GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2005
Problem Set #7

Assigned: 18-Feb-05
Due Date: Week of 28-Feb-05

Reading: In *SP First*, Chapter 5: *FIR Filters*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 7.1*:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = -x[n] + 2x[n-2] + 3x[n-4] - 4x[n-5]$$

(a) Determine the impulse response $h[n]$ for this system.

(b) Determine the filter coefficients b_k in the causal FIR representation: $y[n] = \sum_{k=0}^{M} b_k x[n-k]$.

(c) Determine the *order* of the filter (M), and the *length* of the filter (L).

(d) Make a plot of the shifted unit-step signal $s[n] = 2u[n-3]$; plot enough to show its essential behavior.

(e) Use convolution to determine the output due to the input $2u[n-3]$. Use the convolution table, but look for patterns. Plot the output sequence $y[n]$ for $0 \le n \le 12$.

PROBLEM 7.2*:

For each of the following systems, the signal $x[n]$ is the input and $y[n]$ is the output.

1. $y[n] = n\,x[n-1]$ (Multiplier)
2. $y[n] = x[2^{|n-1|}]$ (Time Distortion)

(a) Find the impulse response for both systems. Give your answers as plots.

(b) Determine if the systems are (1) linear; give a proof or counterexample.

(c) Determine if the systems are (2) time-invariant; give a proof or counterexample.

(d) Determine if the systems are (3) causal; give a proof or counterexample.

PROBLEM 7.3*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

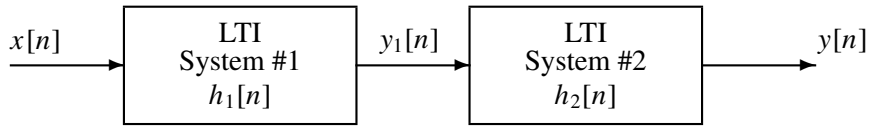

Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a filter described by the difference equation

$$y_1[n] = -2x[n-3] - 2x[n-2] - 2x[n-1]$$

and System #2 is a modified *first-difference* filter described by the impulse response:

$$h_2[n] = 3\delta[n-1] - 3\delta[n-2]$$

(a) Determine the impulse response sequence, $h_1[n]$, of the first system. Plot $h_1[n]$ versus n.

(b) Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(c) If the input signal is

$$x[n] = (-1)^n \left(u[n] - u[n-10]\right)$$

determine the output signal, $y[n]$, and make a plot of $y[n]$ versus n. Indicate regions where the output is zero; and regions where it is nonzero.

PROBLEM 7.4*:

Consider a system defined by $\quad y[n] = \sum_{k=2}^{13} (-1)^k x[n-k]$

(a) Determine the filter length and the filter coefficients, $\{b_k\}$.

(b) Suppose that the input $x[n]$ is non-zero only for $0 \le n \le 128$. Where will the output $y[n]$ first become non-zero? What is the index of the last non-zero value in the output sequence $y[n]$? What is the total length of the input sequence (in samples).

(c) Suppose that the input $x[n]$ is nonzero only for $100 \le n \le 222$, and the impulse response is nonzero for $300 \le n \le 444$, show that $y[n]$ is non-zero at most over a finite interval of the form $N_3 \le n \le N_4$ and determine N_3 and N_4.

Hint: Draw a sketch similar to Fig. 5.5 (on p. 105) to illustrate the zero regions of the output signal.

PROBLEM 7.5*:

Evaluate the following

(a) `yn = conv( [1 0 1 0 1], cos(0.5*pi*(0:8)) );`
Do the calculation by hand, and then check the result with MATLAB.

(b) $y[n] = p[n] * p[n]$, where $p[n] = u[n-7] - u[n]$. Make a plot of $y[n]$ versus n.
Hint: Use the MATLAB GUI `dconvdemo` to check your work.