

$$3.1 (a) 6e^{i\pi} = -6 \quad \text{and}$$

$$3e^{i\frac{2\pi}{3}}e^{-i60\pi t} + 3e^{i\frac{2\pi}{3}}e^{i60\pi t} = 6\cos(60\pi t - \frac{2\pi}{3})$$

$$x(t) = -6 + 6\cos(60\pi t - \frac{2\pi}{3}) = -6(1 + \cos(60\pi t + \frac{\pi}{3}))$$

$$(b) x^3(t) = -216 \left(1 + \cos(60\pi t + \frac{\pi}{3})\right)^3 \quad \text{let } \theta = 60\pi t + \frac{\pi}{3}$$

$$= -216 \left(1 + 3\cos\theta + 3\cos^2\theta + \cos^3\theta\right)$$

$$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$\cos^3\theta = \cos\theta \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right)$$

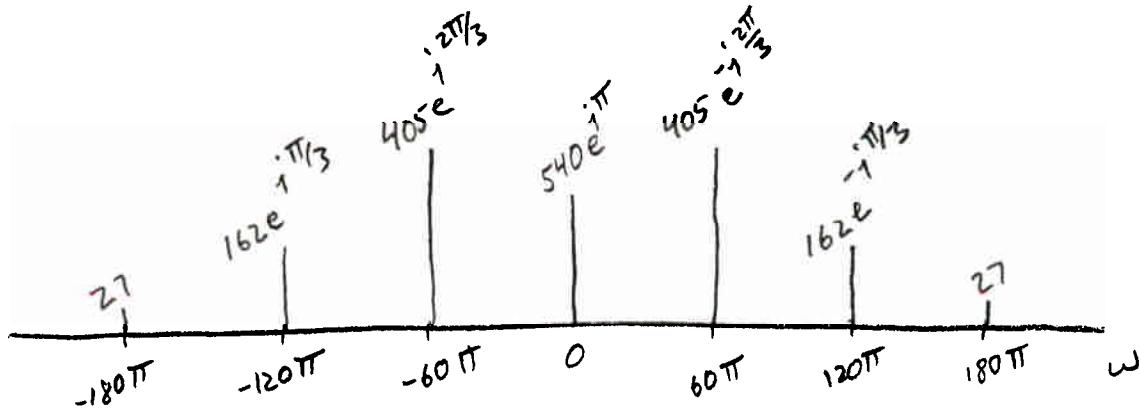
$$= \frac{1}{2}\cos\theta + \frac{1}{2}\cos\theta\cos 2\theta$$

$$= \frac{1}{2}\cos\theta + \frac{1}{4}\cos(2\theta - \theta) + \frac{1}{4}\cos(2\theta + \theta)$$

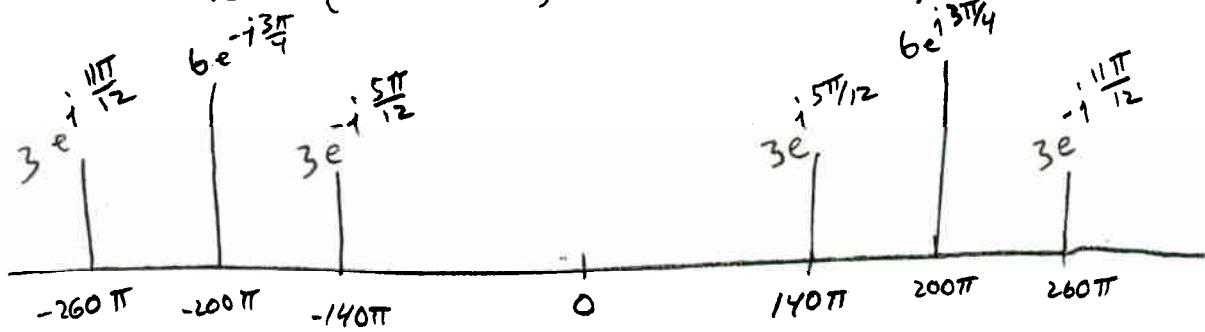
$$= \frac{3}{4}\cos\theta + \frac{1}{4}\cos(3\theta)$$

$$= -216 \left(1 + 3\cos\theta + \frac{3}{2} + \frac{3}{2}\cos(2\theta) + \frac{3}{4}\cos\theta + \frac{1}{4}\cos(3\theta)\right)$$

$$x^3(t) = -54 \left(10 + 15\cos(60\pi t + \frac{\pi}{3}) + 6\cos(120\pi t + \frac{2\pi}{3}) - \cos(180\pi t)\right)$$



$$\begin{aligned}
 3.1(c) \quad z(t) &= z(-6) \left(1 + \cos(60\pi t + \pi_3) \right) \left(\cos(200\pi t - \pi_4) \right) \\
 &= -12 \left(\cos(200\pi t - \pi_4) + \cos(60\pi t + \pi_3) \cos(200\pi t - \pi_4) \right) \\
 &= 12 \cos(200\pi t + 3\pi_4) - 6 \cos(140\pi t - 7\pi_{12}) - 6 \cos(260\pi t + \pi_{12})
 \end{aligned}$$



3.2 (a)

$$X(t) = 3 \left(\frac{e^{j150\pi t} + e^{-j150\pi t}}{2} \right) \left(\frac{e^{j200\pi t} - e^{-j200\pi t}}{2j} \right)$$

$$= -\frac{3j}{4} \left(e^{j350\pi t} + e^{-j350\pi t} + e^{j50\pi t} - e^{-j50\pi t} \right)$$

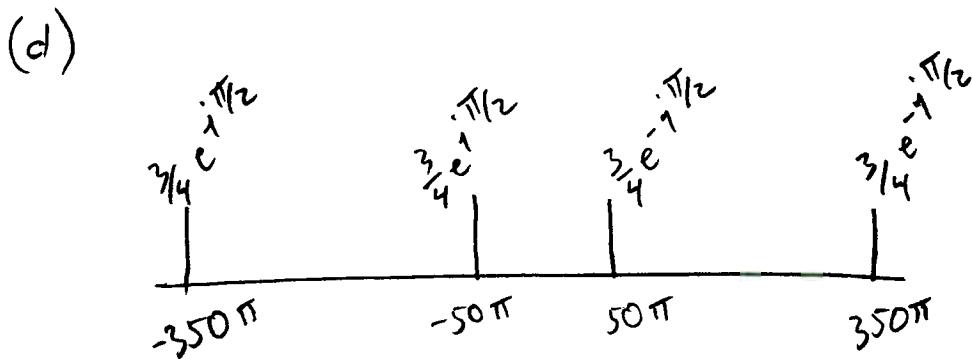
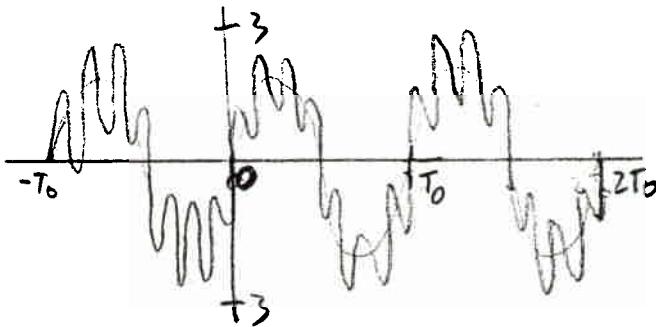
(b)

$$x(t) = \frac{3}{2} \sin(350\pi t) + \frac{3}{2} \sin(50\pi t)$$

$$= \frac{3}{2} \cos(350\pi t - \pi/2) + \frac{3}{2} \cos(50\pi t - \pi/2)$$

(c) The GCF of 350π & 50π is $50\pi \rightarrow f = 25$

$$T_0 = \frac{1}{25}$$



$$3.3 (a) x(t) = \operatorname{Re} \left\{ e^{i(2\pi(259)t + \frac{\pi}{4})} + e^{i(2\pi(262)t - \frac{3\pi}{4})} + e^{i(2\pi(265)t + \frac{\pi}{4})} \right\} \quad (1)$$

$$= \operatorname{Re} \left\{ \hat{e}(t) e^{i(2\pi(262)t + \phi)} \right\} \quad (2)$$

since the terms in the first expression are all complex exponentials, we assume that $\hat{e}(t)$ is also-

$$\text{let } \hat{e}(t) = e^{i(2\pi f_1 t + \psi_1)} + e^{i(2\pi f_2 t + \psi_2)} + e^{i(2\pi f_3 t + \psi_3)}$$

then eq (2) becomes

$$\operatorname{Re} \left\{ e^{i(2\pi(262+f_1)t + \phi + \psi_1)} + e^{i(2\pi(262+f_2)t + \phi + \psi_2)} + e^{i(2\pi(262+f_3)t + \phi + \psi_3)} \right\}$$

which matches (1) if

$$\begin{aligned} f_1 &= -3, \quad \psi_1 + \phi = \frac{\pi}{4} \\ f_2 &= 0, \quad \psi_2 + \phi = -\frac{3\pi}{4} \\ f_3 &= 3, \quad \psi_3 + \phi = \frac{\pi}{4} \end{aligned} \quad \left. \begin{array}{l} \text{we now have 3 equations} \\ \text{and 4 unknowns but we} \\ \text{simplify by assuming } \phi = \phi_2 \\ \text{or } \phi = \phi_2 + \pi \text{ to make the DC} \\ \text{term real} \end{array} \right\}$$

$$\phi = -\frac{3\pi}{4}, \quad \psi_1 = \pi, \quad \psi_2 = 0, \quad \psi_3 = \pi$$

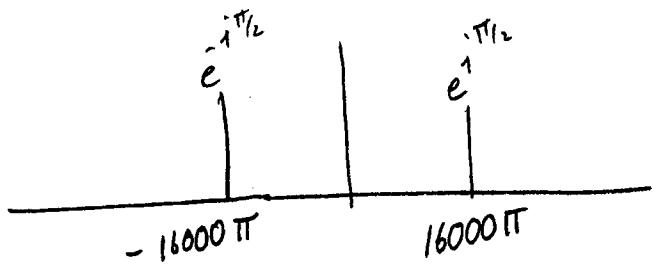
$$\text{or } \phi = \frac{\pi}{4}, \quad \psi_1 = 0, \quad \psi_2 = \pi, \quad \psi_3 = 0$$

$$\text{so } e(t) = 1 - 2\cos(2\pi(3)t)$$

$$\text{or } e(t) = 2\cos(2\pi(3)t) - 1$$

$$(b) f_e = 3 \text{ Hz}, \quad \boxed{T = \frac{1}{3} \text{ s}}$$

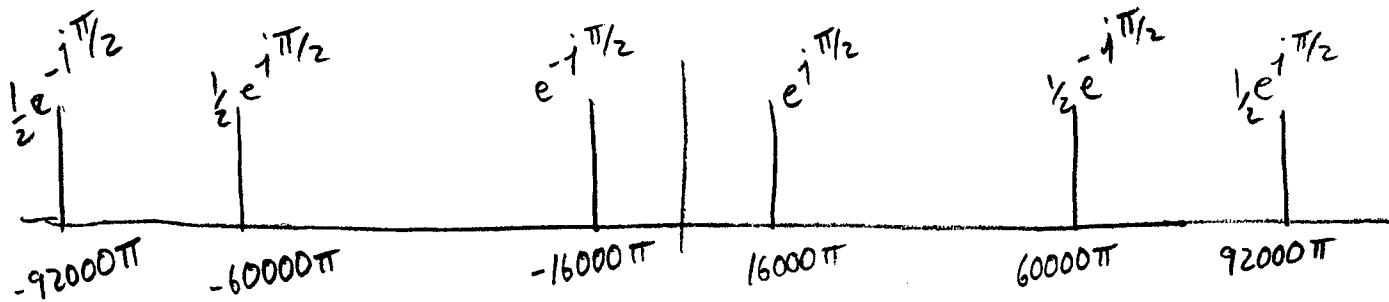
3.4 (a)



$$(b) \quad x_s(t) = X_d(t) = X_L(t) = 2 \cos(2\pi(8000)t + \pi/2)$$

$$X_i(t) = X_L(t) + X_L(t) \cos(2\pi(38000)t)$$

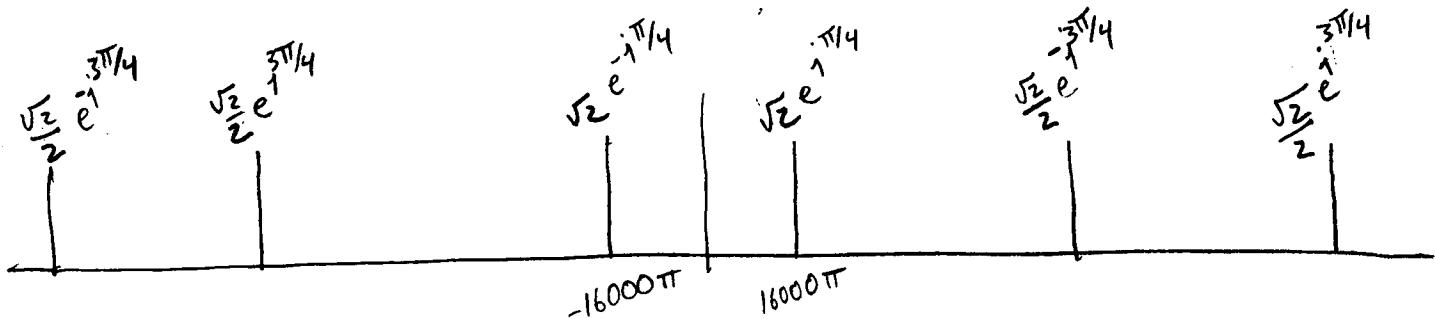
$$= 2 \cos(2\pi(8000)t + 0.5\pi) + \cos(2\pi(30000)t - 0.5\pi) \\ + \cos(2\pi(46000)t + 0.5\pi)$$



$$(c) \quad x_s(t) = 2\sqrt{2} \cos(\omega t + \pi/4)$$

$$X_d(t) = 2\sqrt{2} \cos(\omega t + 3\pi/4) \quad \text{where } \omega = 2\pi 8000$$

The same frequencies are present as in (b) but with different phases and amplitudes.



3.5 (P.3.19 from text)

$$(a) \leftrightarrow (3)$$

$$(b) \leftrightarrow (5)$$

$$(c) \leftrightarrow (1)$$

$$(d) \leftrightarrow (2)$$

$$(e) \leftrightarrow (4)$$