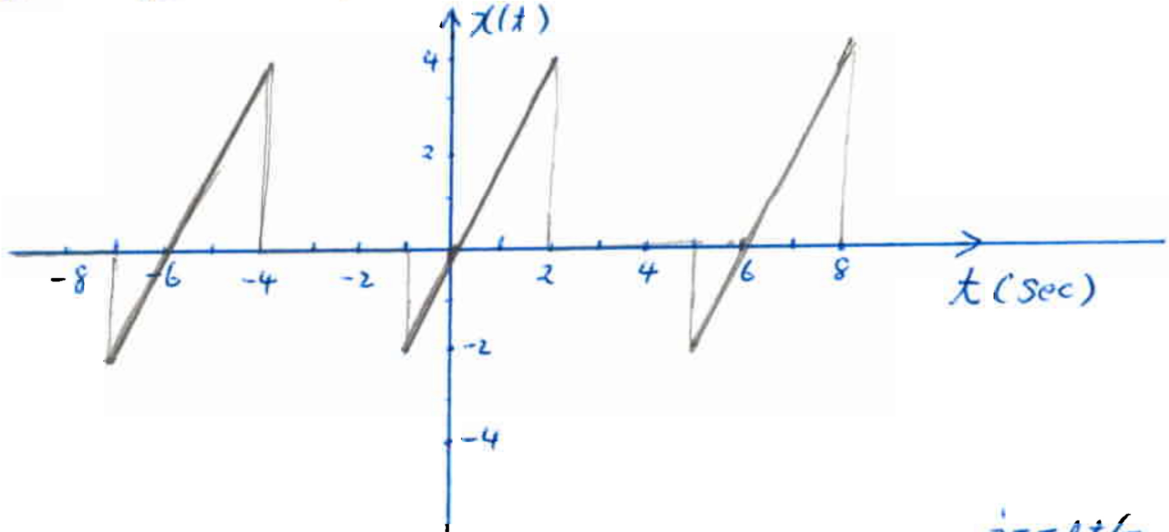
ECE 2025 Spring 2005

Homework #4 Solutions

Problem 4.1

(a) plot of $x(t) = \begin{cases} 2t & \text{for } -1 \le t < 2 \\ 0 & \text{for } 2 < t < 5 \end{cases}$

over $-8 \le t \le 8$, $T_0 = 6$ sec



(b) $a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt \quad \left(\because a_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-j2\pi k t/T_0}\,dt \;\Big|_{k=0}\right)$

$= \frac{1}{6}\int_{-1}^{2} 2t\,dt = \frac{1}{6}\left[t^2 \Big|_{-1}^{2} \right]$

$= \frac{1}{6}[4 - 1] = \frac{1}{2}$

$a_0 = \frac{1}{2}$

(c) $a_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-jk2\pi t/T_0}\,dt$

$= \frac{1}{6}\int_{-1}^{2} 2t\, e^{-j(2\pi/6)kt}\,dt$ *

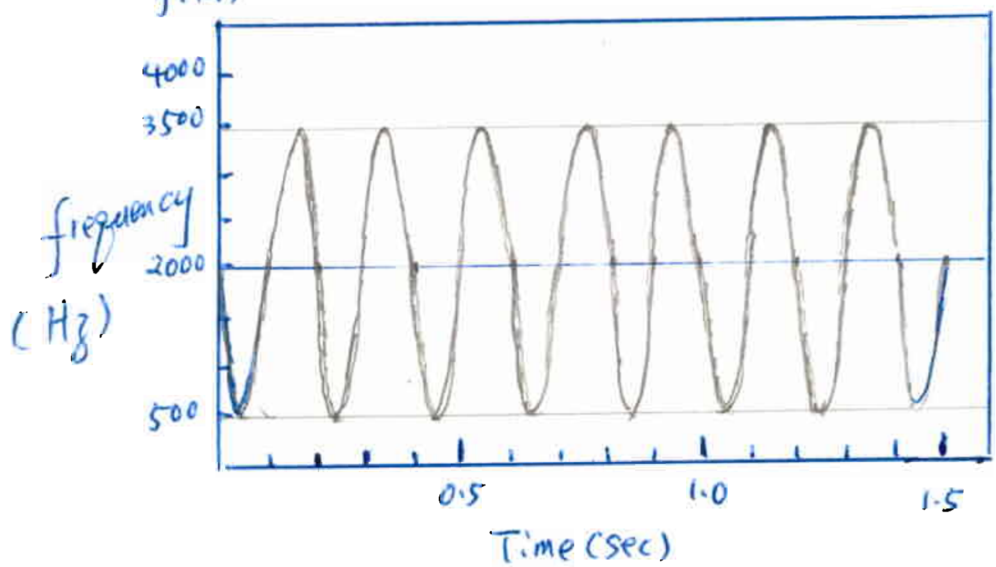
Problem 4.2

(a) From the MATLAB code, the angle function psi is $\varphi(t)$

$$\varphi(t) = 4000\pi t + 300\cos(10\pi t)$$

$$f_i(t) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt} = \frac{(4000\pi - 3000\pi\sin(10\pi t))}{2\pi}$$

$$f_i(t) = 2000 - 1500\sin(10\pi t)$$



(b) The instaneous frequency of an FM signal is

$$\omega_i(t) = 3000\pi + 800\pi\cos(5\pi t)$$

over a duration of 1.0 sec

$$\varphi(t) = 3000\pi t + \frac{800\pi}{5\pi}\sin(5\pi t) + \phi_2$$

$$= 3000\pi t + 160\sin(5\pi t) + \phi_2$$

The MATLAB code is:

```
tt = 0 : 0.0001 : 1.0 ;
BB = 160 ;
W2 = 5*pi ;
φ2 = 0 ;
w0 = 3000*pi ;
psi = w0*tt + BB*sin(w2*tt + φ2);
xx = real(exp(j*psi))
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problem 4.3

(a) Determine the foundamental frequency f_0 in Hz

$$f_0 = GCD(60, 150)$$
$$= 30 Hz$$

(b) Determine the foundamental period, T_0

$$T_0 = \frac{1}{f_0} = \frac{1}{30} \text{ sec}$$

(c) Determine the DC value of this signal

$$A_0 = 12 e^{j\pi}$$
$$= 12(\cos\pi + j\sin\pi)$$
$$= 12(-1 + 0) = -12$$

the DC value is -12 ※

problem 4.4

$x(t)$ is periodic with $T_0 = 4$ sec

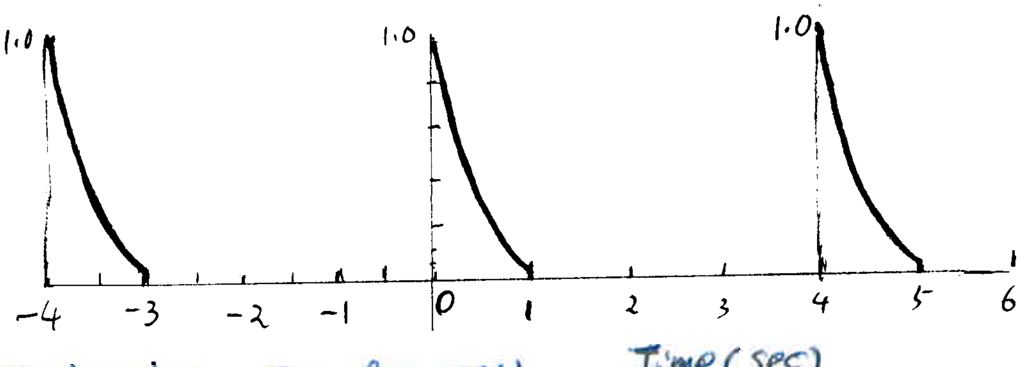
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/4)kt}$$

$$a_k = \frac{1}{4}\int_0^1 e^{-3t} e^{-j(2\pi/4)kt}\,dt$$

(a) Equation for $x(t)$

$$x(t) = \begin{cases} e^{-3t} & \text{for } 0 \le t < 1 \\ 0 & \text{for } 1 \le t < 4 \end{cases}$$

(b) plot of $x(t)$ over the range $-4 \le t \le 6$



(c) Determine a_2 for $x(t)$,

Express the answer in polar form $a_2 = \rho e^{j\varphi}$

$$a_2 = \frac{1}{4}\int_0^1 e^{-3t} e^{-j(2\pi/4)2t}\,dt$$

$$= \frac{1}{4}\int_0^1 e^{-3t} e^{-j\pi t}\,dt = \frac{1}{4}\int_0^1 e^{-(3+j\pi)t}\,dt$$

$$= \frac{1}{4}\,\frac{1}{-(3+j\pi)}\,e^{-t(3+j\pi)}\Big|_0^1 = \frac{1}{(-12-j4\pi)}\left(e^{-(3+j\pi)}-1\right)$$

$$a_2 = \frac{(e^{-3}+1)}{12+j4\pi} = \rho e^{j\varphi}$$

$$= 0.0604\, e^{-j*0.8084}$$

$$\begin{cases} \rho = 0.0604 \\ \varphi = -0.8084 \end{cases}$$

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problem 4.5

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$

Let the ratio be $r = 2^n$, n is an real number

$$r^{12} = 2 = 2^1$$
$$= (2^n)^{12} = 2^{12n}$$
$$12n = 1 \qquad n = \frac{1}{12}$$
$$r = 2^{1/12}$$

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(b)

C	C#	D	Eb	E	F	F#	G	G#	A	Bb	B	C
40	41	42	43	44	45	46	47	48	49	50	51	52
262	277	294	311	330	349	370	392	415	440	466	494	523

$$C = 440 * 2^{40-49} = 261.6256 = 262\ Hz$$
$$C^{\#} = 440 * 2^{41-49} = 277\ Hz$$

⋮

(c) The simple formula is:

$$f_n = 440 \times 2^{(n-49)/12}$$
$$f_n = 440 \times 2^{(n-49)/12}$$

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