Georgia Tech

School of ECE

Problem Set #3

Prob. 3.1

a) Since $x(t)$ is a real signal, the spectrum is symmetric (with complex amplitudes being complex conjugates). Hence, we have

$$\omega_1 = +7\pi \qquad X_{-1} = X_{1}^{*} = \sqrt{32} + j\sqrt{32}$$

$$\omega_2 = 21\pi \qquad X_2 = X_{-2}^{*} = 42\, e^{j\frac{3\pi}{4}}$$

b) $$x(t) = B + (\sqrt{32} - j\sqrt{32})\, e^{j7\pi} + (\sqrt{32} + j\sqrt{32})\, e^{-j7\pi} + 42\, e^{j\frac{3\pi}{4}} \cdot e^{j21\pi} + 42\, e^{-j\frac{3\pi}{4}}\, e^{-j21\pi}$$

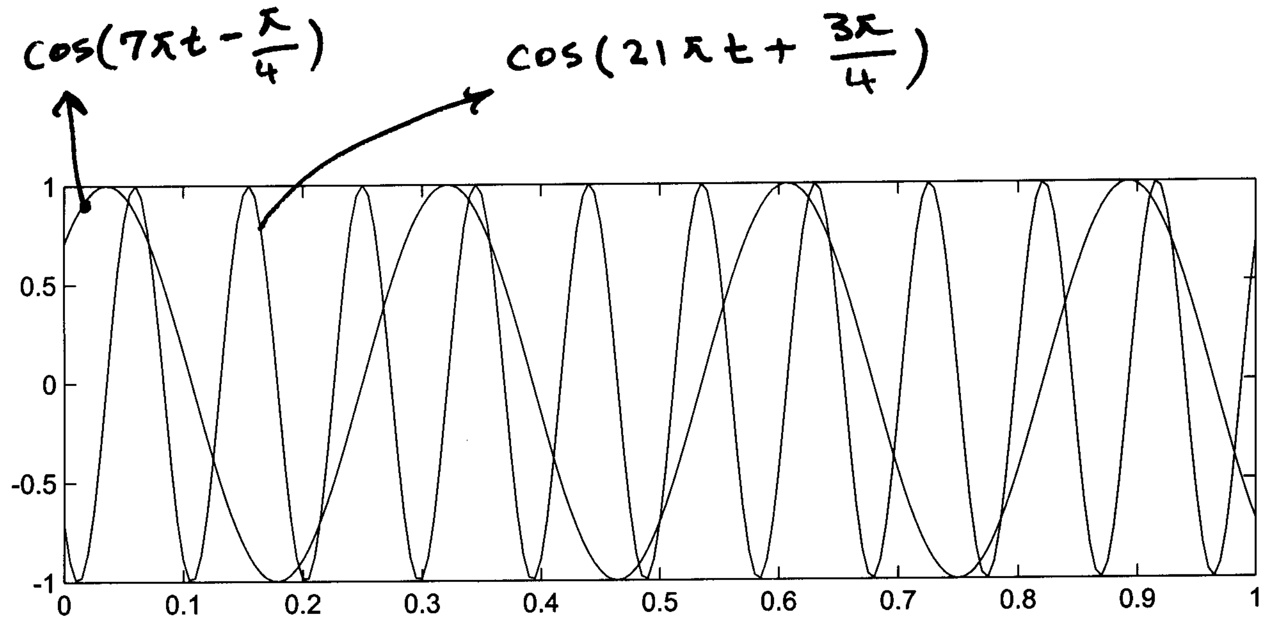
Note: $\sqrt{32} + j\sqrt{32} = 8\, e^{j\frac{\pi}{4}}$

$$\Rightarrow x(t) = B + 16\cos\left(7\pi t - \frac{\pi}{4}\right) + 84\cos\left(21\pi t + \frac{3\pi}{4}\right)$$

If we assume that both of cosines will have maximum at some $t = t_0$, then $B + 16 + 84 = 100$ $\Rightarrow B = 0$. But, in the following we show that B has to be chosen a positive value:

Prob. 3.1 C

The plot shows that the maximums of two cosines never occur simultaneously.



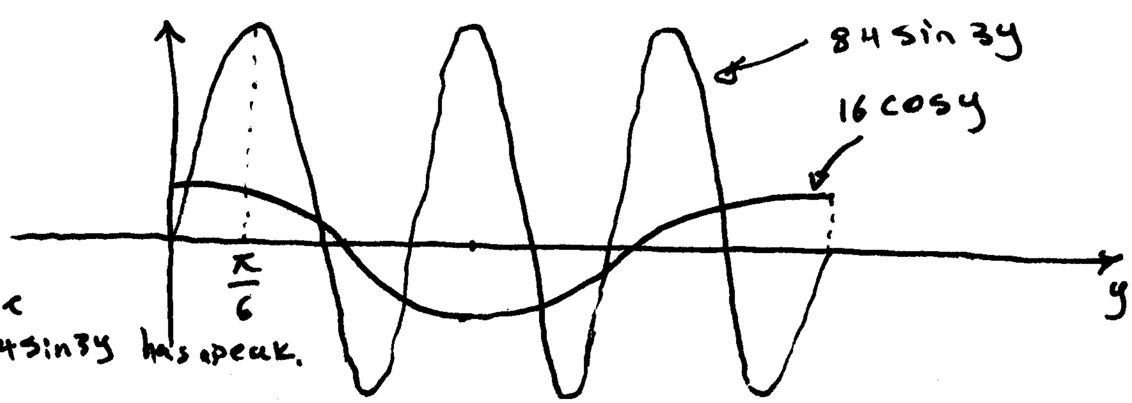
Here, we try to find the maximum for $f(t) = 16\cos(7\pi t - \frac{\pi}{4}) + 84\cos(21\pi t + \frac{3\pi}{4})$

$$f(t) = 16\cos(\underbrace{7\pi t - \tfrac{\pi}{4}}_{y}) + 84\sin(\underbrace{3(7\pi t - \tfrac{\pi}{4})}_{3y}) \Rightarrow 16\cos y + 84\sin 3y = f(y)$$

$\frac{df}{dy} = 0 \qquad \sin y = \frac{63}{4}\cos 3y \;\rightarrow$ one can computationally find the value of y.

$$\Rightarrow \max f(y) = 97.898 \;\longrightarrow\; B \simeq 2.1$$

One may also estimate the maximum point as following



Take $y \simeq \frac{\pi}{6}$

since roughly the max is where $84\sin 3y$ has a peak.

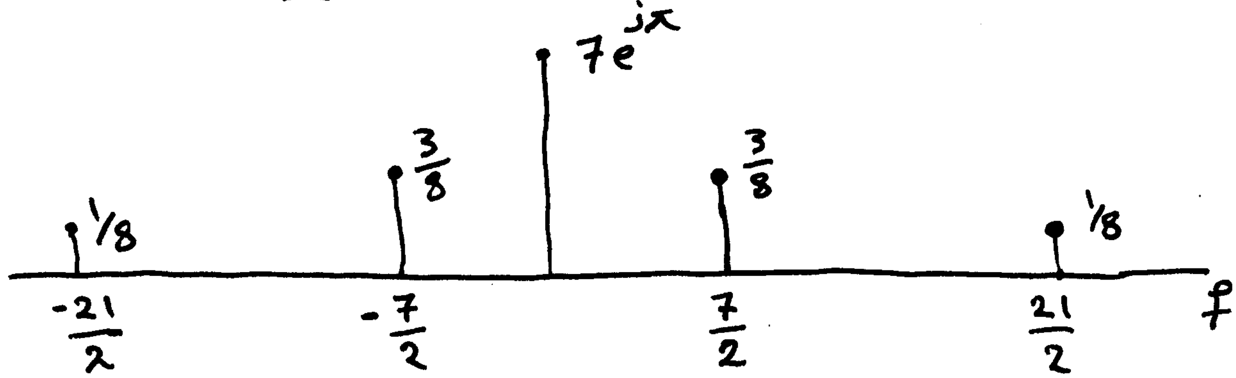
Prob. 3.2

a) $x(t) = \cos^3 7\pi t - 7 = \cos(7\pi t)\left\{\dfrac{1+\cos(14\pi t)}{2}\right\} - 7$

$$= \frac{1}{2}\cos(7\pi t) + \frac{1}{2}\cos(7\pi t)\cdot\cos(14\pi t) - 7$$

$$= \frac{1}{2}\cos(7\pi t) + \frac{1}{4}\cos(21\pi t) + \frac{1}{4}\cos(7\pi t) - 7$$

$$= \frac{3}{4}\left(\frac{e^{j7\pi t} + e^{-j7\pi t}}{2}\right) + \frac{1}{4}\left(\frac{e^{j21\pi t} + e^{-j21\pi t}}{2}\right) - 7$$

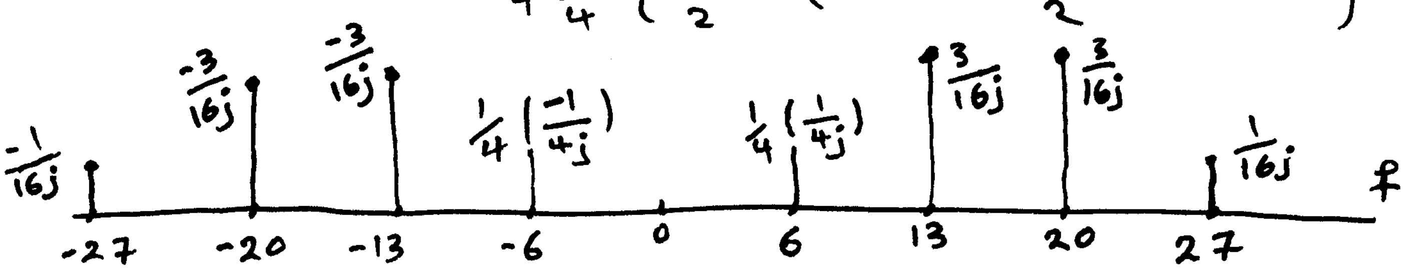


b) using part (a) $y(t) = \left(\frac{3}{4}\cos 7\pi t + \frac{1}{4}\cos 21\pi t\right)\cdot \sin(33\pi t)$

since $\sin\omega_1 \cdot \cos\omega_2 \triangleq \frac{1}{2}\left\{\sin(\omega_1+\omega_2) + \sin(\omega_1-\omega_2)\right\}$

we can write $y(t) = \frac{3}{4}\left\{\frac{1}{2}\sin(40\pi t) + \frac{1}{2}\sin(26\pi t)\right\} +$

$$+ \frac{1}{4}\left\{\frac{1}{2}\sin(54\pi t) + \frac{1}{2}\sin(12\pi t)\right\}$$



Prob. 3.3

a.) $x(t) = (29)e^{j\pi} + \left(19e^{-j\frac{3\pi}{4}} \cdot e^{j3\pi t} + 19e^{j\frac{3\pi}{4}} e^{-j3\pi t}\right) +$

$+ \left(9e^{j\frac{\pi}{3}} \cdot e^{j7\pi t} + 9e^{-j\frac{\pi}{3}} e^{-j7\pi t}\right)$

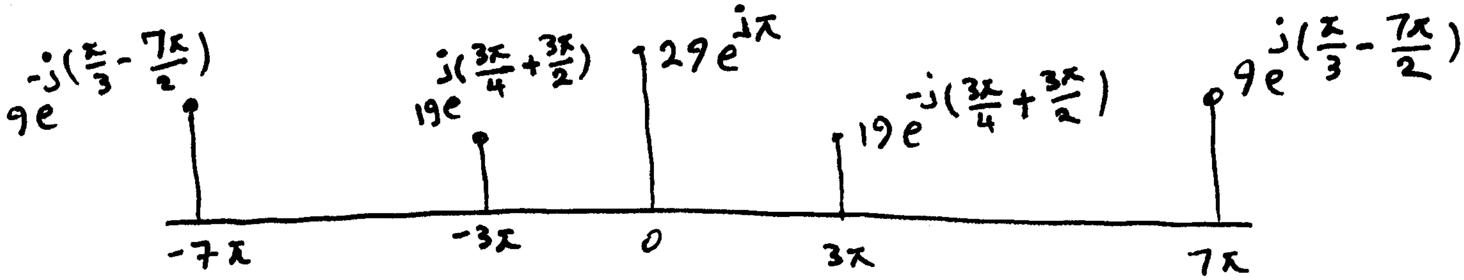
$$x(t) = -29 + 38\cos\left(3\pi t - \frac{3\pi}{4}\right) + 18\cos\left(7\pi t + \frac{\pi}{3}\right)$$

b) we note that:

if $x(t) = \cos(\omega_0 t + \varphi)$, then $y(t) = x(t - t_0) = \cos(\omega_0 t - \omega_0 t_0 + \varphi)$

So, the spectrum of $y(t)$ looks like the spectrum of $x(t)$ except that the coef. are multiplied to $e^{-j\omega_0 t_0}$ or $e^{j\omega_0 t_0}$ (i.e., simply replace φ by $(\varphi - \omega_0 t_0)$ every where in the spectrum of $x(t)$).

Thus the spectrum of $y(t)$ is given by



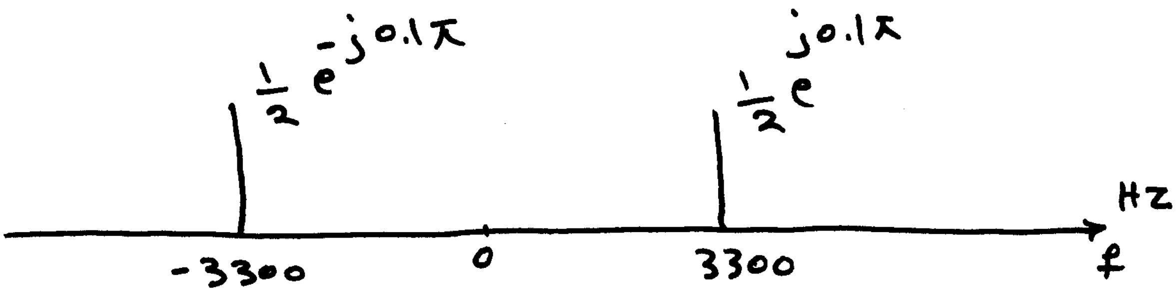
c) Period of $z(t)$ is the same as period of $x(t)$ because if $x(t+T) = x(t)$ then $z(t+T) = 8\underbrace{x(t+T-1)}_{=x(t-1)} + 13$

$\Rightarrow z(t+T) = 8x(t-1) + 13$

$f_0 = \text{G.c.d.}\left(\frac{7}{2}, \frac{3}{2}\right) = \frac{1}{2} \Rightarrow \text{Period} = T = \frac{1}{f_0} = 2$

Prob. 3.4

a)

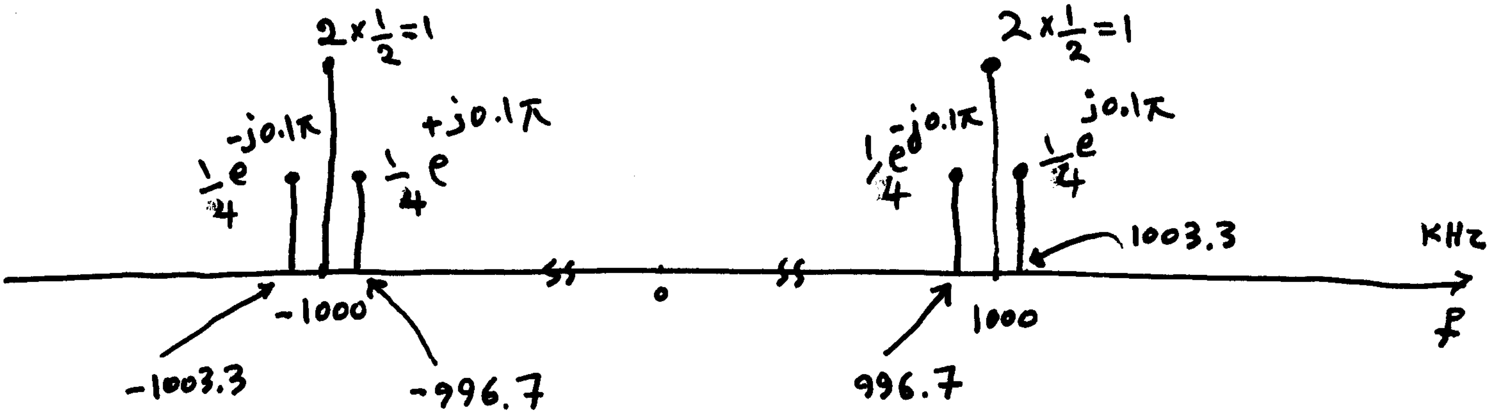


b) $x(t) = \left[\cos\left(2\pi(3.3^{\text{KHz}})t + 0.1\pi\right) + 2\right]\cdot\cos\left(2\pi(\overset{\text{KHz}}{1000})t\right)$

$= \cos\left(2\pi(\overset{\text{KHz}}{3.3})t + 0.1\pi\right)\cdot\cos\left(2\pi t \times 10^{3\,\text{KHz}}\right) + 2\cos\left(2\pi t \times 10^{3\,\text{KHz}}\right)$

$= \frac{1}{2}\cos\left(2\pi(\overset{\text{KHz}}{1003.3})t + 0.1\pi\right) + \frac{1}{2}\cos\left(2\pi(996.7^{\text{KHz}})t - 0.1\pi\right)$

$+ 2\cos\left(2\pi(10^{3\ \text{KHz}})t\right)$



Problem 3.5*:

(a) #3. Period of (a) is $\frac{5}{6}$ s, because there are 6 periods within 5 secs. DC value is +2, and time shift is positive, so $\phi < 0$; frequency of #3 is $f = 1.2$ Hz with negative phase.

(b) #5. Period of (b) is $\frac{2}{3}$ s, because there are 3 periods in 2 seconds. The frequency of #5 is $f_0 = 1.5$ Hz, and it has no DC.

(c) #1. Period of (c) is $\frac{5}{6}$ s, the DC value is +2, and the time shift is negative, so $\phi > 0$. The frequency of #1 is $f = 1.2$ Hz with positive phase.

(d) #2. Period of (d) is $\frac{10}{3}$ s (estimated). The fundamental frequency of #2 is $f_0 = \gcd(0.6, 1.5) = 0.3$ Hz.

(e) #4. Period of (e) is 2.5s; fundamental frequency of #4 is $f_0 = \gcd(1.2, 2.0) = 0.4$ Hz.

1. $x(t) = 2 + 3\cos(2\pi(1.2)t + \pi/2)$
2. $x(t) = 3\cos(2\pi(0.6)t - 0.25\pi) + 3\cos(2\pi(1.5)t + \pi)$
3. $x(t) = 2 + 3\cos(2\pi(1.2)t - 0.25\pi)$
4. $x(t) = 3\cos(2\pi(1.2)t - 0.25\pi) + 3\cos(4\pi t + \pi)$
5. $x(t) = 3\cos(2\pi(1.5)t + \pi)$