10.2

(a) $\left(e^{-t/2}u(t-1.5) - 2\sin(7\pi t)u(t)\right)\delta(t+0.1) =$

$$\left(e^{+0.1/2}\,\overset{0}{\cancel{u(-0.1-1.6)}} - 2\sin(7\pi(-0.1))\,\overset{0}{\cancel{u(-0.1)}}\right)\delta(t+0.1) = \underline{0}$$

(b) $\left(\cos(1.5\pi(t-1)) + u(t-1)\right) * \delta(t-3) = \underline{\cos(1.5\pi(t-4)) + u(t-4)}$

(c) $\int_{-\infty}^{t+0.002} \delta(\tau-0.001)\cos\left(\frac{\pi}{2}\tau\right)u(\tau)\,d\tau =$

(this is non-zero only when $t+0.002 \geq 0.001$ or $t \geq -0.001$)

$$\int_{-\infty}^{t+0.002} \delta(\tau-0.001)\cos\left(\frac{\pi}{2}\cdot 0.001\right)\overset{1}{\cancel{u(0.001)}}\,d\tau$$

$$= \underline{\cos\left(\frac{\pi}{2}\cdot 0.001\right)u(t+0.001)}$$

(d) $\int_{-\infty}^{2} e^{\tau}\delta(\tau+1)\,d\tau = \underline{e^{-1}}$

(e) $\frac{d}{dt}\left\{3\cos\left(\frac{3}{2}\pi t\right)\left[u(t)-u(t-2)\right]\right\} =$

$$\underline{-\frac{9\pi}{2}\sin\left(\frac{3}{2}\pi t\right)\left[u(t)-u(t-2)\right] + 3\cos\left(\frac{3\pi}{2}t\right)\left[\delta(t)-\delta(t-2)\right]}$$

10.3

(a) $y(t) = e^{x(t+2)}$

not linear $\Rightarrow$ $a \cdot x(t) \rightarrow (y(t))^a$ not $a y(t)$

time-invariant $\Rightarrow$ $x_1(t-t_1) \rightarrow e^{x_1(t+2-t_1)} = e^{x_1((t-t_1)+2)} = y_1(t-t_1)$

stable $\Rightarrow$ the system is a memory-less operator so if $\max\{|x(t)|\} < M$ then $y(t)$ is bounded by e^M.

not causal $\Rightarrow$ there is a time shift of -2 so if the input starts at $t=0$ (e.g. $x(t) = u(t)$) then the output starts <u>before</u> that at $t=-2$.

(b) $y(t) = \cos(\omega_c t + x(t))$

not linear $\Rightarrow$ $a x(t) \rightarrow \cos(\omega_c t + a x(t)) \neq a y(t)$

not time-invariant $\Rightarrow$ let $x_1(t) = \pi u(t)$, $x_2(t) = x_1(t-1)$

$y_1(t) = \cos(\omega_c t + \pi u(t)) = \cos(\omega_c t) u(-t) - \cos(\omega_c t) u(t)$

$y_2(t) = \cos(\omega_c t + \pi u(t-1)) =$
$\cos(\omega_c t) u(1-t) - \cos(\omega_c t) u(t-1)$

but $y_1(t-1) = \cos(\omega_c (t-1)) u(1-t) - \cos(\omega_c (t-1)) u(t-1)$

$y_2(t) \neq y_1(t-1)$

stable $\Rightarrow$ $|y(t)| \leq 1$ for all $x(t)$

causal $\Rightarrow$ the output $y(t)$ does not depend on any future values of $x(t)$; $y(t)$ only depends on $x(t)$ at the present time.

10.3 cont.

(c) $y(t) = [A + x(t)] \cos(\omega_c t)$

not linear: if $x(t)=0$ then $y(t) = A\cos(\omega_c t) \neq 0$

not time-invariant: the system contains a component that does not depend on $x(t)$

$$x(t-1) \rightarrow [A + x(t-1)] \cos(\omega_c t)$$

$$y(t-1) = [A + x(t-1)] \cos(\omega_c (t-1))$$

so $x(t-1) \not\rightarrow y(t-1)$

stable: $y(t)$ is bounded by $|A+M|$ if $|x(t)| \leq M$

causal: $y(t)$ depends only on $x(t)$ at the present time

(d) $y(t) = \dfrac{x(t) + x(-t)}{2}$

linear: $a x_1(t) + b x_2(t) \longrightarrow \dfrac{a x_1(t) + b x_2(t) + a x_1(-t) + b x_2(-t)}{2}$

$$= \frac{a x_1(t) + a x_1(-t)}{2} + \frac{b x_2(t) + b x_2(-t)}{2}$$

$$= a y_1(t) + b y_2(t)$$

not time-invariant: let $x(t) = \delta(t)$

$$x(t) \rightarrow \tfrac{1}{2}\left(\delta(t) + \delta(-t)\right) = \delta(t) = y(t)$$

$$x(t-1) \rightarrow \tfrac{1}{2}\left(\delta(t-1) + \delta(-t-1)\right) = \tfrac{1}{2}\delta(t-1) + \tfrac{1}{2}\delta(t+1)$$

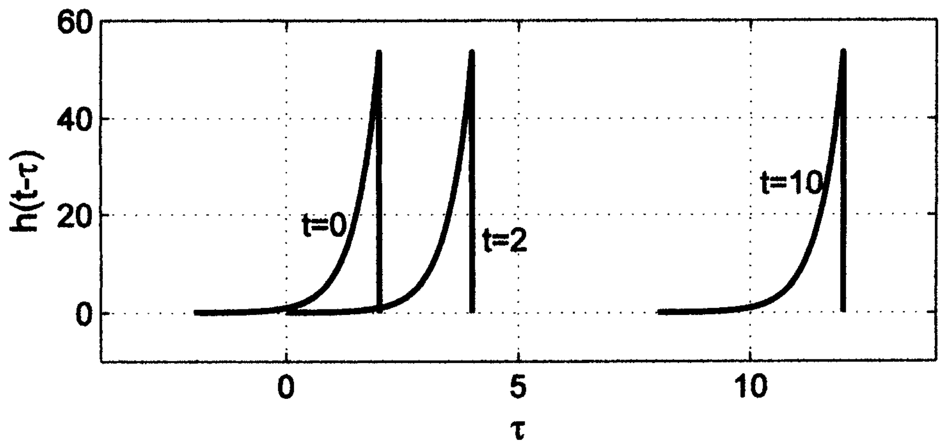
$$x(t-1) \not\rightarrow y(t-1)$$

Stable: if $|x(t)| < M$, then $|y(t)| < \frac{1}{2}M + \frac{1}{2}M = M$

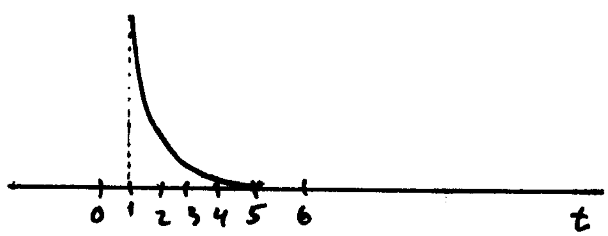
Not causal: If $x(t) = \delta(t-1)$, then $y(t) = \frac{1}{2}\delta(t-1) + \frac{1}{2}\delta(t+1)$ the "$-t$" makes it non-causal.

10.4 $h(t) = e^{-2t}\left[u(t+2) - u(t-2)\right]$

(a)



(b) for $x(t) = \delta(t-3)$, $y(t) = h(t-3) = e^{-2(t-3)}\left[u(t-1) - u(t-5)\right]$



(c) $x(t) = e^{-t/2}\left[u(t+1) - u(t-6)\right]$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Region

① $y(t) = 0$ for $t < -3$

② $y(t) = \int_{-1}^{t+2} e^{-\tau/2} e^{-2(t-\tau)}\, d\tau$ for $-3 \le t < 1$

10.4 cont.

(c) cont.

③ $$y(t) = \int_{t-2}^{t+2} e^{-\tau/2} e^{-2(t-\tau)}\, d\tau \qquad \text{for } 1 \le t < 4$$

④ $$y(t) = \int_{t-2}^{6} e^{-\tau/2} e^{-2(t-\tau)}\, d\tau \qquad \text{for } 4 \le t < 8$$

⑤ $$y(t) = 0 \qquad \text{for } t \geq 8$$

The basic integral is the same for all of these, only the limits are different.

$$\int_a^b e^{-\tau/2} e^{-2(t-\tau)}\, d\tau = e^{-2t}\int_a^b e^{1.5\tau}\, d\tau = \frac{e^{-2t}}{1.5}\left(e^{1.5b} - e^{1.5a}\right)$$

<u>region</u>

② $$y(t) = \frac{2e^{-2t}}{3}\left(e^{\frac{3(t+2)}{2}} - e^{-3/2}\right) = \frac{2}{3}\left(e^{-\frac{1}{2}t+3} - e^{-\left(\frac{3}{2}+2t\right)}\right)$$

③ $$y(t) = \frac{2}{3}e^{-2t}\left(e^{\frac{3(t+2)}{2}} - e^{\frac{3(t-2)}{2}}\right) = \frac{2}{3}e^{-\frac{1}{2}t}\left(e^{3} - e^{-3}\right)$$

④ $$y(t) = \frac{2}{3}e^{-2t}\left(e^{9} - e^{\frac{3(t-2)}{2}}\right) = \frac{2}{3}\left(e^{9-2t} - e^{-t/2-3}\right)$$

①,⑤ $y(t) = 0$

10.5

$$y(t) = \int_{t-3}^{t+1} x(\tau - 1)\, d\tau$$

(a) $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$, determine $h(\tau)$

Let $\gamma = \tau - 1$ for $\tau = t+1$, $\gamma = t$

$d\gamma = d\tau$ $\tau = t-3$, $\gamma = t-4$

then $$y(t) = \int_{t-4}^{t} x(\gamma)\, d\gamma$$

we can see that $h(t-\gamma) = \begin{cases} 0 & \gamma < t-4 \\ 1 & t-4 \le \gamma < t \\ 0 & \gamma \ge t \end{cases}$

when $\gamma < t-4$, $t-\gamma > 4$

$\gamma \ge t$, $t - \gamma \le 0$

so $\underline{h(t) = u(t) - u(t-4)}$

(b) This system is stable because

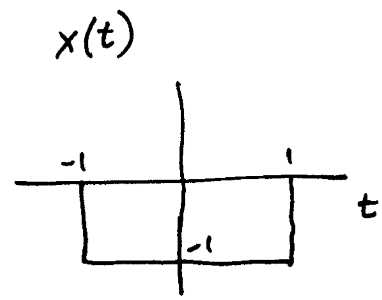
$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau = 4 < \infty$$

(c) This system is causal because

$h(t) = 0$ for $t < 0$

(d) $x(t) = u(t-1) - u(t+1)$

$$y(t) = \int_{-\infty}^{\infty} (u(\tau-1) - u(\tau+1))(u(t-\tau) - u(t-\tau-4))\, d\tau$$



10.5 cont.

(d) cont

Region

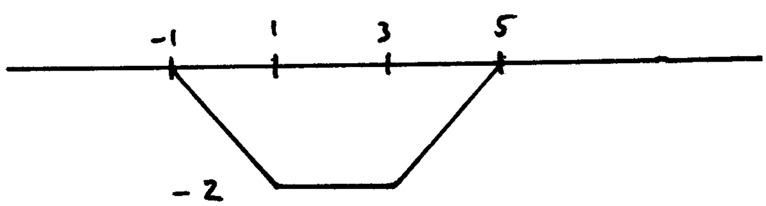
① $y(t) = 0 \qquad t < -1$

② $y(t) = \displaystyle\int_{-1}^{t} -1\, d\tau \qquad -1 \le t < 1$

③ $y(t) = \displaystyle\int_{-1}^{1} -1\, d\tau \qquad 1 \le t < 3$

④ $y(t) = \displaystyle\int_{t-4}^{1} -1\, d\tau \qquad 3 \le t < 5$

⑤ $y(t) = 0 \qquad t \geq 5$



10.6

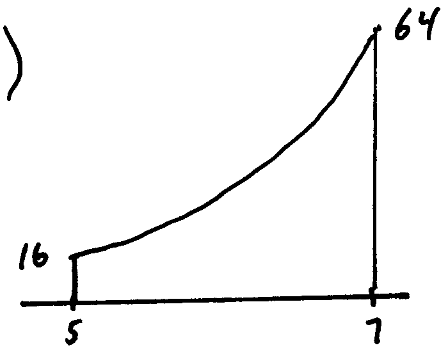
(a) $t_d = 4$

$$h(t) = \left(\delta(t-1) - 4\delta(t-3)\right) * \left(2^t u(t-4)\right)$$

$$= 2^{t-1} u(t-5) - 4\, 2^{t-3} u(t-7)$$

$$= \frac{1}{2}\left(2^t u(t-5)\right) - \frac{1}{2}\left(2^t u(t-7)\right)$$

$$\underline{h(t) = 2^{t-1}\left(u(t-5) - u(t-7)\right)}$$



(b) $t_d \geq -1$ (it can be 5 less than its current value of 4)

(c) #1 - stable, the output is simply a delayed version of the input so if $|x(t)| < M$, $|y_1(t)| < M$

#2 - stable, same as above except $|x(t)| < M$ implies $|y_2(t)| < 4M$

#3 - not stable, $\int_{-\infty}^{\infty} |2^t u(t-t_d)|\, dt \to \infty$

the overall system is stable as can be seen from (a) because ~~the~~ $h(t)$ is of finite duration and amplitude for finite values of t_d.