8.2) We have two cascaded systems described by

$h_1[n] = \delta[n] + \delta[n-7]$ $h_2[n] = u[n] - u[n-7]$

a) Determine $H_1(e^{j\hat{\omega}})$:

$$H_1(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}7}$$

b) Determine $H_2(e^{j\hat{\omega}})$:

$$H_2(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}} + e^{-2j\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} + e^{-j5\hat{\omega}} + e^{-j\hat{\omega}6}$$

or, you can observe the similarity to ~~a~~ a 7-point running average:

$$H_2(e^{j\hat{\omega}}) = 7 \sum_{k=0}^{6} \frac{1}{7} e^{-jk\hat{\omega}}$$

$$= \frac{\sin(\frac{7}{2}\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j3\hat{\omega}}$$

c) Using numerical convolution, show $h[n] = h_1[n] * h_2[n] = u[n] - u[n-14]$

$$h[n] = \sum_{k=0}^{7} h_1[k]\, h_2[n-k]$$

$$= h_1[0]\, h_2[n-0] + h_1[7]\, h_2[n-7]$$

Since $h_1 = \begin{cases} 1 & , n = 0, 7 \\ 0 & , \text{otherwise} \end{cases}$

$$h[n] = u[n] - u[n-7] + u[n-7] - u[n-7-7]$$

$$= u[n] - u[n-14]$$

d) Determine $H(e^{j\hat{\omega}})$:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{13} e^{-j\hat{\omega}k}$$

$$= \frac{\sin(\frac{13}{2}\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j6\hat{\omega}}$$

~~E~~8.2) e) Show $H_1(e^{j\hat{\omega}})\, H_2(e^{j\hat{\omega}}) = H(e^{j\hat{\omega}})$ as found in 8.2.d)

$$H(e^{j\hat{\omega}}) = \left(1 + e^{-j7\hat{\omega}}\right)\left(\sum_{k=0}^{6} e^{-j\hat{\omega}k}\right)$$

$$= \sum_{k=0}^{6} e^{-j\hat{\omega}k} + \sum_{k=0}^{6} e^{-j\hat{\omega}k} e^{-j7\hat{\omega}}$$

$$= \sum_{k=0}^{6} e^{-j\hat{\omega}k} + \sum_{k=0}^{6} e^{-j\hat{\omega}(k+7)}$$

$$= \sum_{k=0}^{6} e^{-j\hat{\omega}k} + \sum_{k=7}^{13} e^{-j\hat{\omega}k}$$

$$= \sum_{k=0}^{13} e^{-j\hat{\omega}k}$$

which matches our result ~~fo~~ from part(d).

Solutions ECE 2025 PS#8

8.4) Given three cascaded systems:

$$S_1 : y_1[n] = -x_1[n] + x_1[n-2]$$

$$S_2 : h_2[n] = 5\delta[n-3] + 5\delta[n-4]$$

$$S_3 : H_3(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} - e^{-j3\hat{\omega}}$$

a) Give the difference equation for S_3:

$$y_3[n] = x[n-1] - x[n-3]$$

b) Determine the frequency response for S_1 and S_2:

$$H_1(e^{j\hat{\omega}}) = -1 + e^{-j2\hat{\omega}}$$

$$H_2(e^{j\hat{\omega}}) = 5e^{-j3\hat{\omega}} + 5e^{-j4\hat{\omega}}$$

c) Find the frequency response of the overall system S:

$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})\, H_2(e^{j\hat{\omega}})\, H_3(e^{j\hat{\omega}})$$

$$= (-1 + e^{-j2\hat{\omega}})(5e^{-j3\hat{\omega}} + 5e^{-j4\hat{\omega}})(e^{-j\hat{\omega}} - e^{-j3\hat{\omega}})$$

$$= (-5e^{-j3\hat{\omega}} - 5e^{-j4\hat{\omega}} + 5e^{-j5\hat{\omega}} + 5e^{-j6\hat{\omega}})(e^{-j\hat{\omega}} - e^{-j3\hat{\omega}})$$

$$= -5e^{-j4\hat{\omega}} - 5e^{-j5\hat{\omega}} + 5e^{-j6\hat{\omega}} + 5e^{-j7\hat{\omega}} + 5e^{-j6\hat{\omega}} + 5e^{-j7\hat{\omega}} - 5e^{-j8\hat{\omega}} - 5e^{-j9\hat{\omega}}$$

$$= -5e^{-j4\hat{\omega}} - 5e^{-j5\hat{\omega}} + 10e^{-j6\hat{\omega}} + 10e^{-j7\hat{\omega}} - 5e^{-j8\hat{\omega}} - 5e^{-j9\hat{\omega}}$$

d) Write one difference equation for the entire system:

$$y[n] = -5x[n-4] - 5x[n-5] + 10x[n-6] + 10x[n-7] - 5x[n-8] - 5x[n-9]$$

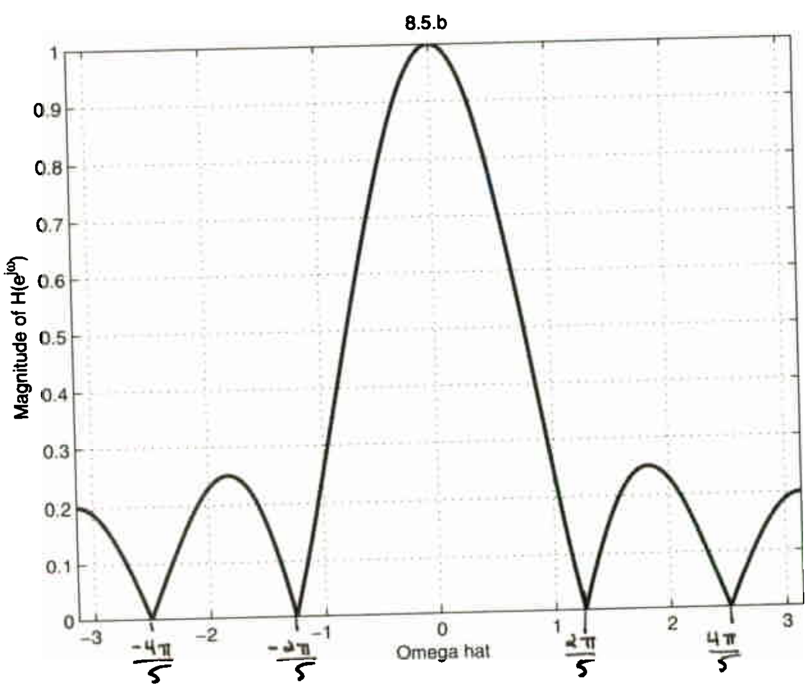
8.5) Evaluating the given Matlab program:

a) Determine $H(e^{j\hat{\omega}})$ for the FIR filter bb:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{4} \frac{1}{5} e^{-j\hat{\omega}k}$$

$$= \frac{\sin\left(\frac{5}{2}\hat{\omega}\right)}{5\sin\left(\frac{1}{2}\hat{\omega}\right)} e^{-j2\hat{\omega}}$$

b) Plot the magnitude of $H(e^{j\hat{\omega}})$ and label all frequencies where $|H(e^{j\hat{\omega}})|$ is zero.



$$|H(e^{j\hat{\omega}})| = \left|\frac{\sin\left(\frac{5}{2}\hat{\omega}\right)}{5\sin\left(\frac{1}{2}\hat{\omega}\right)}\right|$$

8.5) c) Use convolution to find $y[n]$

Give individual values for $n=0,1,2,3,4$

(Remember $x[n]=0$ for $n<0$)

$y[n]$	n	$x[n]$
0	0	0
-0.4	1	-2
-1.0928	2	-3.4641
-1.8928	3	-4
-2.5856	4	-3.4641

$n=0$: $y[0] = \frac{1}{5}\sum_{k=0}^{4} x[-k]$

$= x[0]$

$= \frac{4}{5}\cos\left(\frac{\pi}{6}(0)+\frac{\pi}{2}\right)$

$= 0$

$y[1] = \frac{1}{5}\sum_{k=0}^{4} x[1-k]$

$= \frac{1}{5}\left[x(1) \cancel{+ x(0)}^{0}\right]$

$= \frac{4}{5}\cos\left(\frac{\pi}{6}+\frac{\pi}{2}\right)$

$= -0.4$

$y[2] = \frac{1}{5}\sum_{k=0}^{4} x[2-k]$

$= \frac{1}{5}\left[x(2)+x(1)+x(0)\right]$

$= \frac{4}{5}\cos\left(\frac{\pi}{3}+\frac{\pi}{2}\right) + -0.4$

$= -1.0928$

$y[3] = \frac{1}{5}\sum_{k=0}^{4} x[3-k]$

$= \frac{1}{5}x[3] + y[2]$ (since $y[2] = \frac{1}{5}(x[2]+x[1]+x[0])$)

$= \frac{4}{5}\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right) - 1.0928$

$= -1.8928$

$y[4] = \frac{1}{5}\sum_{k=0}^{4} x[4-k]$

$= \frac{1}{5}x[4] + y[3]$ (since $y[3] = \frac{1}{5}(x[3]+x[2]+x[1]+x[0])$)

$= \frac{4}{5}\cos\left(\frac{2\pi}{3}+\frac{\pi}{2}\right) - 1.8928$

$= -2.5856$

8.5) c)

Provide a general formula for $y[n]$ when $n \geq 4$:

$$y[n] = \sum_{k=0}^{4} \frac{1}{5}(4)\cos\left(\frac{\pi}{6}(n-k)+\frac{\pi}{2}\right)$$

$$= \frac{4}{5}\sum_{k=0}^{4}\cos\left(\frac{\pi}{6}n + \left(\frac{\pi}{2}-\frac{\pi k}{6}\right)\right)$$

Notice this is a sum of five sinusoids with the same frequency but different phases, so we can use phasor addition to combine them:

$$Ae^{j\phi} = \sum_{k=0}^{4} e^{j\left(\frac{\pi}{2}-\frac{\pi k}{6}\right)}$$

$$= e^{j\frac{\pi}{2}}\sum_{k=0}^{4} e^{j\frac{\pi k}{6}}$$

$$1+e^{j\frac{\pi}{6}}+e^{j\frac{\pi}{3}}+e^{j\frac{\pi}{2}}+e^{j\frac{2\pi}{3}}$$

$$= 1 + 0.866 - 0.5j + 0.5 - 0.866j - j - 0.5 - 0.866j$$

$$= 1.8660 - 3.2321j$$

$$= 3.7321 e^{-j\frac{\pi}{3}}$$

$$Ae^{j\phi} = 3.7321\, e^{j\pi/6}$$

$$y[n] = \frac{4}{5}(3.7321)\cos\left(\frac{\pi}{6}n+\frac{\pi}{6}\right)$$

$$= 2.9857\cos\left(\frac{\pi}{6}n+\frac{\pi}{6}\right) \qquad \text{for } n \geq 4 \text{ and } n \leq 4000$$

d) Give at least one value of $\hat{\omega}$ such that $y[n]$ would be zero for $4 \leq n \leq 4000$.

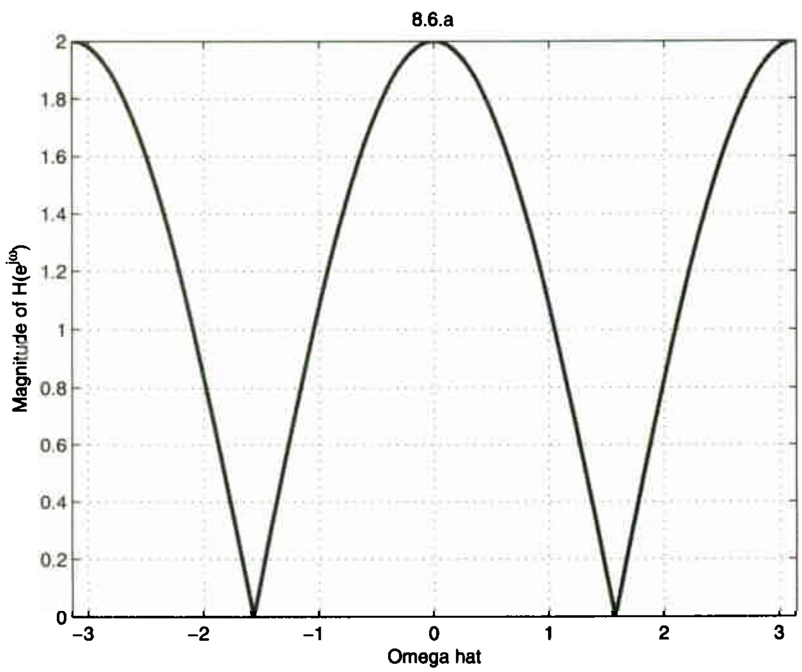
From parts (a) and (b), we get

$$\hat{\omega} = \pm\frac{2\pi}{5} \text{ and } \pm\frac{4\pi}{5} \qquad (\text{for } -\pi < \hat{\omega} \leq \pi)$$

8.6) We are given a system for discrete-time filtering of a continuous-time signal, where the filter is described by:

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} + e^{-j3\hat{\omega}}$$

a) Plot $|H(e^{j\hat{\omega}})|$ over $-\pi < \hat{\omega} \le \pi$



$$|H(e^{j\hat{\omega}})| = |2\cos(\hat{\omega})| \qquad \text{(See (b))}$$

b) For a sampling rate $f_s = 500$ Hz, determine the frequency of an input sinusoid $x(t) = \cos(\omega t)$ such that the output is zero.

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} + e^{-j3\hat{\omega}} = e^{-j2\hat{\omega}}\left(e^{j\hat{\omega}} + e^{-j\hat{\omega}}\right)$$

$$= e^{-j2\hat{\omega}} \cos(\hat{\omega}) \cdot 2$$

$H(e^{j\hat{\omega}}) = 0$ when $\hat{\omega} = \pm\frac{\pi}{2} = \frac{\pi}{2} + \pi l$ (l = integer)

when $f_s = 500$ Hz $\quad \omega = \hat{\omega} f_s = \frac{\pi}{2}(500) = 250\pi = 2\pi(125)$

8.6) c) Given the input signal

$$x(t) = 5 + 3\cos(250\pi t + \pi/4) \qquad -\infty < t < \infty$$

and $f_s = 500$ Hz, determine $y(t)$ for $-\infty < t < \infty$

Because of superposition, we can handle each component of $x(t)$ separately.

$$H(e^{j0}) = 2\cos(0)e^{-j2(0)} = 2 \qquad \hat{\omega} = \frac{0}{500} = 0$$

$$H(e^{j\pi/2}) = 2\cos\left(\frac{\pi}{2}\right)e^{-j2\left(\frac{\pi}{2}\right)} = 0 \qquad \hat{\omega} = \frac{250\pi}{500} = \frac{\pi}{2}$$

$$y(t) = 2\cdot 5 + 0\cdot 3\cos(250\pi t + \pi/4)$$

$$= 10$$

8.7) We are given an LTI filter with the frequency response

$$H(\hat{\omega}) = (1-e^{-j\hat{\omega}})(1+e^{-j\pi/2}e^{-j\hat{\omega}})(1+e^{j\pi/2}e^{-j\hat{\omega}})$$

a) Write the difference equation for this filter.

$$H(e^{j\hat{\omega}}) = (1+e^{-j\pi/2}e^{-j\hat{\omega}} - e^{-j\hat{\omega}} - e^{-j\pi/2}e^{-j2\hat{\omega}})(1+e^{j\pi/2}e^{-j\hat{\omega}})$$

$$= 1+e^{-j\pi/2}e^{-j\hat{\omega}} - e^{-j\hat{\omega}} - e^{-j\pi/2}e^{-j2\hat{\omega}}$$

$$+e^{j\pi/2}e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{j\pi/2}e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$= 1+ e^{-j\hat{\omega}}(e^{-j\pi/2} - 1 + e^{j\pi/2}) + e^{-j2\hat{\omega}}(-e^{-j\pi/2} + 1 - e^{j\pi/2}) - e^{-j3\hat{\omega}}$$

$$e^{j\pi/2} + e^{-j\pi/2} = 2\cos(\pi/2) = 0 = -2\cos(+\tfrac{\pi}{2}) = -(e^{j\pi/2} + e^{-j\pi/2})$$

$$H(e^{j\hat{\omega}}) = 1 + -e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$y[n] = x[n] - x[n-1] + x[n-2] - x[n-3]$$

b) What is the impulse response?

$$h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$$

c) For a sinusoidal input $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$ for what values of $\hat{\omega}$ will $y[n]$ be zero? $(-\pi \le \hat{\omega} \le \pi)$

$$H(e^{j\hat{\omega}}) = e^{-j\frac{3}{2}\hat{\omega}}[e^{j\frac{3}{2}\hat{\omega}} - e^{j\frac{1}{2}\hat{\omega}} + e^{-j\frac{1}{2}\hat{\omega}} - e^{-j\frac{3}{2}\hat{\omega}}]$$

$$= e^{-j\frac{3}{2}\hat{\omega}}\left[2j\sin(\tfrac{3}{2}\hat{\omega}) - 2j\sin(\tfrac{1}{2}\hat{\omega})\right]$$

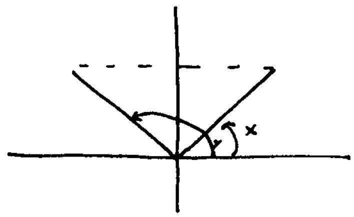
$$= 2e^{j\pi/2}e^{-j3/2\,\hat{\omega}}\left[\sin(\tfrac{3}{2}\hat{\omega}) - \sin(\tfrac{1}{2}\hat{\omega})\right]$$

$$\sin(\tfrac{3}{2}\hat{\omega}) = \sin(\tfrac{1}{2}\hat{\omega})$$

$$\tfrac{3}{2}\hat{\omega} = \tfrac{1}{2}\hat{\omega}$$

$\hat{\omega} = 0$ and $\hat{\omega} = \pm\frac{\pi}{2}$ (see next page)

8.7) c) We also know that $\sin(x) = \sin(y)$ when $y = \pi - x$

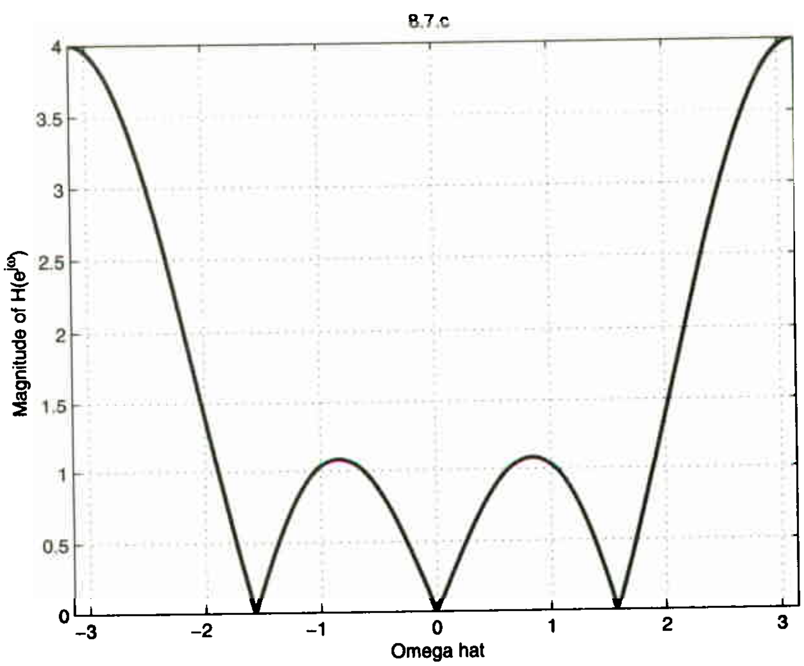


$$\frac{3}{2}\hat{\omega} = \pi - \frac{1}{2}\hat{\omega}$$

$$2\hat{\omega} = \pi$$

$$\hat{\omega} = \frac{\pi}{2}$$

similar analysis for $\hat{\omega} < 0$ yields $\hat{\omega} = -\frac{\pi}{2}$



8.7) d) Find the output of the system when

$$x[n] = 1 + 2\delta[n-3] + 7\cos(0.5\pi n) \qquad -\infty < n < \infty$$

Because of superposition we can work on each component of $x[n]$ separately.

$x_1[n] = 1$: $H(e^{j0}) = 0$ as we found in part (c)

$y_1[n] = 0$

$x_3[n] = 7\cos(0.5\pi n)$:

$H(e^{j\pi/2}) = 0$ as in part (c)

$y_3[n] = 0$

$x_2[n] = 2\delta[n-3]$:

$$y_2[n] = x_2[n] - x_2[n-1] + x_2[n-2] - x_2[n-3]$$

$$= 2\delta[n-3] - 2\delta[n-4] + 2\delta[n-5] - 2\delta[n-6]$$

$$= y[n]$$

Note that since $x_2[n]$ is a scaled, shifted impulse, $y_2[n]$ is a scaled, shifted version of the impulse response.