5.1

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j30\pi k t}$$

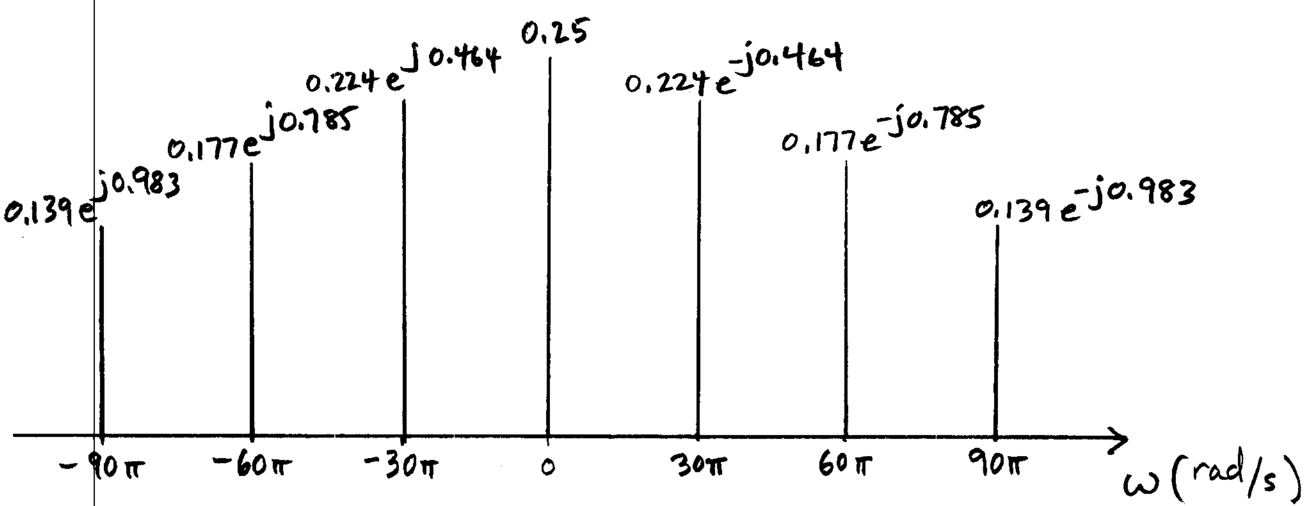
$$a_k = \begin{cases} \dfrac{1}{4+j2k}, & k = -3, -2, \ldots, 2, 3 \\ 0, & |k| > 3 \end{cases}$$

(a) $a_0 = \frac{1}{4}$

$a_1 = \dfrac{1}{4+j2} = 0.224\, e^{-j0.464}$ $\qquad a_{-1} = 0.224\, e^{j0.464}$

$a_2 = \dfrac{1}{4+j4} = 0.177\, e^{-j0.785}$ $\qquad a_{-2} = 0.177\, e^{j0.785}$

$a_3 = \dfrac{1}{4+j6} = 0.139\, e^{-j0.983}$ $\qquad a_{-3} = 0.139\, e^{j0.983}$



(b) $\omega_0 = 30\pi$ rad/s

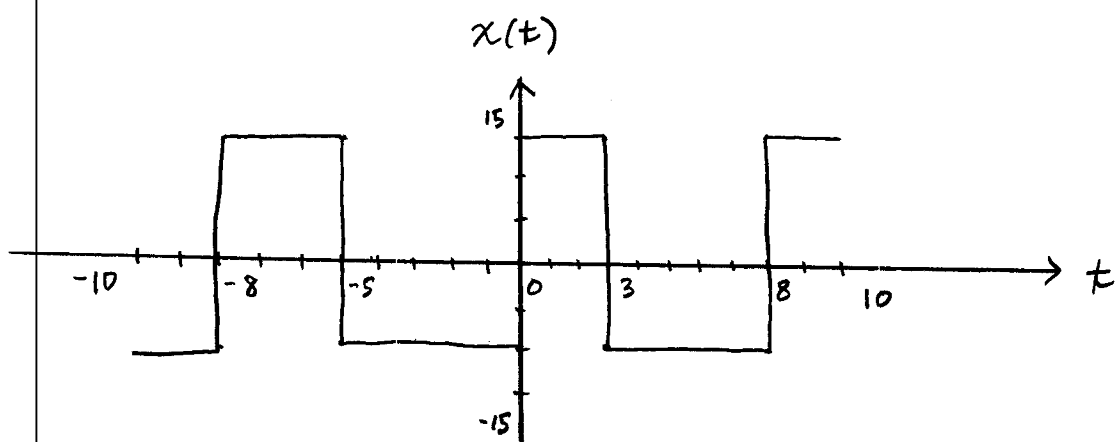
$f_0 = 30\pi / 2\pi = 15$ Hz

$T_0 = 1/15 = 66.67$ ms

5.2

$$x(t) \text{ over one period } = \begin{cases} 15, & 0 \le t \le 3 \\ -9, & 3 < t < 8 \end{cases}$$

(a) $T_0 = 8$



(b)

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

$$= \frac{1}{8}\left(\int_0^3 15\,dt + \int_3^8 (-9)\,dt\right)$$

$$= \frac{1}{8}\left(15t\Big|_0^3 - 9t\Big|_3^8\right)$$

$$= \frac{1}{8}\left(45 - 0 - 72 + 27\right)$$

$$= 0$$

area of positive rectangle

$= 15 \cdot 3 = 45$

area of negative rectangle

$= -9 \cdot 5 = -45$

areas cancel

(c)

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{8}\left(\int_0^3 15e^{-jk\frac{2\pi}{8}t}\,dt + \int_3^8 (-9)e^{-jk\frac{2\pi}{8}t}\,dt\right)$$

$$= \frac{15}{8}\frac{8}{-jk2\pi}\,e^{-jk\frac{\pi}{4}t}\Big|_0^3 - \frac{9}{8}\frac{8}{-jk2\pi}\,e^{-jk\frac{\pi}{4}t}\Big|_3^8$$

$$= \frac{-15\left(e^{-jk\frac{3\pi}{4}}-1\right)+9\left(1-e^{-jk\frac{3\pi}{4}}\right)}{jk2\pi}$$

$$= \frac{24\left(1-e^{-jk\frac{3\pi}{4}}\right)}{jk2\pi}$$

$$= \frac{12\left(1-\cos\left(k\frac{3\pi}{4}\right)+j\sin\left(k\frac{3\pi}{4}\right)\right)}{jk\pi}$$

$$= \frac{12}{k\pi}\left(\sin\left(k\frac{3\pi}{4}\right)+j\left(\cos\left(k\frac{3\pi}{4}\right)-1\right)\right)$$

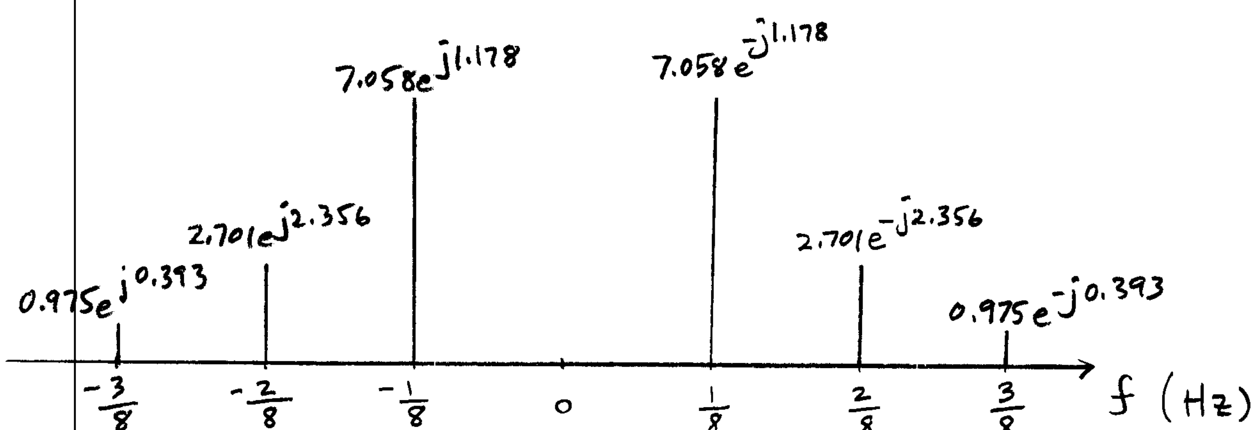
(d) $T_o = 8$, $f_o = \frac{1}{8}$, $-\frac{1}{2} < f < \frac{1}{2} \Leftrightarrow -3 \le k \le 3$

$a_0 = 0$

$a_1 = 7.058\, e^{-j1.178}$

$a_2 = 2.701\, e^{-j2.356}$

$a_3 = 0.975\, e^{-j0.393}$



5.3

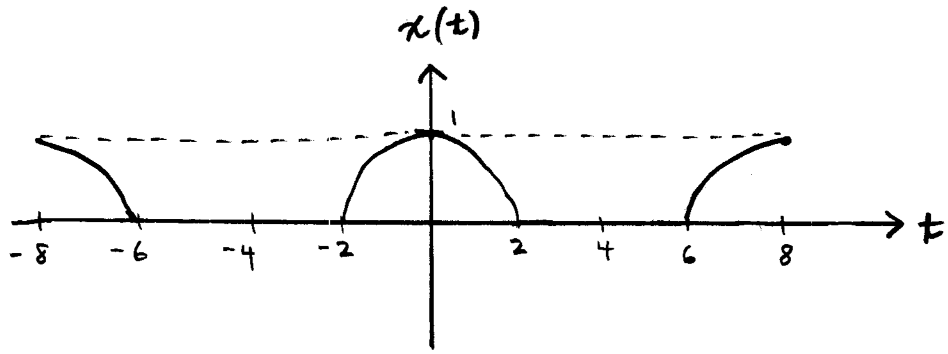
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{\pi}{4}kt}$$

$$a_k = \frac{1}{8}\int_{-2}^{2} \cos\left(\frac{\pi}{4}t\right) e^{-j\frac{\pi}{4}kt}\, dt$$

(a) generally,
$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j\frac{2\pi}{T_0}kt}\, dt$$
$$= \frac{1}{T_0}\int_{0-2}^{T_0-2} x(t)\, e^{-j\frac{2\pi}{T_0}kt}\, dt$$
$$= \frac{1}{T_0}\left(\int_{-2}^{2} x(t)\, e^{-j\frac{2\pi}{T_0}kt}\, dt + \int_{2}^{T_0-2} x(t)\, e^{-j\frac{2\pi}{T_0}kt}\, dt\right)$$

$$\Rightarrow \quad x(t) \text{ over one period} = \begin{cases} \cos\left(\frac{\pi}{4}t\right), & -2 \le t \le 2 \\ 0, & 2 < t < 6 \end{cases}$$

(b)

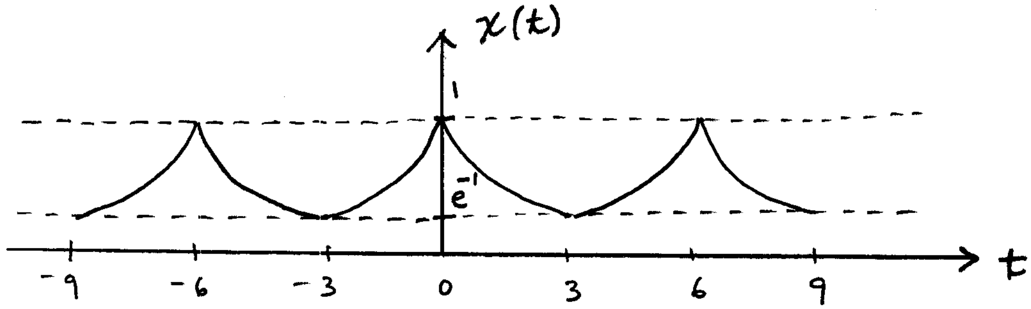


(c)
$$a_0 = \frac{1}{8}\int_{-4}^{4} x(t)\, dt$$
$$= \frac{1}{8}\int_{-2}^{2} \cos\left(\frac{\pi}{4}t\right) dt$$
$$= \frac{1}{8}\,\frac{4}{\pi}\,\sin\left(\frac{\pi}{4}t\right)\Big|_{-2}^{2}$$
$$= \frac{1}{\pi}$$

5.4

$x(t)$ over one period $= e^{-|t|/3}$, $-3 \le t < 3$, $T_0 = 6$

(a)



(b)

$$a_0 = \frac{1}{6}\int_{-3}^{3} e^{-|t|/3}\, dt$$

$$= \frac{1}{6}\int_{-3}^{0} e^{t/3}\, dt + \frac{1}{6}\int_{0}^{3} e^{-t/3}\, dt$$

$$= \frac{1}{6}\, 3 e^{t/3}\Big|_{-3}^{0} - \frac{1}{6}\, 3 e^{-t/3}\Big|_{0}^{3}$$

$$= \frac{1}{2}\left(1 - e^{-1}\right) - \frac{1}{2}\left(e^{-1} - 1\right)$$

$$= 1 - e^{-1}$$

(c)-(d)

$$a_k = \frac{1}{T_0}\int_{0}^{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{6}\int_{-3}^{3} e^{-|t|/3}\, e^{-jk\frac{2\pi}{6}t}\, dt$$

$$= \frac{1}{6}\int_{-3}^{0} e^{t/3}\, e^{-jk\frac{\pi}{3}t}\, dt + \frac{1}{6}\int_{0}^{3} e^{-t/3}\, e^{-jk\frac{\pi}{3}t}\, dt$$

$$= \frac{1}{6}\int_{-3}^{0} e^{\frac{1}{3}(1-jk\pi)t}\, dt + \frac{1}{6}\int_{0}^{3} e^{-\frac{1}{3}(1+jk\pi)t}\, dt$$

$$= \frac{1}{6}\frac{3}{1-jk\pi}\, e^{\frac{1}{3}(1-jk\pi)t}\Big|_{-3}^{0} - \frac{1}{6}\frac{3}{1+jk\pi}\, e^{-\frac{1}{3}(1+jk\pi)t}\Big|_{0}^{3}$$

$$= \frac{1}{2}\frac{1}{1-jk\pi}\left(1 - e^{-(1-jk\pi)}\right) - \frac{1}{2}\frac{1}{1+jk\pi}\left(e^{-(1+jk\pi)} - 1\right)$$

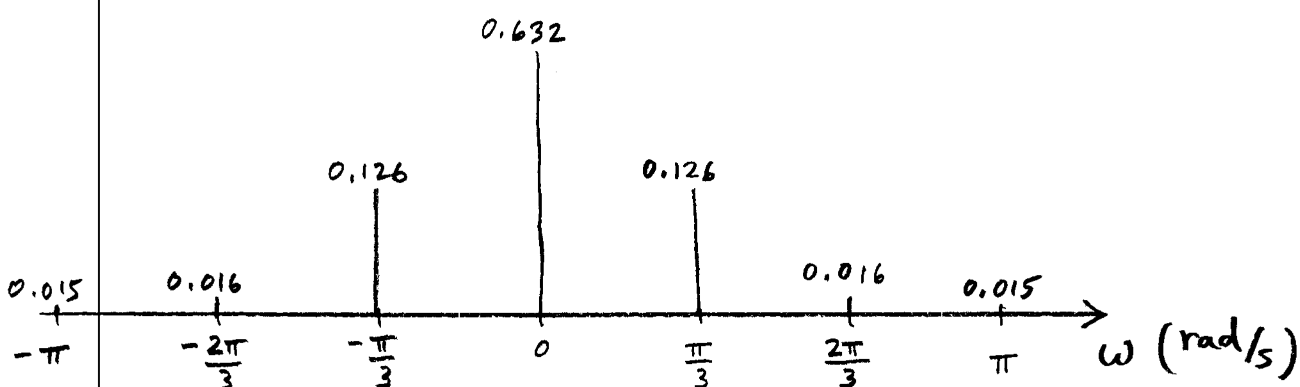
$$= \frac{1}{2}\left(\frac{1}{1-jk\pi} + \frac{1}{1+jk\pi}\right) - \frac{e^{-1}}{2}\left(\frac{e^{jk\pi}}{1-jk\pi} + \frac{e^{-jk\pi}}{1+jk\pi}\right)$$

$$= \frac{1}{2}\left(\frac{1}{1-jk\pi} + \frac{1}{1+jk\pi}\right)\left(1 \mp e^{-1}\right) \qquad \begin{matrix} k = 0, \pm 2, \pm 4, \dots \\ k = \pm 1, \pm 3, \dots \end{matrix}$$

$$= \frac{1}{(1-jk\pi)(1+jk\pi)}\left(1 \mp e^{-1}\right)$$

$$a_k = \begin{cases} \dfrac{1-e^{-1}}{1+(k\pi)^2}, & k = 0, \pm 2, \pm 4, \dots \\[2ex] \dfrac{1+e^{-1}}{1+(k\pi)^2}, & k = \pm 1, \pm 3, \dots \end{cases}$$

(e)



$$\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{3}$$

$$3\omega_0 = \pi$$

5.5

$$x(t) \longrightarrow \underset{\uparrow T_s}{\boxed{\text{C-to-D}}} \longrightarrow \underset{\uparrow T_s}{\boxed{\text{D-to-C}}} \longrightarrow y(t)$$

$$x_1(t) = 5\cos\left(15\pi t + \frac{\pi}{4}\right) \Rightarrow 7.5 \text{ Hz}$$

$$x_2(t) = 3\cos\left(20\pi t - \frac{\pi}{3}\right) \Rightarrow 10 \text{ Hz}$$

$y(t) = x(t)$ requires that $f_s = \frac{1}{T_s}$ be at least twice as large as the highest frequency component in the spectrum of $x(t)$

(a) $x(t) = x_1(t) + x_2(t)$

$\Rightarrow$ highest frequency component @ 10 Hz

$\Rightarrow$ $f_{s,min} = 2(10) = 20$ samples/s

(b) $x(t) = x_1^3(t)$

$\Rightarrow$ highest frequency component @ 22.5 Hz

$\Rightarrow$ $f_{s,min} = 2(22.5) = 45$ samples/s

(c) $x(t) = x_1(t)\,x_2(t)$

$\Rightarrow$ highest frequency component @ $10 + 7.5 = 17.5$ Hz

$\Rightarrow$ $f_{s,min} = 2(17.5) = 35$ samples/s

(d) $x(t) = 2 - 3x_2(t-2)$

$\Rightarrow$ highest frequency component @ 10 Hz

$\Rightarrow$ $f_{s,min} = 2(10) = 20$ samples /s