GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2005
Problem Set #13

Assigned: 8-April-05
Due Date: Week of 18-April-05

Quiz #4 will be given on 22-April. One page ($8\frac{1}{2} \times 11$ in.) of **handwritten** notes allowed.

Reading: In *SP First*, all of Chapter 10: *Frequency Response*; Chapter 11: *Continuous-Time Fourier Transform*, Sections 11-1 through 11-5.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 13.1*:

Determine the *forward* Fourier Transform of the following signals:

(a) $x(t) = 13\cos(77\pi t - \pi/3)$

(b) $x(t) = 7e^{-3t}u(t-2) - 7e^{-3t}u(t)$

(c) $x(t) = \dfrac{\sin(3\pi(t-\frac{1}{2}))}{2t-1}$

(d) $x(t) = u(t-8) - u(t-3)$

PROBLEM 13.2*:

Determine the *inverse* Fourier Transform of the following:

(a) $X(j\omega) = j\sin(4\omega)$

(b) $X(j\omega) = \dfrac{10}{4+3j\omega}e^{-j\omega/4}$

(c) $X(j\omega) = e^{j\omega/5}\{\delta(\omega-7\pi) + \delta(\omega+7\pi)\}$

(d) $X(j\omega) = u(\omega-7\pi) - u(\omega+7\pi)$

PROBLEM 13.3*:

A continuous-time LTI system is defined by the impulse response

$$h(t) = \tfrac{1}{2}\left\{ae^{-at}u(t) - be^{-bt}u(t)\right\}$$

(a) Determine a simple expression for the Fourier transform, $H(j\omega)$, which is also the frequency response of the system.

(b) Make a plot of the magnitude of the frequency response versus ω when $a = 25\pi$ and $b = 100\pi$.

(c) Describe the type of filter in the plot of the previous part (LPF, HPF, or BPF).

(d) Determine the phase of $H(j\omega)$ at $\omega = 0, 50\pi$, and 100π.

(e) When the input signal is $x(t) = 77 + 33\cos(50\pi t - \pi/4) + 44\cos(100\pi t)$, determine the output signal. Use the values given in part (b) for the parameters a and b.

PROBLEM 13.4*:

The frequency responses of two ideal LTI systems are

$$H_1(j\omega) = e^{-j\omega/3}$$
$$H_2(j\omega) = \{u(\omega + 32\pi) - u(\omega - 32\pi)\}\, e^{-j\omega/3}$$

(a) Create a new filter by subtracting the two LTI systems defined above.

$$H_3(j\omega) = H_1(j\omega) - H_2(j\omega)$$

Make a sketch of the magnitude and phase of $H_3(j\omega)$.

(b) Describe the type of filter in the plot of the previous part (LPF, HPF, or BPF).

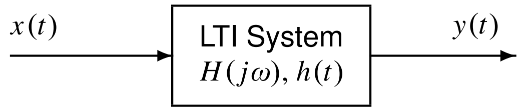
(c) Using the filter from part (a), determine the output of the system[1] when the input signal is

$$x(t) = \sin(44\pi t) + \frac{\sin(24\pi t)}{\pi t}$$

(d) Determine the the impulse response of the system. Express your result in a *simple form.*

[1] Use frequency-domain methods: Determine the Fourier transform of the input signal and then apply the filter in the frequency-domain to determine the corresponding output signal.

PROBLEM 13.5*:



The impulse response of the system (above) is

$$h(t) = \delta(t) - \frac{\sin(\omega_{\text{co}} t)}{\pi t}$$

and the input to this system is a periodic signal (with period $T_0 = 1/5$ sec.) given by a Fourier series:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j 2\pi f_0 k t}$$

where the Fourier coefficients are $a_k = jk(2 - |k|), \quad k = 0, \pm 1, \pm 2, \ldots$

(a) Recall how the spectrum is related to the Fourier series, and then plot the spectrum of the input signal, $x(t)$, over the frequency range $-44\pi < \omega < 44\pi$ in rad/s.

(b) Determine the frequency response $H(j\omega)$ of the system as a general formula. Exploit the fact that $h(t)$ and $H(j\omega)$ are a "Fourier Transform pair." Then, for the case $\omega_{\text{co}} = 12\pi$ rad/s, plot the magnitude $|H(j\omega)|$ vs. ω, and the phase $\angle H(j\omega)$ vs. ω. Use the frequency range $-44\pi < \omega < 44\pi$ rad/s.

(c) Determine the spectrum of the output signal, $y(t)$. Make a plot versus ω over the range $-44\pi < \omega < 44\pi$ rad/s. This will be easy to do if you overlay the plots from parts (a) and (b) on the same frequency axis.