GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2005
Problem Set #9

Assigned: 4-March-05
Due Date: Week of 14-March-05

Reading: In *SP First*, Chapter 7: *z-Transform*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 9.1*:

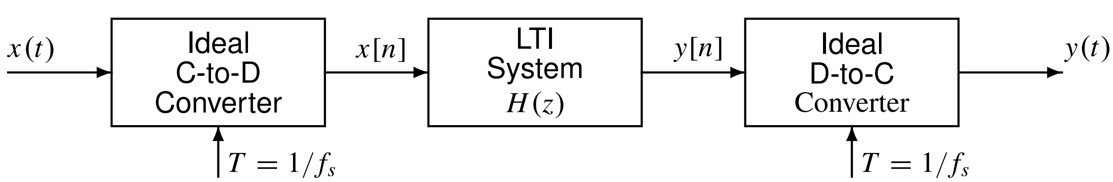
The input to the C-to-D converter in the figure below is

$$x(t) = 7\cos(6000\pi t + 3\pi/4) + 5\cos(10000\pi t + 2\pi/3)$$

The system function for the LTI system is

$$H(z) = 6 - 6z^{-6}$$

If $f_s = 9000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.
Hint: Recall that the frequency response $H(e^{j\hat{\omega}})$ can be obtained directly from $H(z)$.



PROBLEM 9.2*:

We now have ***four ways*** of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $y[n] = \frac{1}{2}(x[n] - x[n-1])$

(b) $h[n] = \frac{1}{7}(u[n-3] - u[n-10])$

(c) $H(e^{j\hat{\omega}}) = [7 - j\sin(5\hat{\omega})]e^{-j5\hat{\omega}}$

PROBLEM 9.3*:
We now have ***four ways*** of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given $H(z)$ and you must find the other three.

(a) $H(z) = 100$

(b) $H(z) = 5 + 6/z - 1/z^2$

(c) $H(z) = (1 - z^{-1})(1 - 2e^{j2\pi/3}z^{-1})(1 - 2e^{j4\pi/3}z^{-1})$

PROBLEM 9.4*:
Consider the linear time-invariant system given by the difference equation

$$y[n] = x[n] - x[n-1] + x[n-2] - x[n-3] + x[n-4] - x[n-5] + x[n-6] = \sum_{k=0}^{6}(-1)^k x[n-k]$$

(a) Determine the z-transform $H(z)$ of the system; simplify using the geometric sum formula for $\sum_k r^k$.

(b) From $H(z)$, determine the frequency response $H(e^{j\hat{\omega}})$, and show that it can be expressed in the form

$$H(e^{j\hat{\omega}}) = \frac{\cos(L\hat{\omega}/2)}{\cos(\hat{\omega}/2)} e^{-j\beta\hat{\omega}}$$

where β and L are constants.

(c) Sketch the frequency response (magnitude only) as a function of frequency from the formula above. You might want to check your plot by doing it in MATLAB with `freekz( )` or `freqz( )`.

(d) Is this FIR filter a lowpass filter (LPF), bandpass filter (BPF), or highpass filter (HPF) ?

PROBLEM 9.5*:
Consider the following MATLAB program:

```
nn = 0:16000;
xx = 7*cos((2/3)*pi*nn + 3pi/4) + 5*cos((10/9)*pi*nn + 2pi/3);
yy = conv([6,0,0,0,0,0,-6],xx);
soundsc(yy,9000)
```

(a) After making the usual correspondence between `xx` and $x[n]$, and between `yy` and $y[n]$, determine the system function $H(z)$ of the FIR filter that is implemented by the `conv( )` statement.

(b) Determine the frequency response of the FIR filter.

(c) Neglecting the end effects in the convolution, determine $y(t)$ that describes the signal produced by the `soundsc( )` statement.
Hint: The result of a previous problem might be useful here.