(12.1) P-11.7

(a) FROM THE TABLE: $\frac{\sin(\omega_b t)}{\pi t} \longleftrightarrow u(\omega+\omega_b) - u(\omega-\omega_b)$

$$\Rightarrow \frac{10\sin(200\pi t)}{\pi t} \longleftrightarrow 10\left[u(\omega+200\pi) - u(\omega-200\pi)\right]$$

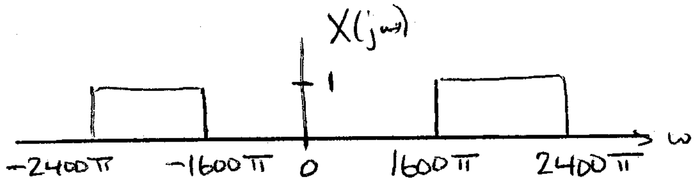
DIFFERENTIATION PROPERTY: $\frac{d}{dt}y(t) \longleftrightarrow j\omega Y(j\omega)$

$$\Rightarrow \frac{d}{dt}\left[\frac{10\sin(200\pi t)}{\pi t}\right] \longleftrightarrow 10 j\omega\left[u(\omega+200\pi) - u(\omega-200\pi)\right]$$

(b) $$\frac{2\sin(400\pi t)}{\pi t} \longleftrightarrow 2\left[u(\omega+400\pi) - u(\omega-400\pi)\right]$$

MODULATION PROPERTY: $y(t)\cos(\omega_o t) \longleftrightarrow \frac{1}{2}Y(j(\omega-\omega_o)) + \frac{1}{2}Y(j(\omega+\omega_o))$

$$\Rightarrow \frac{2\sin(400\pi t)}{\pi t}\cos(2000\pi t) \longleftrightarrow$$



(c) FROM THE TABLE: $\sum_{n=-\infty}^{\infty}\delta(t-nT) \longleftrightarrow \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi}{T}k\right)$

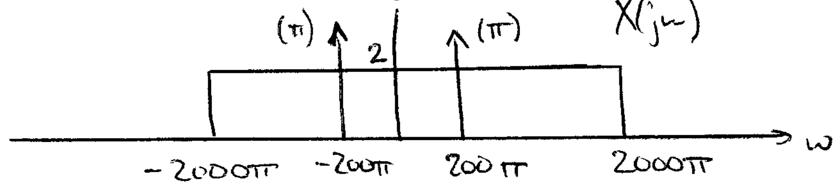
HERE, $T = 10$, SO

$$\sum_{n=-\infty}^{\infty}\delta(t-10n) \longleftrightarrow \frac{\pi}{5}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{\pi k}{5}\right)$$

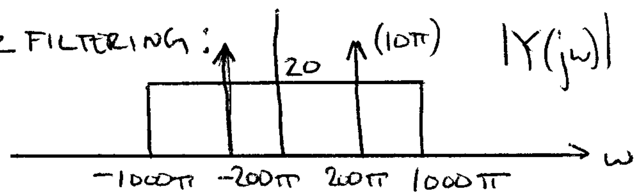
(12.2) P-11.13

(a) $\cos(200\pi t) \longleftrightarrow \pi\delta(\omega - 200\pi) + \pi\delta(\omega + 200\pi)$

$$\frac{2\sin(2000\pi t)}{\pi t} \longleftrightarrow 2\left[u(\omega + 2000\pi) - u(\omega - 2000\pi)\right]$$



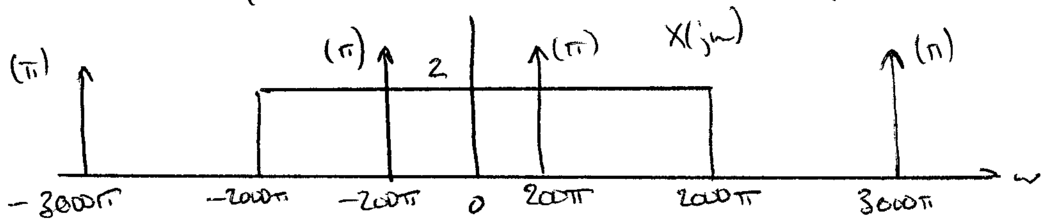
AFTER FILTERING:



$\measuredangle Y(j\omega) = -0.0025\omega$

$$\Rightarrow y(t) = 10\cos\left(200\pi(t - 0.0025)\right) + \frac{20\sin\left[1000\pi(t - 0.0025)\right]}{\pi(t - 0.0025)}$$

(b) $X(j\omega) = \pi\delta(\omega - 200\pi) + \pi\delta(\omega + 200\pi) +$
$2\left[u(\omega + 2000\pi) - u(\omega - 2000\pi)\right] +$
$\pi\delta(\omega - 3000\pi) + \pi\delta(\omega + 3000\pi)$



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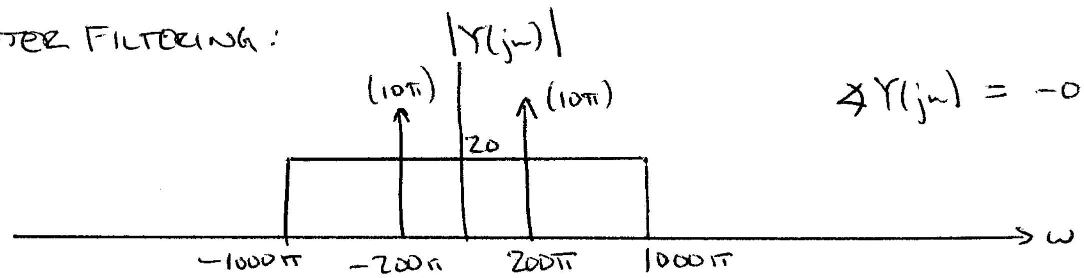
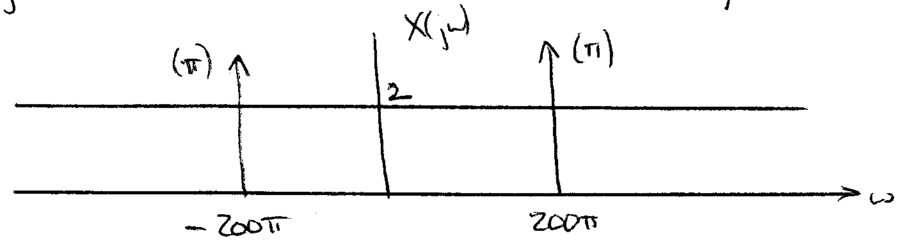


$\measuredangle Y(j\omega) = -0.0025\omega$

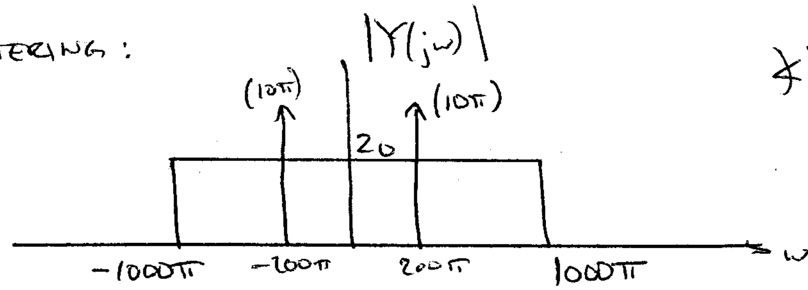
$$\Rightarrow y(t) = 10\cos\left(200\pi(t - 0.0025)\right) + \frac{20\sin\left[1000\pi(t - 0.0025)\right]}{\pi(t - 0.0025)}$$

(same as for (a))

(c) $X(j\omega) = \pi\,\delta(\omega - 200\pi) + \pi\,\delta(\omega + 200\pi) + 2$



AFTER FILTERING:

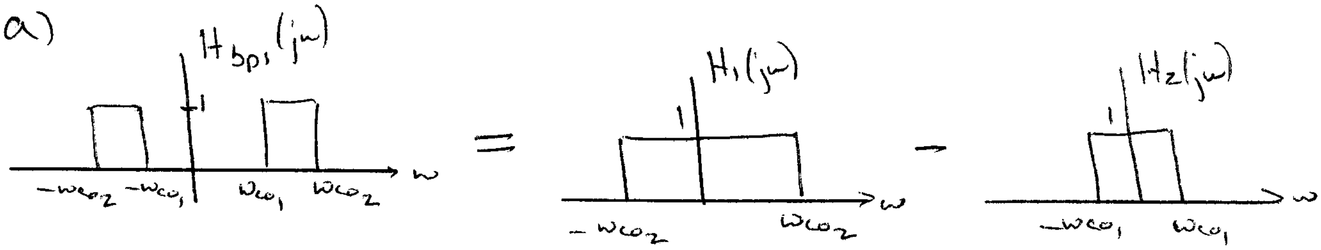


$\measuredangle Y(j\omega) = -0.0025\,\omega$

This is the same $y(t)$ as for (a) and (b).

(c) $H(j\omega)$ is an ideal lowpass filter with cutoff frequency 1000π and a delay of 0.0025 s. Because the input signals in each of these three cases is identical for $|\omega| < 1000\pi$, the output is identical.

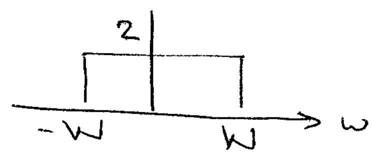
(12.3) (a)



$$\frac{\sin(\omega_{co2}t)}{\pi t} \longleftrightarrow H_1(j\omega) \text{ and } \frac{\sin(\omega_{co1}t)}{\pi t} \longleftrightarrow H_2(j\omega)$$

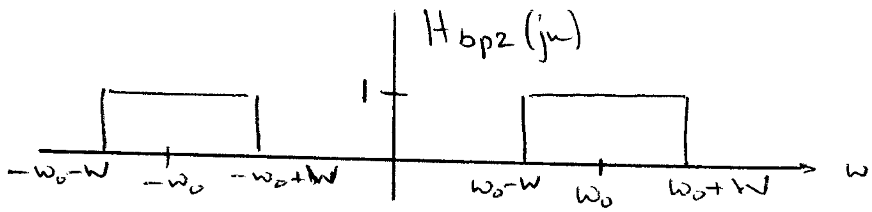
$$\Rightarrow h_{bp1}(t) = \frac{\sin(\omega_{co2}t)}{\pi t} - \frac{\sin(\omega_{co1}t)}{\pi t}$$

(b) $\dfrac{2\sin(Wt)}{\pi t} \longleftrightarrow$



and $x(t)\cos(\omega_0 t) \longleftrightarrow \frac{1}{2}X(j(\omega-\omega_0)) + \frac{1}{2}X(j(\omega+\omega_0))$

Therefore, $H_{bp2}(j\omega)$ has the graph:



(c) ω_0 is the average of ω_{co1} and ω_{co2}:

$$\omega_0 = \frac{\omega_{co1}+\omega_{co2}}{2}$$

W is half the width of the filter passband:

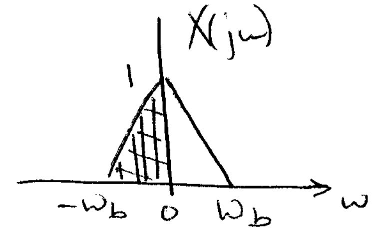
$$W = \frac{\omega_{co2}-\omega_{co1}}{2}$$

12.4

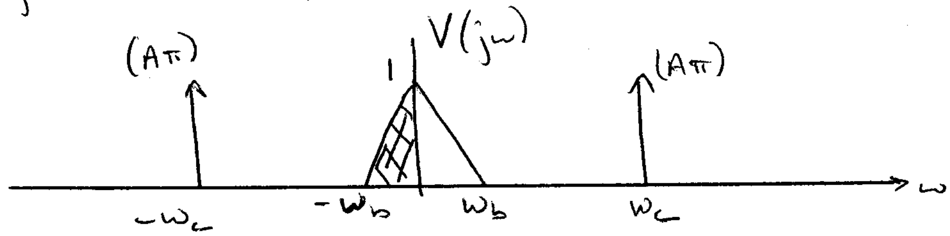
(a) $v(t) = x(t) + A\cos(\omega_c t)$

$$\Rightarrow V(j\omega) = X(j\omega) + A\pi\,\delta(\omega-\omega_c) + A\pi\,\delta(\omega+\omega_c)$$

If $X(j\omega)$ has the spectrum:



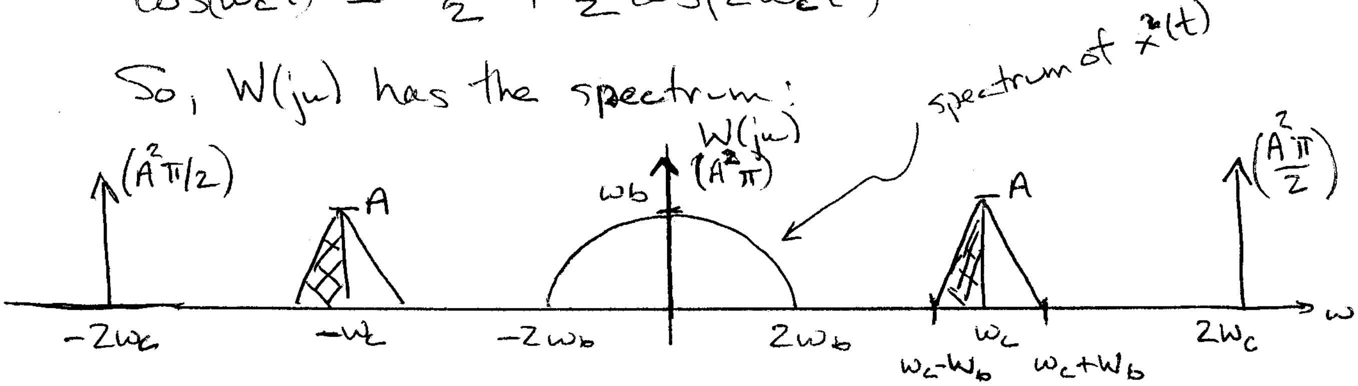
then $V(j\omega)$ has the spectrum:



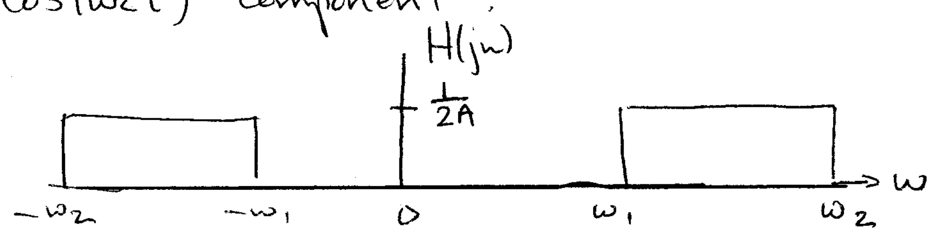
(b) $w(t) = v^2(t) = x^2(t) + 2A\,x(t)\cos(\omega_c t) + A^2\cos^2(\omega_c t)$

$\cos^2(\omega_c t) = \frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)$

So, $W(j\omega)$ has the spectrum:



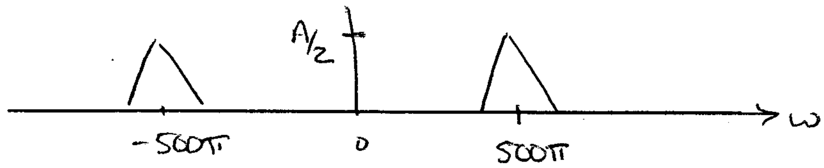
(c) Need to filter $w(t)$ so as to keep only the $x(t)\cos(\omega_c t)$ component:



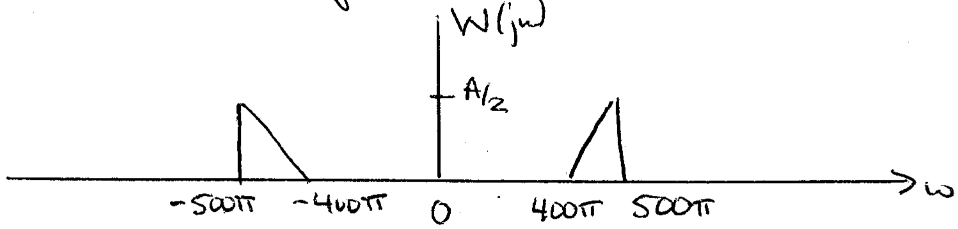
Need $2\omega_b < \omega_1 < \omega_c - W$ and $\omega_c + W < \omega_2 < 2\omega_c$.

To avoid overlap in the spectral components of $W(j\omega)$, we need $\omega_c > 3\omega_b$.

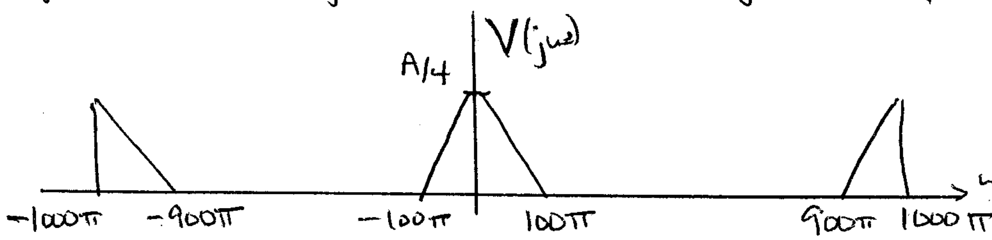
12.5 (a) $x(t)\cos(\omega_c t) \longleftrightarrow \frac{1}{2}X(j(\omega-\omega_c)) + \frac{1}{2}X(j(\omega+\omega_c))$



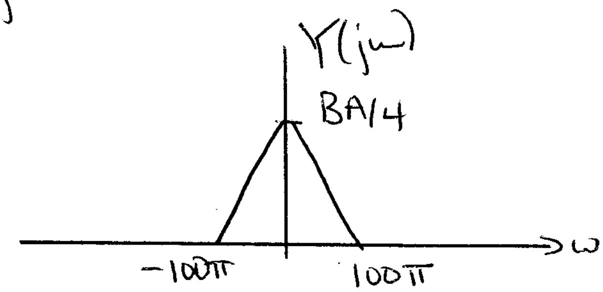
After lowpass filtering:



(b) $V(j\omega) = \frac{1}{2}W(j(\omega-\omega_c)) + \frac{1}{2}W(j(\omega+\omega_c))$



(c) For $Y(j\omega)$ we have



If $B=4$, then $Y(j\omega) = X(j\omega)$.