GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2005
Problem Set #8

Assigned: 25-Feb-05
Due Date: Week of 7-March-05

Quiz #2 on 4-March-2005. Coverage includes Chapters 3, 4, 5 and 6, as well as HW #4, #5, #6 and #7.

Reading: In *SP First*, Chapter 6: *Frequency Response of FIR Filters*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 8.1*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

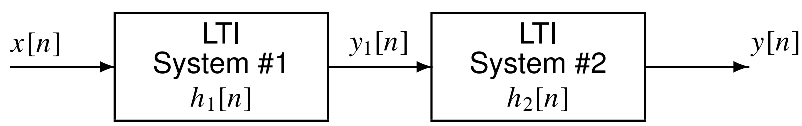


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a filter described by the difference equation

$$y_1[n] = -\tfrac{1}{2}x[n] + 3x[n-1] - \tfrac{1}{2}x[n-2]$$

and System #2 is described by the impulse response

$$h_2[n] = \delta[n-2],$$

(a) Determine the frequency response sequence, $H_1(e^{j\hat{\omega}})$, of the first system.

(b) Determine the frequency response, $H(e^{j\hat{\omega}})$, of the overall cascade system.

(c) Plot the magnitude and phase of the frequency response of the overall cascaded system.

PROBLEM 8.2*:

A discrete-time system is defined by the input/output relation

$$y[n] = (-1)^n x[n]$$

One characteristic of a LTI system is that a sinusoidal input at frequency $\hat{\omega}_0$ will give a sinusoidal output at the same frequency—no new frequency components will appear in the output. If frequencies other than $\hat{\omega}_0$ are contained in the output, then we can conclude that the system is not LTI.

(a) For the system above, determine the output $y_1[n]$ when the input is

$$x_1[n] = \cos(0.5\pi n)$$

Does this input-output pair $\{x_1[n],\ y_1[n]\}$ allow us to conclude that the system is not LTI?
Hint: Which frequencies are present in the output signal, $y_1[n]$?

(b) Exhibit one sinusoidal input signal, $A\cos(\hat{\omega}_0 n + \phi)$, for which the output contains frequency components not contained in the input signal.

(c) Is the system linear? or time-invariant? or neither? Explain.

PROBLEM 8.3*:

The frequency response of a linear time-invariant filter is given by the formula

$$H(e^{j\hat{\omega}}) = \left(1 - je^{-j\hat{\omega}}\right)\left(1 + je^{-j\hat{\omega}}\right)\left(1 - e^{j3\pi/4}e^{-j\hat{\omega}}\right)\left(1 - e^{-j3\pi/4}e^{-j\hat{\omega}}\right) \tag{1}$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.
Hint: Multiply out the factors to obtain a sum of powers of $e^{-j\hat{\omega}}$.

(b) Determine the impulse response of this system, and make a stem plot. Notice that $h[n]$ is finite length.

PROBLEM 8.4*:

The frequency response of a linear time-invariant filter is given by the formula (same as previous problem)

$$H(e^{j\hat{\omega}}) = \left(1 - je^{-j\hat{\omega}}\right)\left(1 + je^{-j\hat{\omega}}\right)\left(1 - e^{j3\pi/4}e^{-j\hat{\omega}}\right)\left(1 - e^{-j3\pi/4}e^{-j\hat{\omega}}\right)$$

(a) If the input is a complex exponential of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for which values of $-\pi \leq \hat{\omega} \leq \pi$ will $y[n] = 0$ for all n?
Hint: In this part, the answer is easy to obtain if you use the factored form of Eq. (1).

(b) Use superposition to determine the output of this system when the input is

$$x[n] = 5 + 9\delta[n-4] + 7\cos(0.5\pi n - 3\pi/4) \qquad \text{for } -\infty < n < \infty$$

Hint: Divide the input into three parts and find the outputs separately each by the easiest method and then add the results. This is what it means to apply the principle of *Superposition*.

PROBLEM 8.5*:

A discrete-time system is known to be LTI, and had been measured at two frequencies, $\hat{\omega} = 0$ and $\hat{\omega} = \pi$.

$$H(e^{j0}) = \tfrac{1}{2} \qquad \text{and} \qquad H(e^{j\pi}) = 2e^{-j11\pi}$$

(a) Determine the output when the input is

$$x_1[n] = \begin{cases} 8 & \text{for } n \text{ even} \\ 4 & \text{for } n \text{ odd} \end{cases}$$

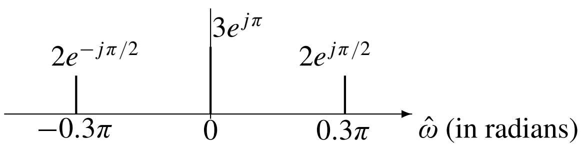
(b) If we also know that $H(e^{j\pi/2}) = H^*(e^{-j\pi/2}) = 3e^{-j11\pi/2}$, then determine the output $y_2[n]$ when the input is a triangle-shaped signal with a period of four:

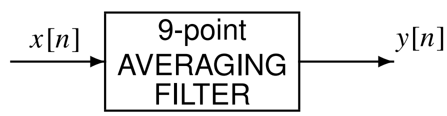
$$x_2[n] = \{\ldots, 1,\ 2,\ 3,\ 2,\ 1,\ 2,\ 3,\ 2,\ 1,\ 2,\ 3,\ 2,\ \ldots\}$$

i.e., $x_2[n+4] = x_2[n]$ with $x_2[0] = 1$, $x_2[1] = 2$, $x_2[2] = 3$, $x_2[3] = 2$, $x_2[4] = 1$, and so on.

PROBLEM 8.6:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

See Problem 6.1 of Spring 1999 for solution to this problem.