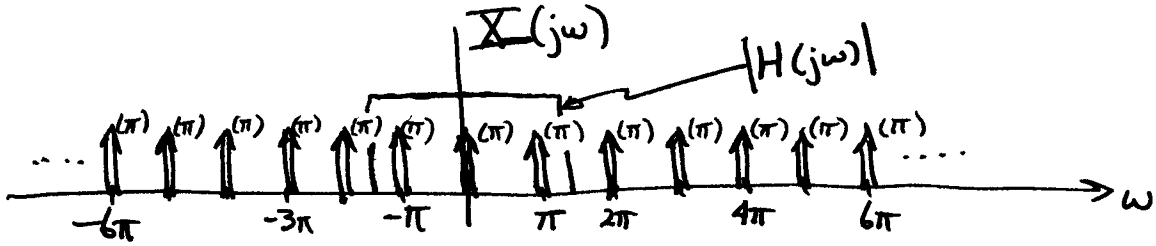
Prob 13.1

(a) $x(t) = \sum_{k=-\infty}^{\infty} \frac{1}{2} e^{j\pi k t} \Rightarrow X(j\omega) = \sum_{k=-\infty}^{\infty} \pi \delta(\omega - k\pi)$



(b) $h(t) = \frac{3\sin(\omega_{co}(t+0.2))}{\pi(t+0.2)} \longrightarrow H(j\omega) = 3e^{j0.2\omega}[\underbrace{u(\omega+\omega_{co}) - u(\omega-\omega_{co})}_{\text{rectangle}}]$

$H(j\omega)$ is an ideal LPF
passband from $-\omega_{co}$ to $+\omega_{co}$

(c) $Y(j\omega) = H(j\omega)X(j\omega)$

$$= H(-j\pi)\pi\delta(\omega+\pi) + H(j0)\pi\delta(\omega) + H(j\pi)\pi\delta(\omega-\pi)$$

$$= 3\pi e^{-j0.2\pi}\delta(\omega+\pi) + 3\pi\delta(\omega) + 3\pi e^{j0.2\pi}\delta(\omega-\pi)$$

$$y(t) = \frac{3}{2}e^{-j0.2\pi}e^{-j\pi t} + \frac{3}{2} + \frac{3}{2}e^{j0.2\pi}e^{j\pi t}$$

$$= \frac{3}{2} + 3\cos(\pi t + 0.2\pi)$$

(d) To get $y(t) = C$, we need a LPF that only passes the impulse at DC ($\omega = 0$). Thus ω_{co} must satisfy $0 < \omega_{co} < \pi$. In that case.

$Y(j\omega) = 3\pi\delta(\omega)$

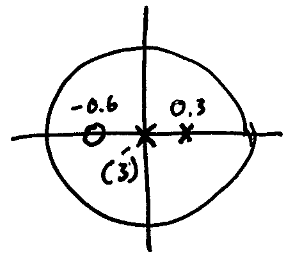
$\Rightarrow y(t) = \frac{3}{2}$ $\quad \therefore \boxed{C = \frac{3}{2}}$

13.2 $$y(n) = 0.3\,y(n-1) + x(n-3) + 0.6\,x(n-4)$$

(a) $$H(z) = \frac{z^{-3} + 0.6 z^{-4}}{1 - 0.3 z^{-1}}$$

(b) poles: $0.3, 0, 0, 0$

zeros: $-0.6, \infty, \infty, \infty$



(c) $x(n) = (-0.6)^n u(n)$

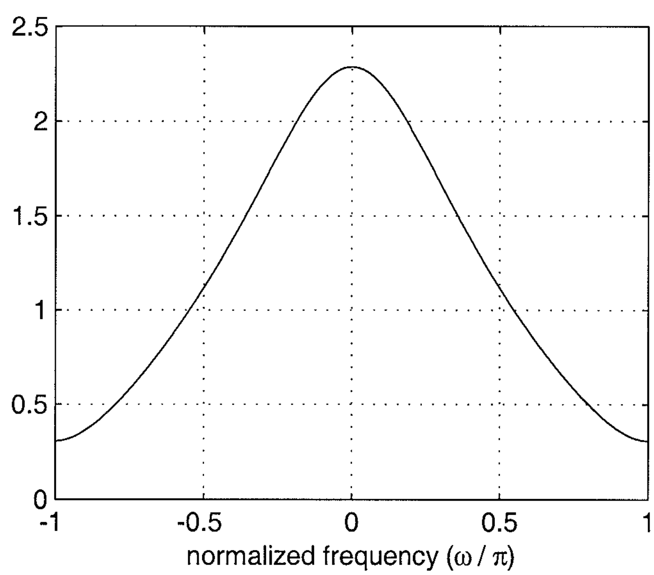
$$X(z) = \frac{1}{1 + 0.6 z^{-1}}$$

$$Y(z) = H(z)X(z) = \frac{z^{-3}(1 + 0.6 z^{-1})}{1 - 0.3 z^{-1}} \cdot \frac{1}{1 + 0.6 z^{-1}} = \frac{z^{-3}}{1 - 0.3 z^{-1}}$$

$$\boxed{y(n) = (0.3)^{(n-3)}\, u(n-3)}$$

(d) $$H(e^{j\omega}) = H(z)\Big|_{z = e^{j\omega}} = \boxed{e^{-j3\omega} \frac{1 + 0.6 e^{-j\omega}}{1 - 0.3 e^{-j\omega}}}$$

(e)



Prob 13.3

(a) $$\frac{Y(z)}{X(z)} = H(z) = \frac{1-0.2z^{-2}+0.3z^{-5}}{1+0.5z^{-3}-0.9z^{-7}}$$

$$\Rightarrow (1+0.5z^{-3}-0.9z^{-7})Y(z) = (1-0.2z^{-2}+0.3z^{-5})X(z)$$

$$\Rightarrow y[n] = -0.5y[n-3]+0.9y[n-7]+x[n]-0.2x[n-2]+0.3x[n-5]$$

(b) aa = [1, 0, 0, 0.5, 0, 0, 0, −0.9]

bb = [1, 0, −0.2, 0, 0, 0.3]

Prob 13.4

(a) $$H_a(z) = \frac{1}{1-0.5z^{-1}} + \frac{z^{-1}}{1-0.5z^{-1}}$$

$$h_a[n] = (0.5)^n u[n] + (0.5)^{n-1} u[n-1]$$

(b) $$H_b(z) = \frac{1/2}{1+3/4\, e^{j0.3\pi}z^{-1}} + \frac{1/2}{1+3/4\, e^{-j0.3\pi}z^{-1}}$$

$$h_b[n] = \frac{1}{2}\left(-\frac{3}{4}e^{j0.3\pi}\right)^n u[n] + \frac{1}{2}\left(-\frac{3}{4}e^{-j0.3\pi}\right)^n u[n]$$

$$= \left(-\frac{3}{4}\right)^n \left\{ \frac{1}{2}e^{j0.3\pi n} + \frac{1}{2}e^{-j0.3\pi n} \right\} u[n]$$

$$= \left(-\frac{3}{4}\right)^n \cos(0.3\pi n)\, u[n]$$

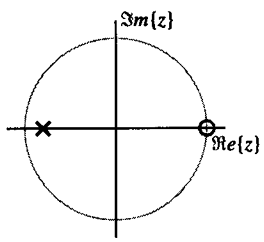
(c) $$H_c(z) = \frac{0.6+z^{-1}}{1+0.6z^{-1}} = 0.6 + \frac{0.64z^{-1}}{1+0.6z^{-1}}$$

$$h_c[n] = 0.6\delta[n] + 0.64(-0.6)^{n-1}u[n-1]$$

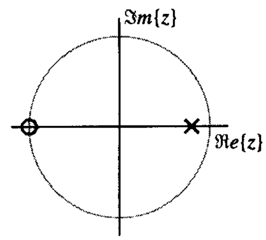
An alternate form is

$$h_c[n] = 0.6(-0.6)^n u[n] + (-0.6)^{n-1}u[n-1]$$

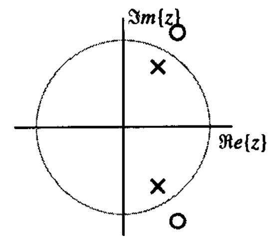
PROBLEM 13.5:



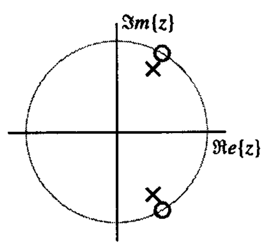
Pole–Zero Plot #1



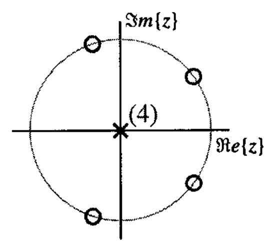
Pole–Zero Plot #2



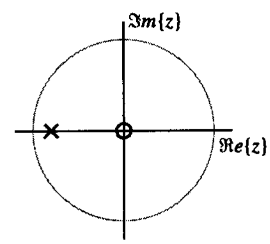
Pole–Zero Plot #3



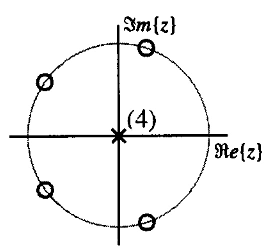
Pole–Zero Plot #4



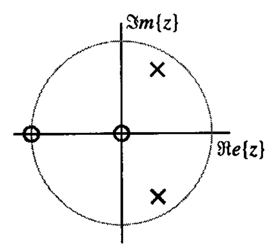
Pole–Zero Plot #5



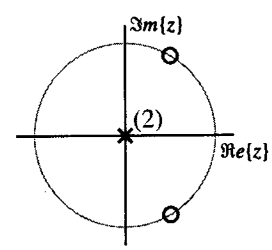
Pole–Zero Plot #6



Pole–Zero Plot #7



Pole–Zero Plot #8



Pole–Zero Plot #9

For each of systems below[2] determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. *Note:* the unit circle is shown for reference. ***Hint:*** You might find it helpful to work the previous problem before this one even though it does not have a star.

#2 – $\mathcal{S}_1$: $H(z) = \dfrac{1+z^{-1}}{1-0.8z^{-1}}$ zero at $z=-1$, pole at $z=0.8$

$H(z) = 2\dfrac{1-z^{-5}}{1-z^{-1}}$

#7 – $\mathcal{S}_2$: $y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4]$ FIR, 4 zeros, running sum

#4 – $\mathcal{S}_3$: $H(z) = \dfrac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$ zeros on U.C., poles at $0.8e^{\pm j\pi/3}$ } Notch filter

#6 – $\mathcal{S}_4$: $y[n] = -0.8y[n-1] + 2x[n]$ $H(z) = \dfrac{2}{1+0.8z^{-1}}$ zero at $z=0$, pole at $z=-0.8$

#8 – $\mathcal{S}_5$: $H(z) = \dfrac{1.8(1+z^{-1})}{1 - 0.8z^{-1} + 0.64z^{-2}}$ zero at $z=-1$, complex poles at $0.8e^{\pm j\pi/3}$

#9 – $\mathcal{S}_6$: $y[n] = \dfrac{10}{3}(x[n] - x[n-1] + x[n-2])$ $H(z) = \frac{10}{3}(1 - z^{-1} + z^{-2})$ Two zeros on U.C. at $e^{\pm j\pi/3}$

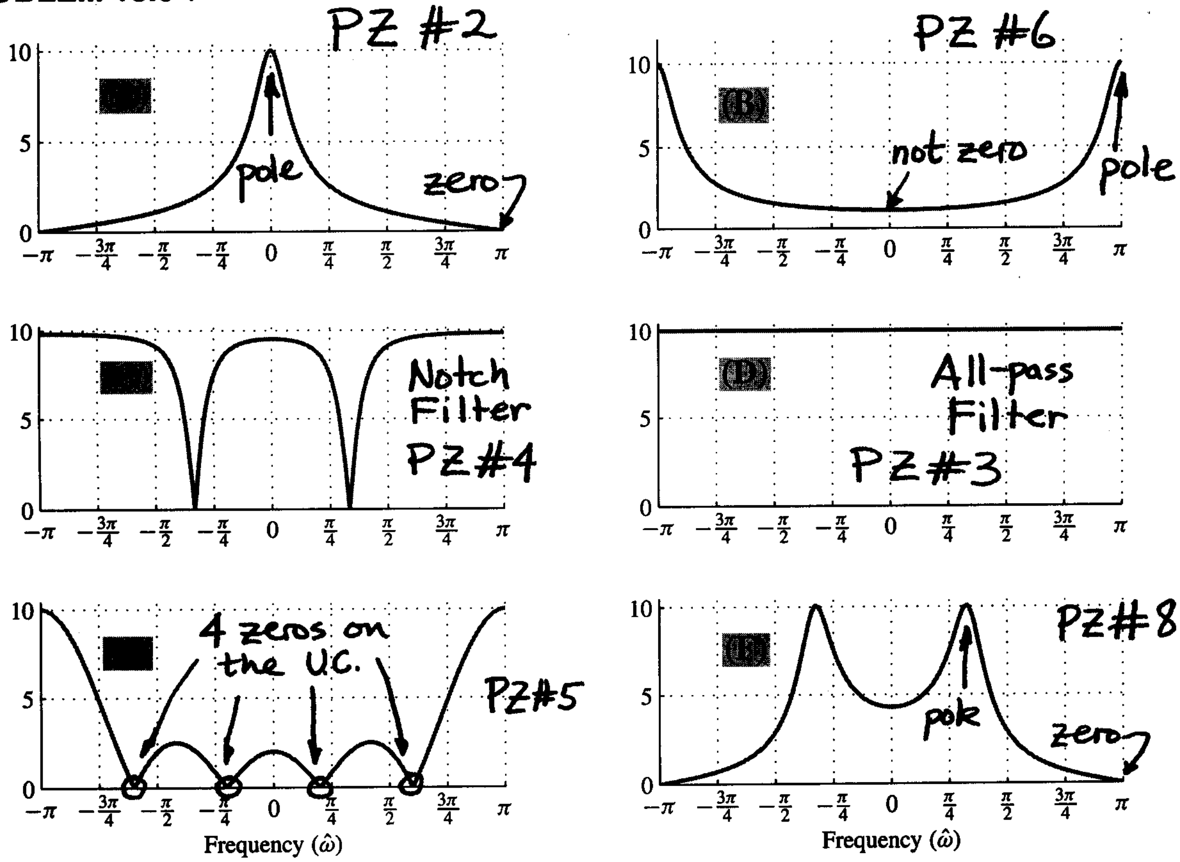
#5 – $\mathcal{S}_7$: $H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$ FIR, 4 zeros $H(z) = 2\dfrac{1+z^{-5}}{1+z^{-1}}$

#3 – $\mathcal{S}_8$: $y[n] = 0.8y[n-1] - 0.64y[n-2] + 6.4x[n] - 8x[n-1] + 10x[n-2]$ complex zeros, outside the U.C.

$H(z) = \dfrac{6.4 - 8z^{-1} + 10z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$

[2] These same systems are also used in the next problem.

PROBLEM 13.6*:



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. *Note:* the frequency axis is $\hat{\omega}$.

(A) $\mathcal{S}_1$: $H(z) = \dfrac{1+z^{-1}}{1-0.8z^{-1}}$ zero at $z=-1=e^{j\pi}$ at $\hat{\omega}=\pi$; pole at $z=0.8 \Rightarrow$ LPF

None $\mathcal{S}_2$: $y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4]$ FIR, 4 zeros on U.C. running sum: zeros at $e^{j2\pi k/5}$

(C) $\mathcal{S}_3$: $H(z) = \dfrac{8-8z^{-1}+8z^{-2}}{1-0.8z^{-1}+0.64z^{-2}}$ zeros on U.C. at $e^{\pm j\pi/3}$ or $\hat{\omega}=\pm\pi/3$ Notch Filter

(B) $\mathcal{S}_4$: $y[n] = -0.8y[n-1] + 2x[n]$ pole at $z=-0.8 \Rightarrow$ HPF (not zero at $\hat{\omega}=0$)

(F) $\mathcal{S}_5$: $H(z) = \dfrac{1.8(1+z^{-1})}{1-0.8z^{-1}+0.64z^{-2}}$ zero at $z=-1 \Rightarrow$ at $\hat{\omega}=\pi$; complex poles give a peak at $\hat{\omega}=\pm\pi/3$

None $\mathcal{S}_6$: $y[n] = \dfrac{10}{3}(x[n] - x[n-1] + x[n-2])$ zeros at $e^{\pm j\pi/3} \Rightarrow$ nulls at $\hat{\omega}=\pm\pi/3$

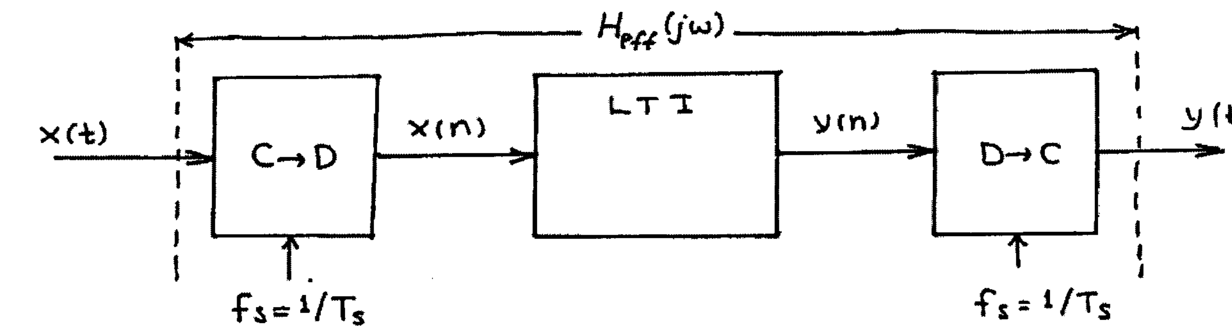
(E) $\mathcal{S}_7$: $H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$ zeros on U.C. at $e^{\pm j\pi/5}$ & $e^{\pm j3\pi/5}$

(D) $\mathcal{S}_8$: $y[n] = 0.8y[n-1] - 0.64y[n-2] + 6.4x[n] - 8x[n-1] + 10x[n-2]$

$H(z) = \dfrac{6.4-8z^{-1}+10z^{-2}}{1-0.8z^{-1}+0.64z^{-2}}$ is all-pass filter

$H(1) = H(e^{j0}) = \dfrac{6.4-8+10}{1-0.8+0.64} = 10$

Problem 13.7:



(a) $y(n) = 0.8\,y(n-1) + x(n) + x(n-2)$

$f_s = 200$ Hz

$$Y(z) = 0.8z^{-1}Y(z) + X(z) + z^{-2}X(z) \Rightarrow$$

$$H(z) = \frac{1 + z^{-2}}{1 - 0.8z^{-1}}$$

$$H(e^{j\hat{\omega}}) = \frac{1 + e^{-j2\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}} \qquad -\pi < \hat{\omega} < \pi$$

$$H_{eff}(j\omega) = \frac{1 + e^{-j2(\omega/200)}}{1 - 0.8e^{-j(\omega/200)}} \qquad -\pi\cdot 200 < \omega < \pi\, 200$$

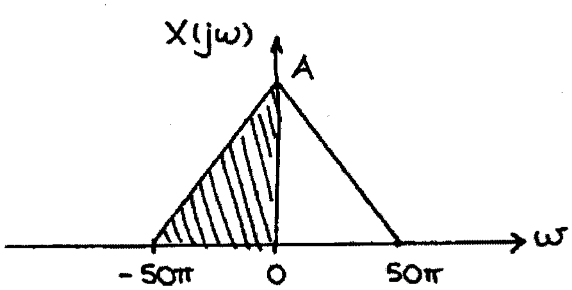
$$\hat{\omega} = 2\pi\hat{f} = 2\pi\frac{f}{f_s} = \omega T_s = \omega/200$$

$$y(t) = 2\,|H_{eff}(j(\omega = 100\pi))|\cos(100\pi t + \angle H_{eff}(\omega = 100\pi))$$

$$H_{eff}(\omega = 100\pi) = \frac{1 + e^{-j2\frac{100\pi}{200}}}{1 - 0.8e^{-j\frac{100\pi}{200}}} = \frac{1 + e^{-j\pi}}{1 - 0.8e^{-j\pi/2}} = 0$$

Therefore $y(t) = 0$

(b)



$f_s \geq 2f_{max}$

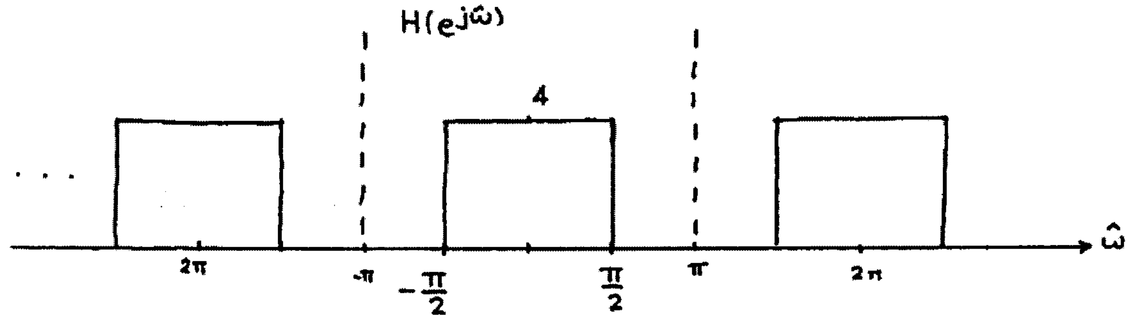
$\omega_{max} = 50\pi = 2\pi(25)$ ~

$f_{max} = 25$ Hz

$f_s \geq 2\cdot 25 = 50$Hz ~

$f_{s\,min} = 50$Hz

(c)

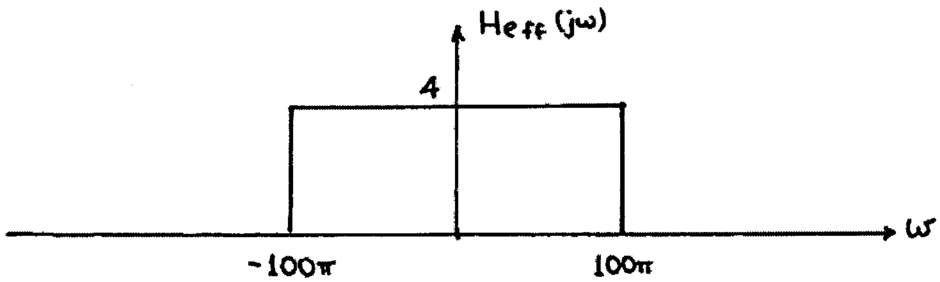


$f_s = 200\,\text{Hz}$

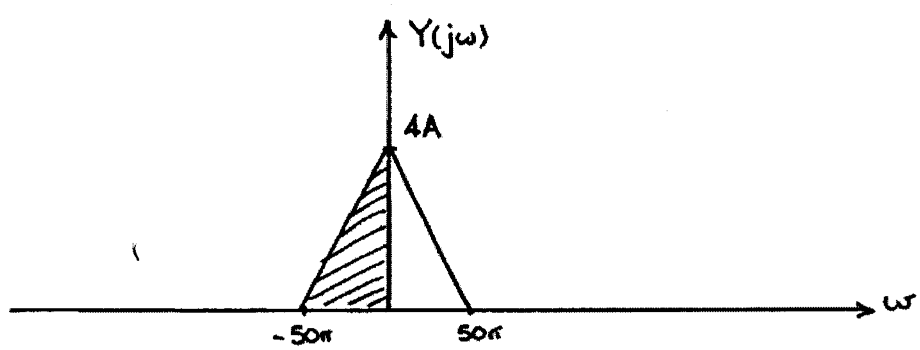
$$H_{eff}(j\omega) = H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\omega T_s} = H(e^{j\omega/200}) \qquad -\frac{\pi}{2} < \omega T_s = \hat{\omega} < \frac{\pi}{2}$$

$$\rightsquigarrow \quad H_{eff}(j\omega) = \begin{cases} 4 & \left|\frac{\omega}{200}\right| < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases} \quad \rightsquigarrow$$

$$H_{eff}(j\omega) = \begin{cases} 4 & |\omega| < 100\pi \\ 0 & |\omega| > 100\pi \end{cases}$$



$$Y(j\omega) = H_{eff}(j\omega)\,X(j\omega)$$



(d) $\quad -\frac{\pi}{2} < \hat{\omega} < \frac{\pi}{2} \quad \rightsquigarrow \quad -\frac{\pi}{2} < \omega T_s < \frac{\pi}{2} \quad \rightarrow \quad -\frac{\pi}{2} f_s < \omega < \frac{\pi}{2} f_s$

For $X(j\omega) = Y(j\omega) \rightsquigarrow \quad 50\pi \leq \frac{\pi}{2} f_s \Rightarrow f_{s\,min} = \dfrac{100\pi}{\pi} = 100\,\text{Hz}$