GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2005
Problem Set #5

Assigned: 4-Feb-05
Due Date: Week of 14-Feb-05

Reading: In *SP First*, Chapter 3: *Spectrum Representation,* all.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 5.1*:

A signal is defined by the equation

$$x(t) = \cos^4(100\pi t + 0.25\pi)$$

(a) Determine the fundamental frequency of the signal.

(b) Determine the Fourier Series coefficients of this signal.

(c) Plot the spectrum of $x(t)$. Most of $\{a_k\}$ are zero, but the nonzero ones can be used to make it is clear which harmonic components are present in the signal's spectrum.
Hint: Use the expansion of $(a+b)^4$ to quickly obtain an exponential form equivalent to $\cos^4$.

PROBLEM 5.2*:

A signal $x(t)$ is periodic with period $T_0 = 4$. Therefore, it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/4)kt}$$

It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

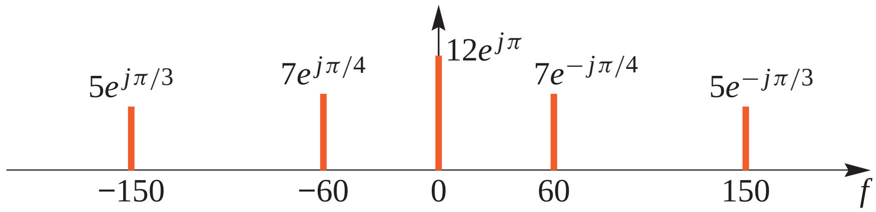
$$a_k = \frac{1}{4}\int_0^1 e^{-3t} e^{-j(2\pi/4)kt}\,dt$$

(a) Determine a general formula for the Fourier coefficients a_k for $x(t)$ above. Express your answer as complex numbers in polar form.

(b) Plot the spectrum for this signal versus frequency. Label the frequency axis in hertz. Show the first three harmonics, i.e., $|k| \le 3$.

PROBLEM 5.3*:

The figure below shows a spectrum plot for the periodic signal $x(t)$. *The frequency axis has units of Hz.*



(a) Determine the fundamental frequency f_0 (in Hz) of this signal.

(b) A periodic signal of this type can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}.$$

where ω_0 is the fundamental frequency in rad/sec. If the Fourier series coefficients of $x(t)$ are denoted by $a_k, \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots,$ determine the indices for which the coefficients are nonzero. List these nonzero Fourier series coefficients and their values in a table.

PROBLEM 5.4*:

A periodic signal $x(t)$ is described over one period $-2 \le t < 2$ by the equation

$$x(t) = \begin{cases} \cos(\frac{1}{2}\pi t) & \text{for } -1 \le t \le 1 \\ 0 & \text{for } -2 \le t < -1 \text{ and } 1 < t < 2 \end{cases}$$

The period of this signal is $T_0 = 4$ sec. We have seen that such a periodic signal can be represented by a Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{where} \quad a_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-jk\omega_0 t}\, dt$$

with $\omega_0 = 2\pi/T_0$.

(a) Sketch the periodic function $x(t)$ for $-6 \le t < 6$.

(b) Determine a_0, the DC coefficient for the Fourier series.

(c) Set up the *Fourier analysis* integral for determining a_k for $k \ne 0$. (Insert proper limits and integrand.)

(d) Evaluate the integral in the previous part and obtain an expression for a_k that is valid for all k. *Note:* In this case, your final formula for a_k can be simplified so that it is purely real.

PROBLEM 5.5*:

The periodic signal $x(t)$ described over one period $-2 \le t < 2$ by the equation

$$x(t) = \begin{cases} \cos(\frac{1}{2}\pi t) & \text{for } -1 \le t \le 1 \\ 0 & \text{for } -2 \le t < -1 \text{ and } 1 < t < 2 \end{cases}$$

can be represented by a Fourier series.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(\pi/2)kt}$$

(a) If we add a constant value to $x(t)$, we obtain a new signal, e.g., $w(t) = x(t) + \frac{1}{2}$. Make a plot of the periodic signal $w(t)$ over the time interval $-6 \le t < 6$.

(b) The new signal $w(t)$ can also be represented by a Fourier series,

$$w(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t},$$

because it is periodic with period T_0. Explain how b_0 and b_k are related to a_0 and a_k.
Hint: You should not have to evaluate any new integrals explicitly to answer this question.

(c) Form another new signal by time-shifting $x(t)$, e.g., $y(t) = x(t-1)$. Sketch the waveform for $y(t) = x(t-1)$ over the time interval $-6 \le t < 6$.

(d) The new signal $y(t)$ can also be represented by a Fourier series,

$$y(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t},$$

because it is periodic with period T_0. Explain how c_0 and c_k are related to a_0 and a_k.
Hint: You should not have to evaluate any new integrals explicitly to answer this question.

(e) From the result of the previous part, what is the relationship between the magnitudes of a_k and c_k?