GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2005
Problem Set #10

Assigned: 11-Mar-05
Due Date: Week of 28-March-05

Quiz #3 will be given on 1-April (Friday). Coverage: Chapters 5, 6, 7, and 8; HW #7, #8, #9, and #10.

Reading: In *SP First*, Chapter 8: *IIR Filters*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 10.1*:

Given a feedback filter defined via the recursion:

$$y[n] = \tfrac{1}{2} y[n-4] + x[n] \qquad\qquad \text{(DIFFERENCE EQUATION)}$$

(a) Determine the impulse response $h[n]$, assuming the "at rest" initial condition.

(b) When the input to the system is the signal: $x[n] = n^2 \, (u[n] - u[n-4])$
determine the output signal $y[n]$, assuming the "at rest" initial condition (i.e., the output signal is zero for $n < 0$). Present your final answer as a plot of all of $y[n]$.

(c) Write two MATLAB statements that would first compute the output in the previous part over the range $0 \le n \le 20$, and then plot it as a stem plot. Consult `help filter`.

PROBLEM 10.2*:

For the system: $\boxed{H(z) = \dfrac{1 + z^{-1}}{1 - \frac{1}{2} z^{-1}}}$ determine various aspects of the time-domain (n) behavior:

(a) The inverse z-transform of $H(z)$ is the impulse response $h[n]$. Determine the inverse z-transform for $H(z)$ as a mathematical formula, and make a plot of the first five values of the impulse response, $h[n]$.

(b) When the input is the signal, $(-1)^n u[n-2]$, determine the output signal (as a simple formula).

(c) Write a one-line MATLAB statement that would compute the output in the previous part over the range $0 \le n \le 20$.

(d) Find a nonzero input signal for which the output is *finite-duration*, i.e., $y[n] = 0$ for $n > n_0$ where n_0 is a fixed number.

PROBLEM 10.3*:

Determine the z-transforms of the following signals. *Express your answer as the ratio of polynomials in* z^{-1} *by placing all terms over a common denominator.*

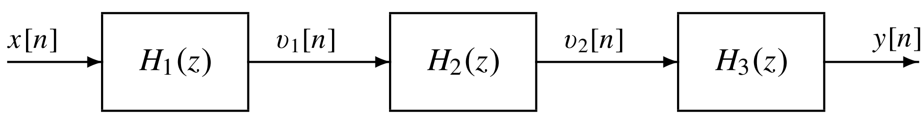
(a) $x_a[n] = \left(-\frac{1}{2}\right)^n u[n-5]$

(b) $x_b[n] = 100(0.6)^n u[n] - 100(-0.6)^n u[n]$

(c) $x_c[n] = \delta[n] + u[n-1]$

PROBLEM 10.4*:

In the following cascade of systems, all of the individual system functions, $H_i(z)$, are known.



$H_1(z) = z^{-1} - z^{-3}$ $\qquad$ $H_2(z) = 2 + z^{-1}$ $\qquad$ $y[n] = a_1 y[n-1] + a_2 y[n-2] + a_3 y[n-3] + b_0 v_2[n]$

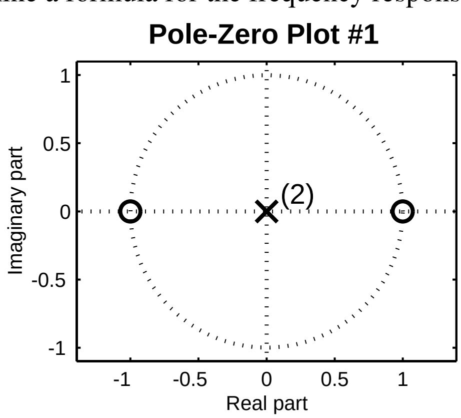
(a) Determine $H_3(z)$, the z-transform of the last system.

(b) When $a_1 = a_2 = \frac{1}{2}$, $a_3 = 0$ and $b_0 = 7$, determine the system function, $H(z)$ for the cascaded system.

(c) Consider the impulse response $h[n]$ of the cascaded system, i.e., the output when the input is $x[n] = \delta[n]$. Determine values for $\{a_1, a_2, a_3, b_0\}$ so that the impulse response will be $h[n] = 8\delta[n - n_d]$. Also determine the value of the time shift n_d.

PROBLEM 10.5*:

(a) Determine a formula for the frequency response of an FIR filter defined by the pole-zero plot below:



(b) For the FIR filter in part (a), write a simplified version of the frequency response $H(e^{j\hat{\omega}})$ and use it to prove that the maximum value of the frequency response magnitude will be at $\hat{\omega} = \pm\pi/2$.

(c) The pole-zero plot does not define the scaling of the frequency response. Therefore, you can rescale $H(e^{j\hat{\omega}})$ with a scaling constant β so that the *maximum* value of the frequency response $\beta H(e^{j\hat{\omega}})$ will be equal to one. Determine the numerical value of the scaling constant β for the frequency response from parts (a) and (b), and then draw a sketch of $|H(e^{j\hat{\omega}})|$.

PROBLEM 10.6:

This problem has been given before. Study it and its solution carefully when preparing for Quiz #3.

We have developed several concepts that are useful in solving problems involving LTI systems. The main concepts are the *difference equation*, the *impulse response*, the *system function*, and the *frequency response* function. Most problem solving demands that you be able to go back and forth among these different mathematical representations of the LTI system because, as simple as it seems, the z-transform is *not* always the best tool for solving problems. Indeed, for a specific problem, one of these representations may be more convenient than the others, or we may need to use more than one of these representations in solving a given problem. The following is a simple problem that might be posed about an LTI system:

Given the input sequence $x[n]$ find the output sequence for all n when the system is an IIR filter:

$$y[n] = 0.8y[n-1] + x[n] + x[n-2].$$

The following is a partial list of possible approaches to solving this problem:

1. *Time-Domain:* Use the difference equation representation of the system to compute the output $y[n]$ for all required values of n. *For example, you could do this using* MATLAB.

2. *Z-Domain:* Multiply the z-transform of the input by the system function and determine $y[n]$ as the inverse z-transform of $Y(z)$.

3. *Frequency-Domain:* Break the input into a sum of complex exponential signals, use the frequency response function to determine the output due to each complex exponential signal separately, and finally, add the individual outputs together to get $y[n]$.

In each of these solution methods you would use one or more of the basic representations of the first-order IIR filter. Which method is easiest will have a lot to do with the nature of the input signal. For example, if you are given the difference equation and you want to use approach #2, you will have to determine the system function $H(z)$ from the difference equation coefficients.

Now in each of the following cases, the input will be given. In each case, determine which representation of the system and which of the above approaches will lead to the easiest solution of the problem, and detail the steps in using that approach to solve the problem. For example, if you choose approach #2 to solve the problem, your answer should be something like the following:

Step 1: Find $X(z)$, the z-transform of $x[n]$.

Step 2: Find $H(z)$, the system function of the first-order IIR filter.

Step 3: Multiply $X(z)H(z)$ to get $Y(z)$.

Step 4: Take the inverse z-transform of $Y(z)$ to get $y[n]$.

Now here are some possible inputs. In each case, state which of the approaches above (#1, #2, or #3) you would use. There may not be a clear cut answer. Give the approach that you *think* will yield the solution with least effort. Then carry out the method to get the output.

(a) $x[n] = u[n]$.

(b) $x[n] = 2\cos(0.5\pi n - \pi/2) + \cos(0.25\pi n - \pi)$ for $-\infty < n < \infty$.

(c) $x[n] = 10\delta[n-5]$.

(d) $x[n]$ is a sampled speech signal. It is represented by a vector of 10000 numbers. In this case, you do not have to find the actual output.