(6.1) (a) $x[n] = 2\cos(0.7\pi n - 3\pi/4) \Rightarrow \hat{\omega} = 0.7\pi$

This normalized frequency is related to the frequency of $x(t)$ by

$$\hat{\omega} = \frac{\omega}{9000} + 2\pi \ell, \quad \ell = 0, \pm 1, \pm 2, \ldots$$

or

$$\hat{\omega} = -\frac{\omega}{9000} + 2\pi \ell, \quad \ell = 0, \pm 1, \pm 2, \ldots$$

$$\Rightarrow \quad \omega = \hat{\omega}(9000) + 2\pi(9000)\ell \qquad , \ \ell = 0, \pm 1, \pm 2, \ldots$$

$$\text{or } \omega = -\hat{\omega}(9000) + 2\pi(9000)\ell \qquad \leftarrow \text{'folding' cases}$$

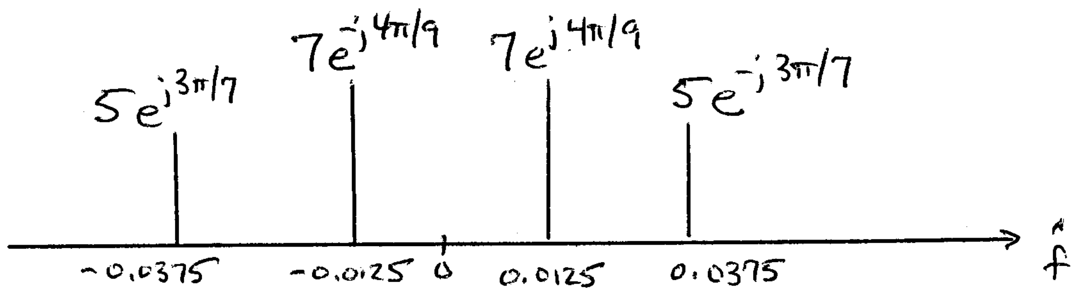
Since we need $36{,}000 < \frac{\omega}{2\pi} < 45{,}000$, we have

$\omega_1 = 39{,}150(2\pi) \Rightarrow \underline{x_1(t) = 2\cos(2\pi(39{,}150)t - 3\pi/4)}$
[$\ell = 4$ in first equation]

$\omega_2 = 41{,}850(2\pi) \Rightarrow \underline{x_2(t) = 2\cos(2\pi(41{,}850)t + 3\pi/4)}$
[$\ell = 5$, in second equation $\Rightarrow$ folding example]

(b) Highest frequency in $x(t)$ is 300 Hz.
$\Rightarrow$ Need $f_s > 600$ Hz.

(c) $\hat{\omega} = \omega/8000$ or $\hat{f} = f/8000$



(6.2) Using $x[n]$ to describe the position of the spoke starting at 0°, we have

$$x[n] = e^{+j\,2\pi\left(\frac{1}{2/3}\right)\left(\frac{1}{30}\right)n} = e^{+j\frac{2\pi}{20}n}$$

$\frac{2}{3}$ = observed period 30 frames/sec = f_s.

So, our normalized frequency is $\hat{\omega}_0 = \pi/10$. However, with 8 spokes the normalized frequency could also be $\pi/10 + \frac{2\pi}{8}\ell$ for any integer ℓ.

Let ω_0 = rotation rate of the wheel in radians/sec, which has to be negative for a clockwise rotation. After sampling we have:

$$e^{j\omega_0 t}\Big|_{t=n/30} = e^{j\omega_0 n/30}$$

Equating these two expressions:

$$e^{j\omega_0 n/30} = e^{+j\left(\pi/10 + \frac{2\pi\ell}{8}\right)n}$$

$$\Rightarrow \quad \omega_0/30 = \frac{\pi}{10} + \frac{2\pi\ell}{8}$$

$$\omega_0 = 2\pi\left(3/2 + \frac{30\ell}{8}\right)$$

$$\Rightarrow \left(\tfrac{3}{2} + \tfrac{15}{4}\ell\right) \text{ revs/sec for } \ell = -1, -2, -3, \ldots$$

How fast? $\left|\left(\frac{3}{2} + \frac{15}{4}\ell\right)\right| \frac{\text{revs}}{\text{sec}} \times 0.6\pi \frac{\text{m}}{\text{rev}} \times \frac{1\text{ km}}{1000\text{ m}} \times \frac{3600\text{ sec}}{\text{hr}}$

$$= \left(25.45\,\ell - 10.18\right)\text{ km/hr}, \quad \ell = 1, 2, 3, \ldots$$

6.3 P-4.12

(a) $\hat{\omega} = 0.13\pi$, $f_s = 1000$ samples/sec

All possible frequencies given by:

$$\left.\begin{aligned} \hat{\omega} &= \frac{\omega}{f_s} + 2\pi l \\ \text{OR}\quad \hat{\omega} &= -\frac{\omega}{f_s} + 2\pi l \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} &\omega = \hat{\omega} f_s + 2\pi f_s l \\ \text{OR}\ &\omega = -\hat{\omega} f_s - 2\pi f_s l \end{aligned}\right. , \quad l = 0, \pm 1, \pm 2, \ldots$$

Folding Cases

So, all possible answers are given by:

$$x(t) = 10\cos\left(\left[130\pi + 2\pi(1000)l\right]t + \pi/13\right)$$

$$\text{AND}\quad x(t) = 10\cos\left(\left[-130\pi - 2\pi(1000)l\right]t - \pi/13\right), \quad l = 0, \pm 1, \ldots$$

(b) $x(t) = 26\cos(2\pi(200)t + \pi/4) + 14\cos(2\pi(500)t + 3\pi/4)$

$$x[n] = x(t)\Big|_{t = n/700} = 26\cos\left(2\pi\left(\tfrac{200}{700}\right)n + \pi/4\right) + 14\cos\left(2\pi\left(\tfrac{500}{700}\right)t + 3\pi/4\right)$$

$$= 26\cos\left(2\pi(2/7)n + \pi/4\right) + 14\cos\left(2\pi\left(\tfrac{5}{7}\right)t + 3\pi/4\right)$$

$$= 26\cos\left(2\pi(2/7)n + \pi/4\right) + 14\cos\left(-2\pi(2/7)t + 3\pi/4\right)$$

$$x[n] = 26\cos\left(2\pi(2/7)n + \pi/4\right) + 14\cos\left(2\pi(2/7)t - 3\pi/4\right)$$

$$26e^{j\pi/4} + 14e^{-j3\pi/4} = 12e^{j\pi/4}$$

$$\Rightarrow x[n] = 12\cos\left(2\pi(2/7)n + \pi/4\right)$$

$$\Rightarrow y(t) = 12\cos\left(2\pi(2/7)(700)t + \pi/4\right)$$

$$= 12\cos\left(2\pi(200)t + \pi/4\right)$$

6.4 (a) $x[n] = x(t)\Big|_{t=\frac{n}{1000}} = 7\cos\left(\frac{1800}{1000}\pi n + \pi/4\right)$

$= 7\cos(1.8\pi n + \pi/4) = 7\cos(-0.2\pi n + \pi/4)$

$x[n] = 7\cos(0.2\pi n - \pi/4)$

$\Rightarrow y(t) = 7\cos(0.2\pi(1000t) - \pi/4)$

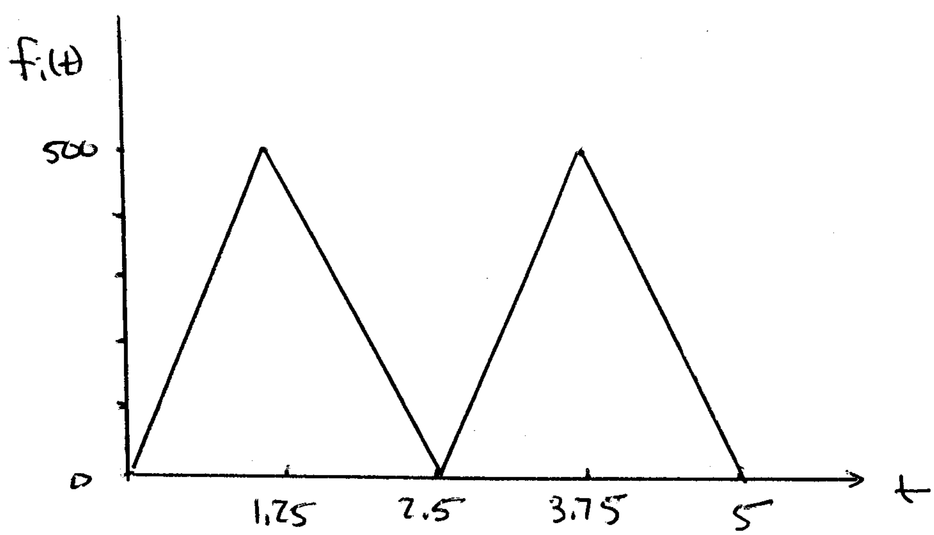
$= 7\cos(200\pi t - \pi/4)$

(b) $x(t) = \cos(2000\pi t - 400\pi t^2) = \cos(400\pi t^2 - 2000\pi t)$

$f_i(t) = \frac{d}{dt}\left[200t^2 - 1000t\right] = 400t - 1000$

At $t=0$, $f_i(0) = 1000$ Hz aliases to 0 Hz for $f_s = 1000$ Hz.

So, instantaneous frequency will start at 0 Hz and increase with a slope of 400 Hz/sec. The highest frequency that can be represented is 500 Hz.



(6.5) (a) To indicate the dimensions of the matrices involved, we can write the expression for $x[n]$ as:

$$x[n] = \mathrm{Re}\left\{ \exp\left(j \begin{bmatrix} 0 \\ 1 \\ 2 \\ \vdots \\ 200{,}000 \end{bmatrix} \begin{bmatrix} 0 & 0.35\pi & 0.7\pi & 1.75\pi \end{bmatrix} \right) \begin{bmatrix} 1 \\ 1+j \\ -7j \\ 2j \end{bmatrix} \right\}$$

Remember that when a matrix is passed to exp(·), it computes the exponent of each element of that matrix.

$$\exp\left(j \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 200{,}000 \end{bmatrix} \begin{bmatrix} 0 & 0.35\pi & 0.7\pi & 1.75\pi \end{bmatrix} \right) = \left[\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} e^{j0.35\pi n} \end{bmatrix} \begin{bmatrix} e^{j0.7\pi n} \end{bmatrix} \begin{bmatrix} e^{j1.75\pi n} \end{bmatrix} \right]$$

Column vectors for $n = 0:200000$.

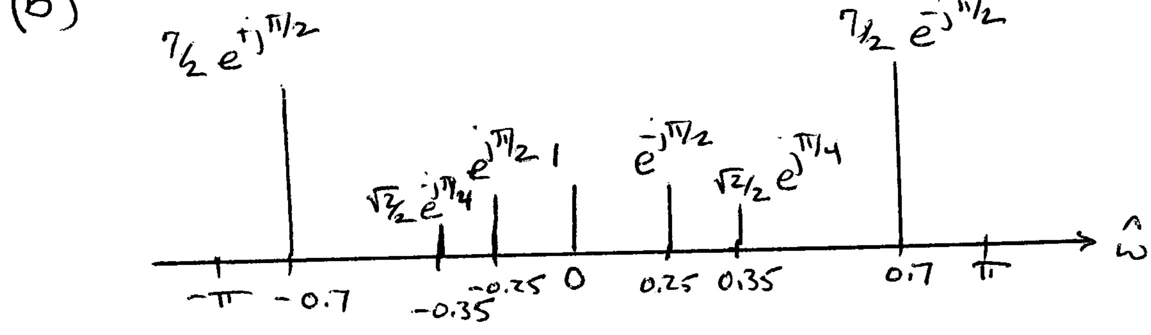
Multiplying this matrix by $\begin{bmatrix} 1 \\ 1+j \\ -7j \\ 2j \end{bmatrix}$ will form a weighted sum of these vectors

$$\Rightarrow x[n] = \mathrm{Re}\left\{ 1 + (1+j)e^{j0.35\pi n} + (-7j)e^{j0.7\pi n} + 2j\, e^{j1.75\pi n} \right\}$$

$$= 1 + \sqrt{2}\cos(0.35\pi n + \pi/4) + 7\cos(0.7\pi n - \pi/2) + 2\cos(1.75\pi n + \pi/2)$$

$$= 1 + \sqrt{2}\cos(0.35\pi n + \pi/4) + 7\cos(0.7\pi n - \pi/2) + 2\cos(0.25\pi n - \pi/2), \quad n = 0, 1, \ldots, 200{,}000$$

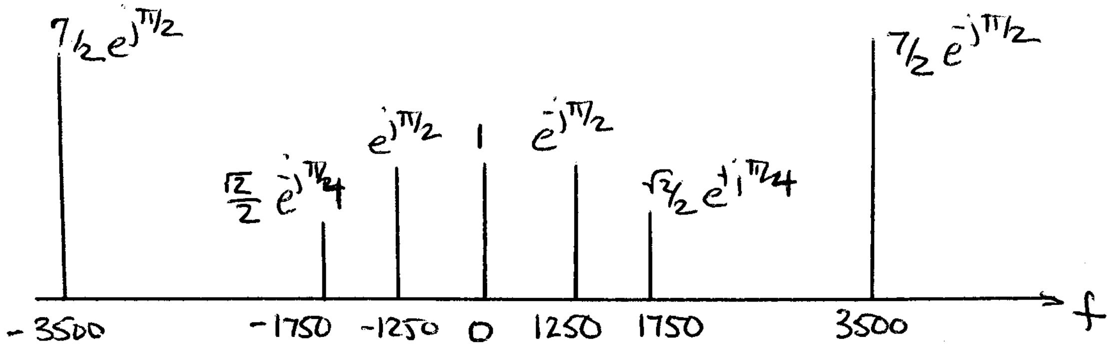
(b)



6.6 (b) $f_s = 10{,}000$ Hz.

$$\Rightarrow x(t) = 1 + \sqrt{2}\cos(3500\pi t + \pi/4) + 7\cos(7000\pi t - \pi/2) + 2\cos(2500\pi t - \pi/2)$$

(a)



(c) $x(t)$ was generated from 200,000 samples at 10,000 samples/sec.

$\Rightarrow$ 20 seconds in duration.