$$x[n] = 2.2\cos(0.3\pi n - \pi/3) \qquad \boxed{f_s = 6000}$$

Compare to

$$x\left(\frac{n}{f_s}\right) = A\cos\left(2\pi f_0 \frac{n}{f_s} + \varphi\right) \leftarrow \boxed{\text{sampled continuous-time signal.}}$$

$$\Rightarrow \quad \frac{2\pi f_0}{f_s} = 0.3\pi, \text{ or } 0.3\pi + 2\pi, \text{ or } 0.3\pi - 2\pi.$$

Solve: $\frac{2\pi f_0}{f_s} = 0.3\pi \Rightarrow f_0 = f_s\left(\frac{0.3}{2}\right) = 6000 \times 0.15$

$$f_0 = 900\text{ Hz}$$

$$\rightarrow x(t) = 2.2\cos(1800\pi t - \pi/3)$$

NOTE: difference is f_s

Then $\frac{2\pi f_0}{f_s} = 2.3\pi \Rightarrow f_0 = f_s\left(\frac{2.3}{2}\right) = 6900\text{ Hz}$

$$\rightarrow x(t) = 2.2\cos(2\pi(6900)t - \pi/3).$$

Finally,

$$\frac{2\pi f_0}{f_s} = -1.7\pi \Rightarrow f_0 = f_s\left(-\frac{1.7}{2}\right) = -5100\text{ Hz}.$$

$$x(t) = 2.2\cos(2\pi(-5100)t - \pi/3)$$

$$\rightarrow x(t) = 2.2\cos(2\pi(5100)t + \pi/3)$$

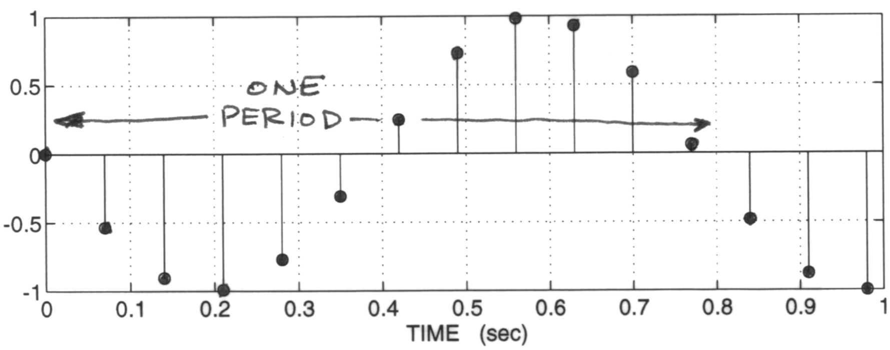
Problem 6.2: **ECE-2025** **Spring-2005**

```
Fo = 13;
Period = 1/Fo;
Ts  = 0.07;
tt = 0 : Ts : (13*Period);
j = sqrt(-1);
xx = real( exp( j*(2*pi*Fo*tt  - pi/2) ) );
%
stem( tt, xx ),  xlabel('TIME   (sec)'),  grid
```

$F_{SAMP} = 1/T_s = \frac{1}{0.07} = 14.28\text{Hz}.$

NOT GREATER THAN $2F_0$

(a)



we observe 1 sec of the signal which is 1.28 periods

$x[n] = \cos(2\pi(13)(0.07n) - \pi/2)$
$= \cos(2\pi(0.91)n - \pi/2)$ ← FOLDING
$= \cos(2\pi(0.09)n + \pi/2)$

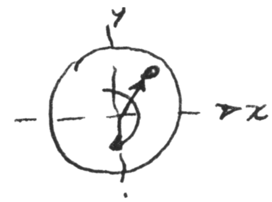
in continuous-time, the folded frequency is $14.28 - 13 = 1.28$ Hz
$\Rightarrow$ period $\approx \frac{1}{1.28} \approx .8$ sec

(b) SAMPLING THM $\Rightarrow$ $F_{SAMP} \geq 2F_0 = 2(13) = 26$

$F_{SAMP} = 1/T_s$

To get $\boxed{\text{SMOOTH}}$ plot need about 20 samples per period, which is a sampling rate of $20F_0$ $\therefore$ $T_s \leq \frac{1}{20F_0} = \frac{1}{20(13)} = \frac{1}{260}$.

(a) 720 rpm = 12 rotations/sec.



If (x,y) = the co-ordinates of the spot we can also use polar co-ords: $r\angle\theta$.

The radius of the spot is constant: r

The angle of the vector from the origin to the spot changes LINEARLY

$$\theta = \varphi_0 + \omega_0 t$$

where φ_0 is the initial phase

ω_0 = freq of rotation in rad/sec.

$\therefore \quad \omega_0 = 2\pi(12) = 24\pi.$

So the position of the spot $= re^{-j24\pi t + j\varphi_0}$

The minus sign is for clockwise rotation

(b) Disk spot will stand still if flash rate is once per rotation, once every 2 rotations etc. $\Rightarrow$ possible flash rate to hold spot still $= \dfrac{12}{\ell}$, $\ell = 1, 2, 3, \ldots$

So we get an answer of $\boxed{\text{rate} = 1, 2, 3, 4, 6, \; \& \; 12 \text{ per sec}}$

The rate must be a factor of 12

(c) $f_s = 13$ per sec. $\Rightarrow$ sample at $n/13$.

$x[n] = re^{j\varphi_0} e^{-j24\pi n/13} \quad \Rightarrow \omega_0 = \dfrac{-24\pi}{13}$

But ω_0 is same as: $-\dfrac{24\pi}{13} + 2\pi = 2\pi/13$

$x[n] = re^{j\varphi_0} e^{+j2\pi n/13} \longleftarrow$ (will take 13 flashes per revolution)

Spot will move counter-clockwise (due to + sign)

At what rate? once per sec = 60 rpm

Problem 6.4: **ECE-2025** **Spring-2005**

(a) $\theta[n] = \pi(0.7\times10^{-3})n^2$ ← This is $\theta[n]$ in $\Re e\{e^{j\theta[n]}\}$

For $n=10$:

$$\theta[10] = \pi(0.7\times10^{-3})\,10^2 = 0.07\pi = 12.6^\circ$$

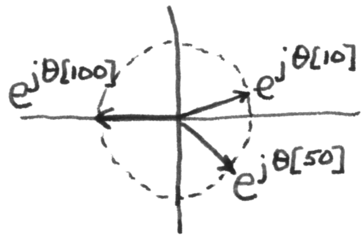
For $n=50$:

$$\theta[50] = \pi(0.7\times10^{-3})(50)^2 = \pi(0.7\times10^{-3}\times25\times10^2) = 1.75\pi$$

315°

For $n=100$:

$$\theta[100] = \pi(0.7\times10^{-3})10^4 = 7\pi = \pi \text{ rads, or } 180^\circ$$



(c) Work part (c) before part (b)

$v[n] = \cos(0.7\pi n)$ $\quad f_s = 8000\,\text{Hz}$

Ideal D/A ⇒ replace n with $f_s t$

$$v(t) = v[n]\Big|_{n=8000t} = \cos(0.7\pi\times8000t) = \cos(2\pi(2800)t)$$

Freq is 2800 Hz

(b) $x[n] = \cos(\pi(0.7\times10^{-3})n^2)$ ← Replace n with $8000t$

$$x(t) = \cos(\pi(0.7\times10^{-3})\times64\times10^6\,t^2)$$

$$= \cos(\pi(44.8\times10^3)\,t^2)$$

$\underbrace{n=0,1,\ldots 200}$

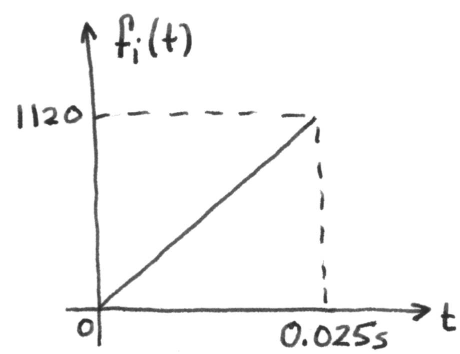
$0 \le t \le \frac{200}{f_s} = 0.025\,\text{sec.}$

$$\psi(t) = \pi(44.8\times10^3)t^2$$

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\psi(t)$$

$$= \frac{1}{2\pi}(2\pi)(44.8\times10^3)t$$

$$= 44800t \text{ Hz}$$

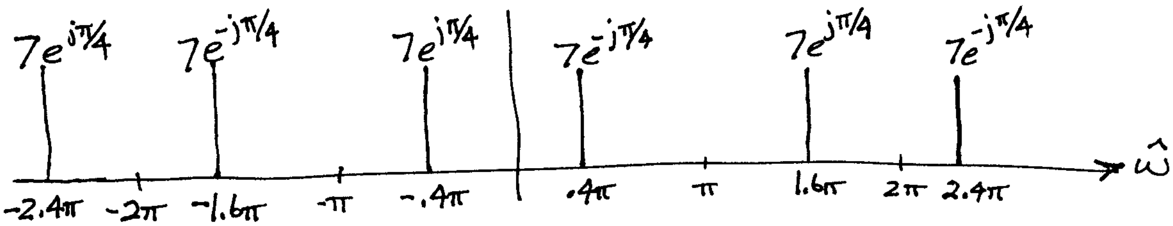


Prob 6.5 $\quad x(t) = 7e^{j(24\pi t - \pi/4)} + 7e^{j(16\pi t + \pi/4)}$

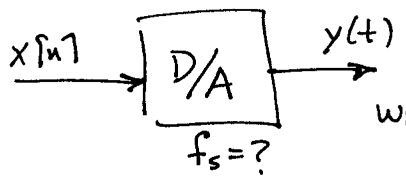
(a) $f_s = 10$ samples/sec

$$x[n] = x(n/10) = 7e^{j(24\pi \frac{n}{10} - \pi/4)} + 7e^{j(16\pi \frac{n}{10} + \pi/4)}$$

$$= 7e^{j(2.4\pi n - \pi/4)} + 7e^{j(1.6\pi n + \pi/4)}$$



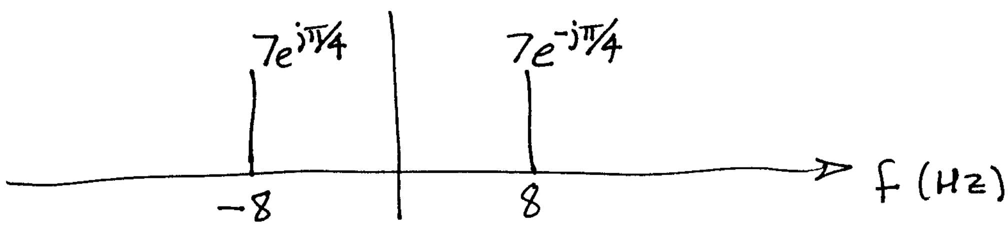
(b)



want ± 8Hz.

The lines at $\hat{\omega} = \pm 0.4\pi$ rads will be reconstructed by the D/A Converter:

$$\hat{\omega} = \frac{2\pi f}{f_s} \implies f_s = \frac{2\pi f}{\hat{\omega}} = \frac{2\pi(8)}{0.4\pi} = 40\text{ Hz}$$



Notice that $y(t) = 14\cos(16\pi t - \pi/4)$.

The output is real, even though the input is complex.