ZCE 2025 HW#7 Solutions

prob. 7.1

(a) $y_2[n] = y_1[n-1] + \alpha y_1[n-3]$

$\therefore \{b_k\} = \{0, 1, 0, \alpha\}$

$\underline{H_2}(e^{j\hat{\omega}}) = \sum_{k=0}^{3} b_k e^{-j\hat{\omega}\cdot k} = e^{-j\hat{\omega}} + \alpha e^{-j3\hat{\omega}}$ ※

(b) $h_1[n] = \alpha\delta[n] + \delta[n-2]$

$\therefore \{b_k\} = \{\alpha, 0, 1\}$

$H_1(e^{j\hat{\omega}}) = \alpha e^{-j\hat{\omega}\cdot 0} + e^{-j2\hat{\omega}} = \alpha + e^{-j2\hat{\omega}}$

$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}}) H_2(e^{j\hat{\omega}}) = (\alpha + e^{-j2\hat{\omega}})(e^{-j\hat{\omega}} + \alpha e^{-j3\hat{\omega}})$

$= \alpha e^{-j\hat{\omega}} + e^{-j3\hat{\omega}} + \alpha^2 e^{-j3\hat{\omega}} + \alpha e^{-j5\hat{\omega}}$

$\therefore H(e^{j\hat{\omega}}) = \alpha e^{-j\hat{\omega}} + (1+\alpha^2) e^{-j3\hat{\omega}} + \alpha e^{-j5\hat{\omega}}$ ※

(c) when $\alpha = \frac{1}{2}$

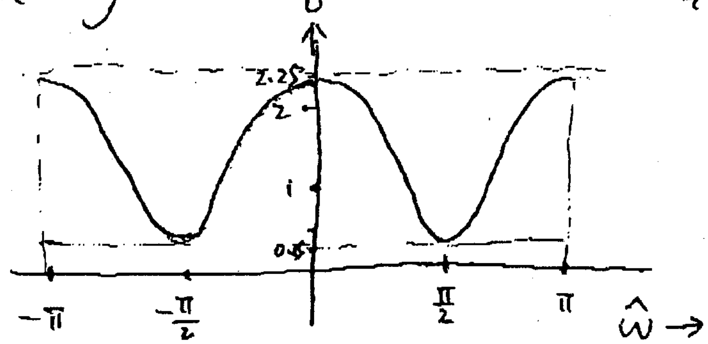
$H(e^{j\hat{\omega}}) = \frac{1}{2} e^{-j\hat{\omega}} + (1+\frac{1}{4}) e^{-j3\hat{\omega}} + \frac{1}{2} e^{-j5\hat{\omega}}$

$= \frac{5}{4} e^{-j3\hat{\omega}} (1 + \frac{2}{5} e^{j2\hat{\omega}} + \frac{2}{5} e^{-j\cdot 2\hat{\omega}})$

$= \frac{5}{4} e^{-j3\hat{\omega}} (1 + \frac{4}{5}\cos 2\hat{\omega})$

$H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}} (\frac{5}{4} + \cos 2\hat{\omega})$

The magnitude of $H(e^{j\hat{\omega}})$ is $\frac{5}{4} + \cos 2\hat{\omega}$



prob. 7.1(d)

when $\alpha = \frac{1}{2}$ and the input $x[n] = 10\cos(\frac{1}{2}\pi n + 0.25\pi)$

$\because H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}(\frac{5}{4} + \cos 2\hat{\omega})$

$H(e^{j\frac{\pi}{2}}) = e^{-j\frac{3}{2}\pi}(\frac{5}{4} + \cos\pi) = \frac{1}{4}e^{-j\frac{3}{2}\pi}$

$y[n] = \frac{1}{4}\times 10 \cos(\frac{1}{2}\pi n + 0.25\pi - \frac{3}{2}\pi) = \frac{5}{2}\cos(\frac{1}{2}\pi n - \frac{5}{4}\pi)$

$\therefore y[n] = \frac{5}{2}\cos(\frac{1}{2}\pi n + \frac{3}{4}\pi)$ ✱

prob. 7.2

LTI system

$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5] + x[n-6] + x[n-7] = \sum_{k=0}^{7} x[n-k]$

(a) $\{b_k\} = \{1, 1, 1, 1, 1, 1, 1, 1\}$

$H(e^{j\hat{\omega}}) = \sum_{k=0}^{7} b_k e^{-j\hat{\omega}\cdot k}$

$\therefore H(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} + e^{-j5\hat{\omega}} + e^{-j6\hat{\omega}} + e^{-j7\hat{\omega}}$ ✱

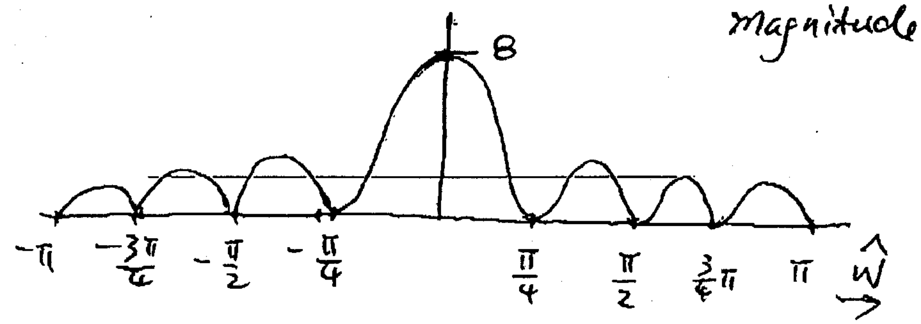
(b) using summation formula $\sum_{k=0}^{M} r_k = \frac{1-r^{M+1}}{1-r}$ and $r = e^{-j\hat{\omega}}$

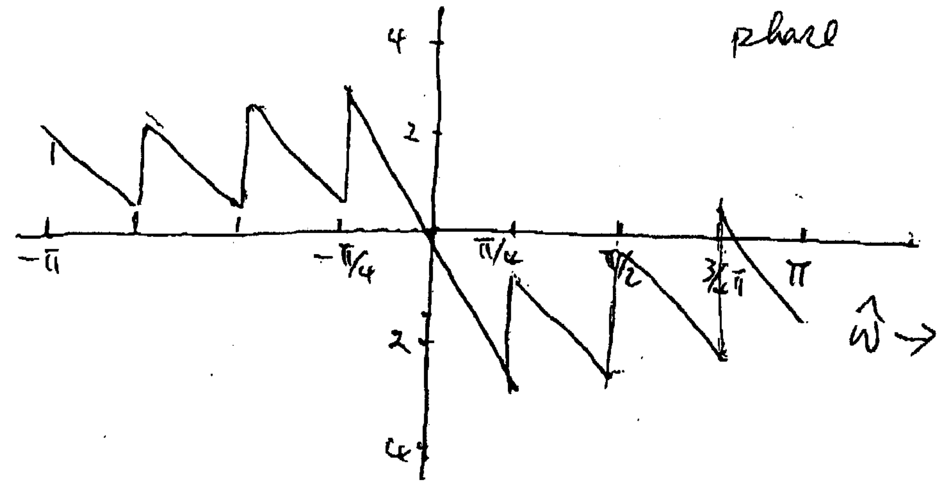
$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} e^{-j\hat{\omega}k} = \frac{1-e^{-j8\hat{\omega}}}{1-e^{-j\hat{\omega}}} = \frac{e^{-j4\hat{\omega}}(e^{+j4\hat{\omega}} - e^{-j4\hat{\omega}})}{e^{-j\hat{\omega}/2}(e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2})}$

$\therefore H(e^{j\hat{\omega}}) = \frac{e^{-j4\hat{\omega}}\sin 4\hat{\omega}}{e^{-j\hat{\omega}/2}\sin(\hat{\omega}/2)} = e^{-j3.5\hat{\omega}}\frac{\sin(8\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$ ✱

prob. 7.3 $H(e^{j\hat{\omega}}) = \dfrac{\sin(8\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j3.5\hat{\omega}}$

ⓒ Sketch magnitude and phase of $H(e^{j\hat{\omega}})$





ⓓ $x[n] = 100 + 200\cos(\hat{\omega}_0 n)$ for $-\infty < n < +\infty$

Find all possible non-zero $\hat{\omega}_0$, $0 < \hat{\omega}_0 < \pi$

such that $y[n] = c =$ constant

$\because$ $H(e^{j\hat{\omega}}) = \dfrac{\sin(8\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j3.5\hat{\omega}}$

$x[n] = x_1[n] + x_2[n] = 100 + 200\cos(\hat{\omega}_0 n)$

for $x_1[n]$, $\hat{\omega} = 0$

$H(e^{j\cdot 0}) = \dfrac{8/2}{1/2} = 8$

for $x_2[n]$, $\hat{\omega} = \hat{\omega}_0$

$H(e^{j\hat{\omega}_0}) = \dfrac{\sin(8\hat{\omega}_0/2)}{\sin(\hat{\omega}_0/2)} = 0 \Rightarrow \hat{\omega}_0 = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3}{4}\pi$

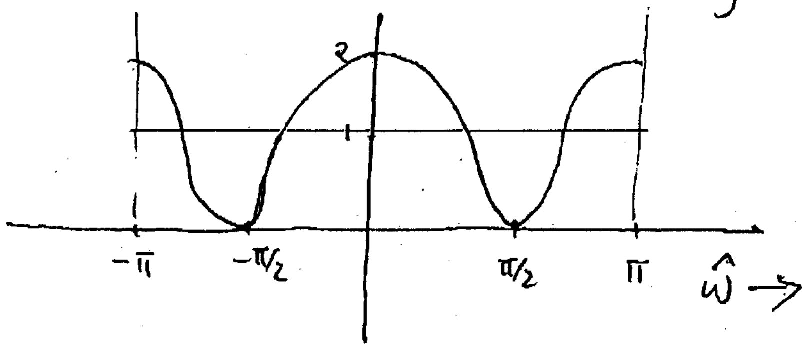
$\therefore$ $y[n] = 8 \times 100 = 800$ ※

Prob. 7.3

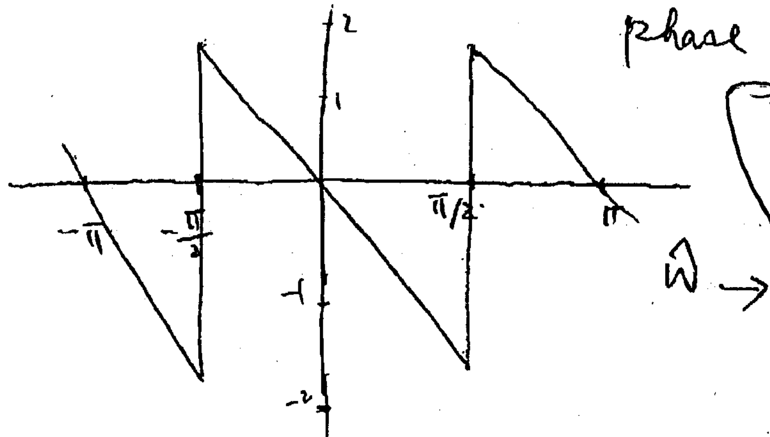
(a) $H(e^{j\hat{\omega}}) = 1 + e^{-j2\hat{\omega}} = e^{-j\hat{\omega}}(e^{+j\hat{\omega}} + e^{-j\hat{\omega}})$

$H(e^{j\hat{\omega}}) = 2e^{-j\hat{\omega}} \cos\hat{\omega}$

magnitude $= |2\cos\hat{\omega}|$



phase $= -\hat{\omega}$



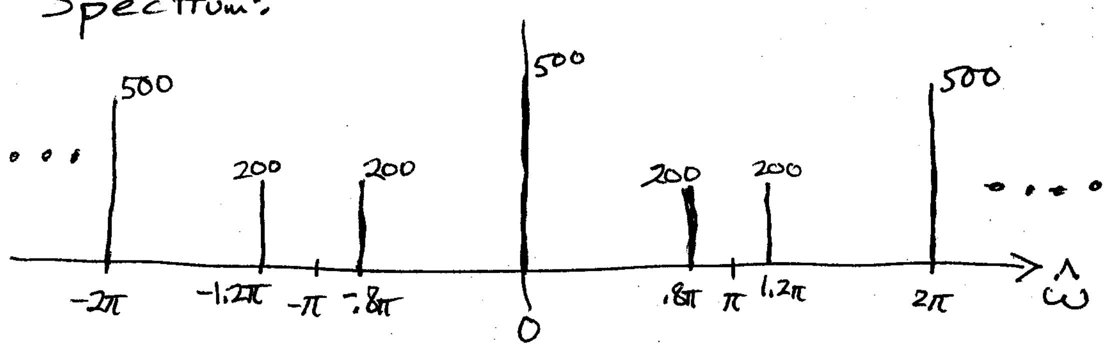
There is a phase jump of π at $\hat{\omega} = \pi/2$, because $2\cos\hat{\omega}$ goes negative at $\hat{\omega} = \pi/2$.

(b) $x(t) = 500 + 400(800\pi t)$ for $-\infty < t < \infty$

$f_s = 1000$ samples/sec

$x[n] = 500 + 400\left(\frac{800\pi}{1000} n\right) = 500 + 400\cos(0.8\pi n)$

Spectrum:



prob. 7.3(e)

$\because H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(2\cos\hat{\omega})$

$x[n] = 500 + 400\cos(0.8\pi n)$

$= x_1[n] + x_2[n]$

on $x_1[n]$, $H(e^{j\cdot 0}) = 1\times(2) = 2 \Rightarrow y_1[n] = 1000$

on $x_2[n]$, $H(e^{j\cdot 0.8\pi}) = e^{-j0.8\pi}(2\cos 0.8\pi) = -0.1618\,e^{-j0.8\pi}$

$= 0.1618\,e^{j(\pi - 0.8\pi)} = 0.1618\,e^{j0.2\pi}$

$\Rightarrow y_2[n] = 400\times 0.1618\cos(0.8\pi n + 0.2\pi)$

$= 647.2\cos(0.8\pi n + 0.2\pi)$

$y[n] = 1000 + 647.2\cos(0.8\pi n + 0.2\pi)$

$\because n = f_s t = 1000t$

$\therefore y(t) = 1000 + 647.2\cos(800\pi t + 0.2\pi)$ ※

prob. 7.4 $y[n] = b_1 x[n-1] + b_3 x[n-3]$

(a) when $x_1[n] = 10\cos(0.5\pi n) \Rightarrow y_1[n] = \cos(0.5\pi(n-1))$

$x_2[n] = 20 \Rightarrow y_2[n] = 0$

$H(e^{j\hat{\omega}}) = b_1 e^{-j\hat{\omega}} + b_3 e^{-j3\hat{\omega}}$

$H(e^{j0}) = b_1 + b_3 = 0$ —(1)

$H(e^{j0.5\pi}) = b_1 e^{-j0.5\pi} + b_3 e^{-j1.5\pi} = b_1(-j) + b_3(j)$

$= (b_3 - b_1)j = (b_3 - b_1)e^{j\frac{\pi}{2}} = (b_1 - b_3)e^{-j\frac{\pi}{2}}$

$y_1[n] = 10\times(b_1 - b_3)\cos(0.5\pi n - 0.5\pi)$

$\Rightarrow 10\times(b - b_3) = 1$ (2)

From eq(1) & eq(2) $\Rightarrow b_1 = 1/20$, $b_3 = -\frac{1}{20}$ ※

prob. 7.4(b)

$$H(e^{j\hat{\omega}}) = \frac{1}{20}e^{-j\hat{\omega}} - \frac{1}{20}e^{-j3\hat{\omega}} = \frac{1}{20}e^{-j\hat{\omega}\cdot 2}(e^{j\hat{\omega}} - e^{-j\hat{\omega}})$$

Since $b_1 = \frac{1}{20}$, $b_3 = -\frac{1}{20}$

$$H(e^{j\hat{\omega}}) = \frac{1}{20}e^{j\hat{\omega}\cdot 2}\, 2j\sin\hat{\omega} = \frac{1}{10}e^{-j2\hat{\omega}}e^{j\pi/2}\sin\hat{\omega}$$

$$H(e^{j\hat{\omega}}) = \frac{1}{10}\sin\hat{\omega}\; e^{-j(2\hat{\omega}-\pi/2)}$$

7.4(c)

$$\because x_4[n] = 13 + 17(-1)^n = 13 + 17\cos(\pi n)$$

when $\hat{\omega} = 0$, $H(e^{j\cdot 0}) = \frac{1}{10}(\sin 0)\, e^{-j(0-\pi/2)} = 0$

when $\hat{\omega} = \pi$, $H(e^{j\cdot\pi}) = \frac{1}{10}(\sin\pi)\, e^{-j(2\pi-\pi/2)} = 0$

$$\Rightarrow y_4[n] = 0$$

prob. 7.5(a) $H(e^{j\hat{\omega}}) = (1+e^{-j\hat{\omega}})(1-je^{-j\hat{\omega}})(1+je^{-j\hat{\omega}})$

$$= (1+e^{-j\hat{\omega}})(1-j^2e^{-j2\hat{\omega}})$$

$$H(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}}$$

$$\{b_k\} = \{1, 1, 1, 1\}$$

(a) $y[n] = \sum_{k=0}^{3} b_k\, x[n-k] = x[n] + x[n-1] + x[n-2] + x[n-3]$

(b) $h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$

prob. 7(c)

$$x[n] = 10 + 9\delta[n-2] + 8\cos(0.5\pi n)$$
$$= x_1[n] + x_2[n] + x_3[n]$$

when $\hat{w} = 0$, $H(e^{j\hat{w}}) = H(e^{j\cdot 0}) = 1 + 1 + 1 + 1 = 4$

when $\hat{w} = 0.5\pi$, $H(e^{j\pi/2}) = 1 + e^{-j\pi/2} + e^{-j\pi} + e^{-j3\pi/2}$

$$= 1 - j - 1 + j = 0$$

$$h[n] * 9\delta[n-2] = 9\delta[n-2] + 9\delta[n-3] + 9\delta[n-4] + 9\delta[n-5].$$

$$y[n] = H(e^{j\cdot 0}) \times 10 + h[n] * 9\delta[n-2] + H(e^{j\pi/2}) \times 8\cos(0.5\pi n)$$
$$= 40 + 9\delta[n-2] + 9\delta[n-3] + 9\delta[n-4] + 9\delta[n-5] + 0$$

$$\therefore y[n] = 40 + 9\delta[n-2] + 9\delta[n-3] + 9\delta[n-4] + 9\delta[n-5]$$

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