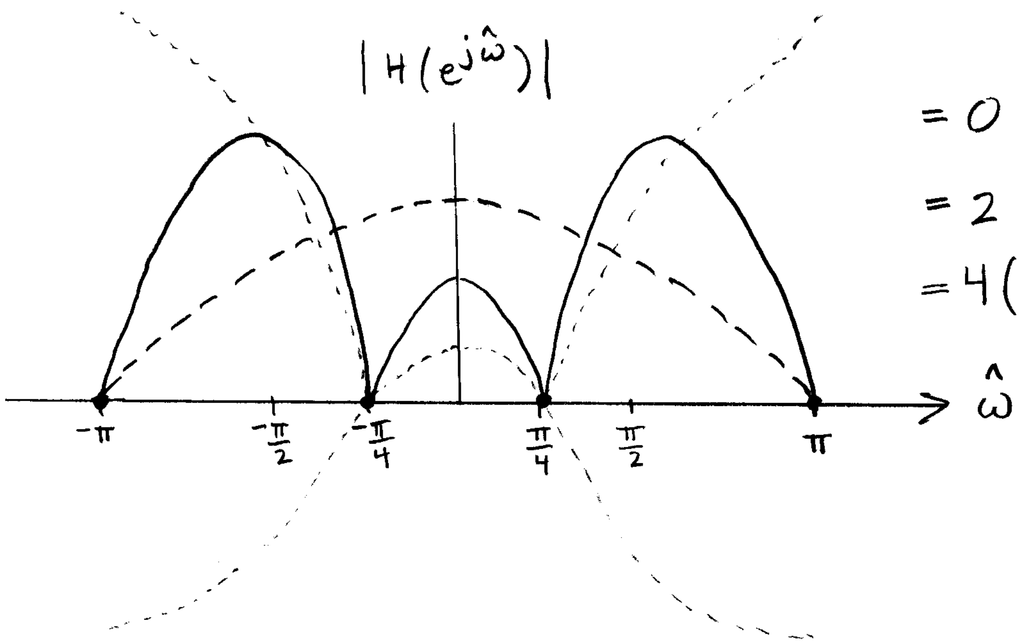
(a) $\quad 1+z^{-1} = 0 \Rightarrow z = -1$

$1-e^{j\frac{\pi}{4}}z^{-1} = 0 \Rightarrow z = e^{j\frac{\pi}{4}}$

$1-e^{-j\frac{\pi}{4}}z^{-1} = 0 \Rightarrow z = e^{-j\frac{\pi}{4}}$



(b)
$$|H(e^{j\hat{\omega}})| = \left|1+e^{-j\hat{\omega}}\right| \cdot \left|1-\sqrt{2}\,e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right|$$
$$= \left|e^{-j\frac{\hat{\omega}}{2}}\left(e^{j\frac{\hat{\omega}}{2}} + e^{-j\frac{\hat{\omega}}{2}}\right)\right| \cdot \left|e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} - \sqrt{2} + e^{-j\hat{\omega}}\right)\right|$$
$$= \left|2\cos\left(\frac{\hat{\omega}}{2}\right)\right| \cdot \left|2\cos\hat{\omega} - 2\cos\frac{\pi}{4}\right|$$



$= 0 \;@\; \hat{\omega} = \pm\frac{\pi}{4}, \pm\pi$

$= 2 \;@\; \hat{\omega} = \pm\frac{\pi}{2}$

$= 4\left(1-\frac{1}{\sqrt{2}}\right) @\; \hat{\omega} = 0$

(c) $\quad x[n] = 3\cos\left(\frac{\pi}{2}n - \frac{\pi}{10}\right) \Rightarrow \hat{\omega} = \frac{\pi}{2}$

$H(e^{j\hat{\omega}}) = 2\cos\left(\frac{\hat{\omega}}{2}\right)\left(2\cos\hat{\omega} - 2\cos\frac{\pi}{4}\right)e^{-j\frac{3}{2}\hat{\omega}}$

$H(e^{j\frac{\pi}{2}}) = 4\cos\left(\frac{\pi}{4}\right)\left(\cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{4}\right)\right)e^{-j\frac{3\pi}{4}} = -2e^{-j\frac{3\pi}{4}} = 2e^{j\frac{\pi}{4}}$

$y[n] = 3(2)\cos\left(\frac{\pi}{2}n - \frac{\pi}{10} + \frac{\pi}{4}\right) = 6\cos\left(\frac{\pi}{2}n + \frac{3\pi}{20}\right)$

8.4

(a) $H(z) = \frac{1}{2} z^{-2} + \frac{1}{2} z^{-6}$

$= \frac{1}{2} \frac{z^4+1}{z^6}$

Zeros of $H(z)$:

$z^4 + 1 = 0$

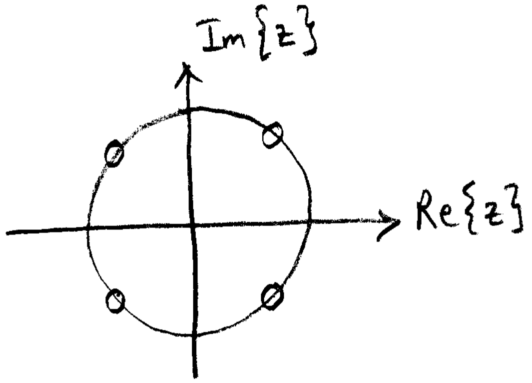
$z^4 = -1$

$(re^{j\theta})^4 = (e^{j\pi}) e^{j2\pi l}$

$r^4 = 1 \Rightarrow r = 1$

$4\theta = \pi + 2\pi l \Rightarrow \theta = \begin{cases} \pi/4 \\ 3\pi/4 \\ 5\pi/4 \\ 7\pi/4 \end{cases}$

(Zeros @ $z = e^{j\pi/4}, e^{j3\pi/4}, e^{j5\pi/4}, e^{j7\pi/4}$)



(b) $x(t) = 10\cos(6000\pi t - \frac{\pi}{4}) + 8\cos(9000\pi t - \frac{\pi}{5})$

$f_s = 8000$ samples/sec

$8000 < 2(4500) \Rightarrow$ aliasing will occur

$$x[n] = x(t)\big|_{t=n/f_s} = 10\cos\left(\frac{3\pi}{4}n - \frac{\pi}{4}\right) + 8\cos\left(\frac{9\pi}{8}n - \frac{\pi}{5}\right)$$

$$= 10\cos\left(\frac{3\pi}{4}n - \frac{\pi}{4}\right) + 8\cos\left(\left(\frac{9\pi}{8} - 2\pi\right)n - \frac{\pi}{5}\right)$$

$$= 10\cos\left(\frac{3\pi}{4}n - \frac{\pi}{4}\right) + 8\cos\left(\frac{-7\pi}{8}n - \frac{\pi}{5}\right)$$

$$= 10\cos\left(\frac{3\pi}{4}n - \frac{\pi}{4}\right) + 8\cos\left(\frac{7\pi}{8}n + \frac{\pi}{5}\right)$$

$$H(e^{j\hat{\omega}}) = \frac{1}{2}e^{-j2\hat{\omega}} + \frac{1}{2}e^{-j6\hat{\omega}}$$

$$= \frac{1}{2}e^{-j4\hat{\omega}}\left(e^{j2\hat{\omega}} + e^{-j2\hat{\omega}}\right)$$

$$= \cos(2\hat{\omega})\, e^{-j4\hat{\omega}}$$

$$H(e^{j\frac{3\pi}{4}}) = \cos\left(\frac{3\pi}{2}\right)e^{-j3\pi} = 0$$

$$H(e^{j\frac{7\pi}{8}}) = \cos\left(\frac{7\pi}{4}\right)e^{-j\frac{7\pi}{2}} = \frac{1}{\sqrt{2}}\, e^{j\frac{\pi}{2}}$$

$$y[n] = 8\left(\frac{1}{\sqrt{2}}\right)\cos\left(\frac{7\pi}{8}n + \frac{\pi}{5} + \frac{\pi}{2}\right) = 4\sqrt{2}\cos\left(\frac{7\pi}{8}n + \frac{7\pi}{10}\right)$$

$$y(t) = y[n]\Big|_{n=f_s t} = 4\sqrt{2}\cos\left(7000\pi t + \frac{7\pi}{10}\right)$$

8.5

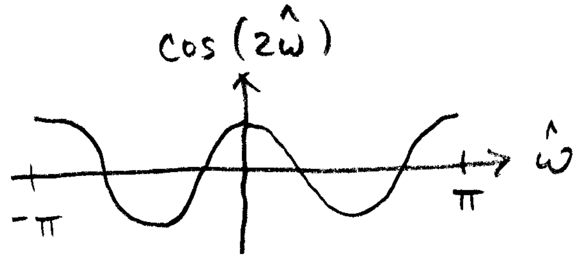
(a) $x[n] = 50\cos\left(\frac{3\pi}{2}n - \frac{\pi}{4}\right) + 40\cos\left(\frac{\pi}{4}n - \frac{\pi}{5}\right)$

$h[n] = 2\delta[n-2] + 2\delta[n-6]$

$H(z) = 2z^{-2} + 2z^{-6}$

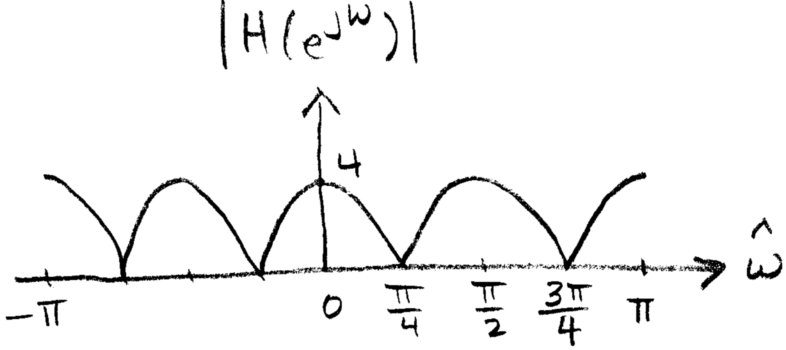
(b) $H(z) = 4\, H(z)\Big|_{\text{from } 8.4(a)}$

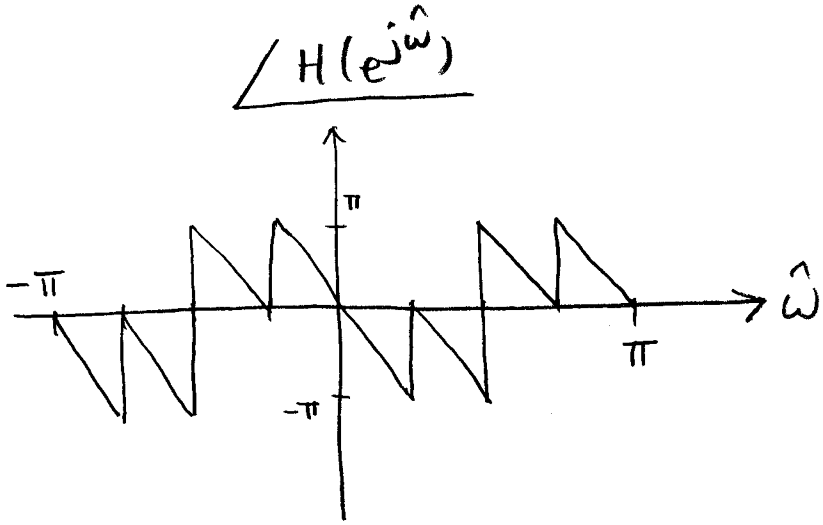
$\Rightarrow H(e^{j\hat{\omega}}) = 4\cos(2\hat{\omega})\, e^{-j4\hat{\omega}}$ from 8.4 (b)



$$|H(e^{j\hat{\omega}})| = \begin{cases} -4\cos(2\hat{\omega}), & \text{if } \frac{-3\pi}{4} < \hat{\omega} < \frac{-\pi}{4} \text{ or } \frac{\pi}{4} < \hat{\omega} < \frac{3\pi}{4} \\ 4\cos(2\hat{\omega}), & \text{elsewhere} \end{cases}$$

$$\angle H(e^{j\hat{\omega}}) = \begin{cases} \pi - 4\hat{\omega}, & \text{if } \frac{-3\pi}{4} < \hat{\omega} < \frac{-\pi}{4} \text{ or } \frac{\pi}{4} < \hat{\omega} < \frac{3\pi}{4} \\ -4\hat{\omega}, & \text{elsewhere} \end{cases}$$





(c)

$$x[n] = 50\cos\left(\tfrac{3\pi}{2}n - \tfrac{\pi}{4}\right) + 40\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{5}\right)$$

$$= 50\cos\left(\left(\tfrac{3\pi}{2} - 2\pi\right)n - \tfrac{\pi}{4}\right) + 40\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{5}\right)$$

$$= 50\cos\left(-\tfrac{\pi}{2}n - \tfrac{\pi}{4}\right) + 40\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{5}\right)$$

$$= 50\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{4}\right) + 40\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{5}\right)$$

$$H(e^{j\frac{\pi}{2}}) = 4e^{j0} \qquad H(e^{j\frac{\pi}{4}}) = 0$$

$$y[n] = 50(4)\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{4}\right)$$

$$y(t) = y[n]\Big|_{n = f_s t} = 200\cos\left(4000\pi t + \tfrac{\pi}{4}\right)$$