Solutions ECE 2025 Problem Set #2

2.1)

a) $X_1 = 5e^{j\frac{3\pi}{2}} = -j5$

$X_2 = 4e^{j\frac{2\pi}{3}} = -2 + j2\sqrt{3}$

$X_3 = 4e^{j\frac{\pi}{3}} = 2 + j2\sqrt{3}$

$X = X_1 + X_2 + X_3$

$= j(4\sqrt{3} - 5)$

$= (4\sqrt{3} - 5)e^{j\pi/2}$

$x(t) = (4\sqrt{3} - 5)\cos(\omega t + \pi/2)$

$= 1.9282\cos(\omega t + 1.5708)$

Express

$x(t) = 5\cos(\omega t + \frac{3\pi}{2}) + 4\cos(\omega t + \frac{2\pi}{3}) + 4\cos(\omega t + \frac{\pi}{3})$

in the form

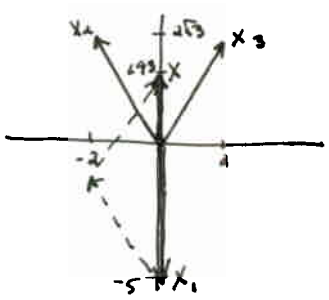
$x(t) = A\cos(\omega t + \phi)$

using complex phasor manipulations, finding numerical values for A and ϕ.

$A = (4\sqrt{3} - 5) \approx 1.9282$

$\phi = \frac{\pi}{2} \approx 1.5708$

b)



Plot all the phasors used to solve the problem in the complex plane.

(See last page for Matlab plot.)

Solutions ECE 2025 PS#2

2.2) Suppose the given Matlab code is used to plot a sinusoidal signal.

$$x(t) = \mathrm{Re}\left\{20 e^{j2\pi(50)(t-0.005)}\right\}$$

$$= 20\cos(100\pi(t-0.005))$$

$t \in [-0.01 : 0.0001 : 0.04]$

Determine the amplitude (A) and phase (ϕ) and period (T_0).

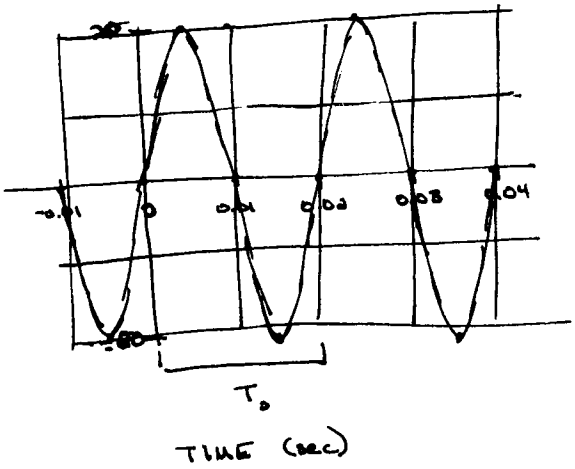
$A = 20$

$$\phi = \frac{-2\pi t_d}{T_0} = -2\pi F_0 t_d = -2\pi(50)(0.005) = -\frac{\pi}{2}$$

$$T_0 = \frac{1}{50\text{Hz}} = 0.02\text{ s} = 20\text{ ms}$$

Draw a sketch of the plot done by Matlab. Label the period on your plot.

SECTION OF A SINUSOID



(See last page for Matlab plot.)

Solutions ECE 2025 PS #2

2.3) Given $z(t) = j4e^{j(-\pi/12)(t-4)}$ evaluate the following definite integral:

a) $\int_0^{30} j4e^{j(-\frac{\pi}{12})(t-4)}\,dt$

$= j4e^{j\frac{\pi}{3}} \int_0^{30} e^{j(-\frac{\pi}{12})t}\,dt$

$= j4e^{j\frac{\pi}{3}} \left(-\frac{12}{\pi j}\right)\left[e^{j(-\frac{\pi}{12})t}\right]_0^{30}$

$= -\frac{48}{\pi} e^{j\frac{\pi}{3}}\left[e^{j(-\frac{5\pi}{2})} - 1\right]$

$= \frac{-48}{\pi}\left[e^{j(-\frac{13\pi}{6})} - e^{j\frac{\pi}{6}}\right]$

$= \frac{-48}{\pi}\left[\frac{\sqrt{3}}{2} - j\frac{1}{2} - \frac{1}{2} - j\frac{\sqrt{3}}{2}\right]$

$= \frac{-24}{\pi}\left[(\sqrt{3}-1) - j(\sqrt{3}+1)\right]$

$= -5.5925 + j20.8713$

$= 21.6076\, e^{j1.8326}$ Express as a complex constant in polar form.

Find all values of $u > 0$ such that $\int_3^u z(t)\,dt = 0$:

b) From above:

$\int_3^u z(t)\,dt = -\frac{48}{\pi} e^{j\frac{\pi}{3}}\left[e^{j(-\frac{\pi}{12})t}\right]_3^u$

$= \frac{-48}{\pi} e^{j\frac{\pi}{3}}\left[e^{j(-\frac{\pi}{12})u} - e^{j(-\frac{\pi}{4})}\right]$

$0 = e^{j(-\frac{\pi}{12})u} - e^{j(-\frac{\pi}{4})}$

$e^{j(-\frac{\pi}{12})u} = e^{j(-\frac{\pi}{4})}$

because $e^{j\theta}$ is periodic with period 2π

$e^{j(-\frac{\pi}{12})u} = e^{j\left[-\frac{\pi}{12}u + 2\pi n\right]} = e^{j(-\frac{\pi}{4})}$

$u\left(-\frac{\pi}{12}\right) + 2\pi n = -\frac{\pi}{4}$

$u = \frac{3 + 24\pi n}{3 + 24n}$ $\forall\, n \geq 0$

Solutions ECE 2025 PS #2

2.3) We are asked to evaluate the following definite integral:

c) $\int_0^{30} z^*(t)\, z(t)\, dt$

$z^*(t) = -j4e^{+j(+\frac{\pi}{12})(t-4)}$

$\int_0^{30} \left[j4e^{-j(\frac{\pi}{12})(t-4)}\right]\left[-j4e^{j(\frac{\pi}{12})(t-4)}\right] dt$

$= \int_0^{30} 16\, dt$

$= 16t\Big|_0^{30}$

$= 480$

$= 480e^{j0}$

Solutions ECE 2025 PS#2

2.4) Solve the following equation for all possible combinations of M and ψ, using the phasor method.

$$5\cos(\omega_0 t) = M\cos\left(\omega_0 t - \frac{\pi}{6}\right) + 5\cos(\omega_0 t + \psi)$$

$X = 5e^{j0}$ $\quad X_1 = Me^{j(-\frac{\pi}{6})}$ $\quad X_2 = 5e^{j\psi}$

$z = 5$ $\quad z_1 = \frac{\sqrt{3}}{2}M - \frac{1}{2}Mj$ $\quad z_2 = 5\cos\psi + j5\sin\psi$

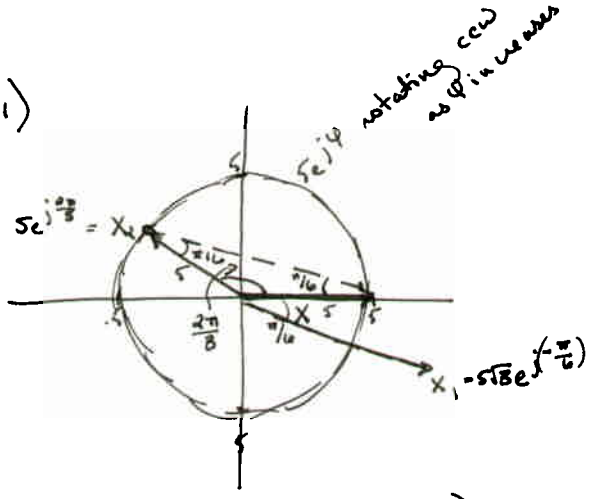
$$5 = \frac{\sqrt{3}}{2}M + 5\cos\psi$$
$$0 = -\frac{1}{2}M + 5\sin\psi$$
$$M = 10\sin\psi$$

$$5 = 5\sqrt{3}\sin\psi + 5\cos\psi$$
$$1 = \sqrt{3}\sin\psi + \cos\psi$$

$$\psi = \frac{2\pi}{3} + 2\pi n$$

$$M = 10\left(\sin\frac{2\pi}{3}\right)$$
$$= 5\sqrt{3}$$

1)



(See last page for Matlab plot.)

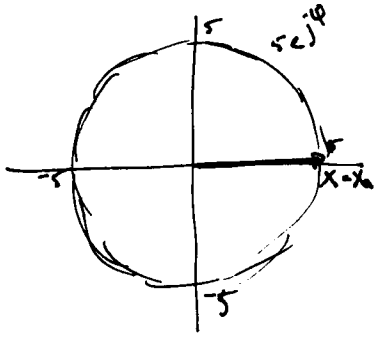
1) $\psi = \frac{2\pi}{3} + 2\pi n$, $M = 5\sqrt{3}$ $\quad \forall n$

2) $M = 0$, $\psi = 2\pi n$ $\quad \forall n$

for all integers n

Provide a geometrical diagram to explain the answer.

2)



if $\psi = 2\pi n$
then $X_2 = X$
$\Rightarrow X_1 = 0e^{j(-\frac{\pi}{6})}$

Solutions ECE 2025 PS #2

2.5) Determine a formula for $x(t)$ in terms of complex exponentials

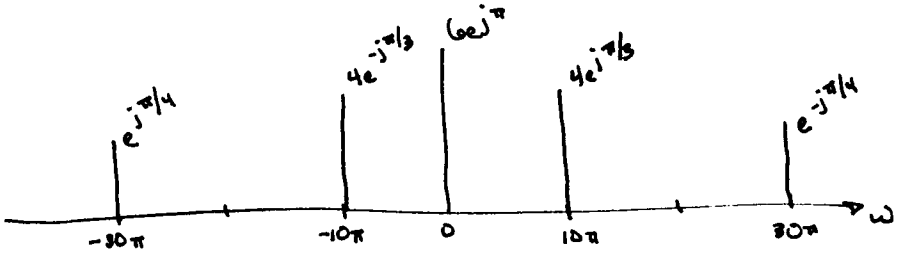
a) $x(t) = -6 + 8\cos(10\pi t + \pi/3) + 2\cos(30\pi t - \pi/4)$

$$= 6e^{j\pi} + \frac{8}{2}e^{j\pi/3}e^{j10\pi t} + \frac{8}{2}e^{-j\pi/3}e^{-j10\pi t}$$

$$+ \frac{2}{2}e^{-j\pi/4}e^{j30\pi t} + \frac{2}{2}e^{j\pi/4}e^{-j30\pi t}$$

$$= 6e^{j\pi} + 4e^{j\pi/3}e^{j10\pi t} + 4e^{-j\pi/3}e^{-j10\pi t} + e^{-j\pi/4}e^{j30\pi t} + e^{j\pi/4}e^{-j30\pi t}$$

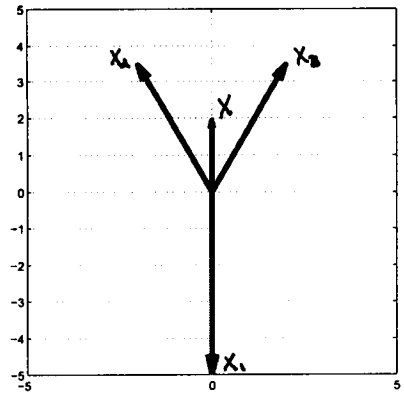
b)



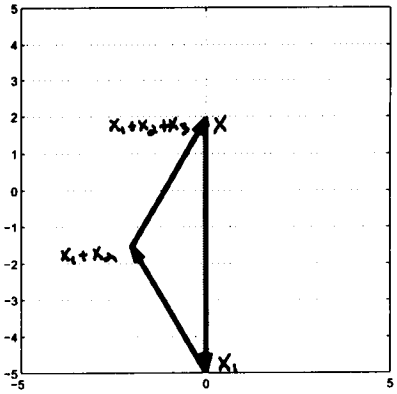
Plot the spectrum representation for $x(t)$.

Solutions ECE 2025 PS#2

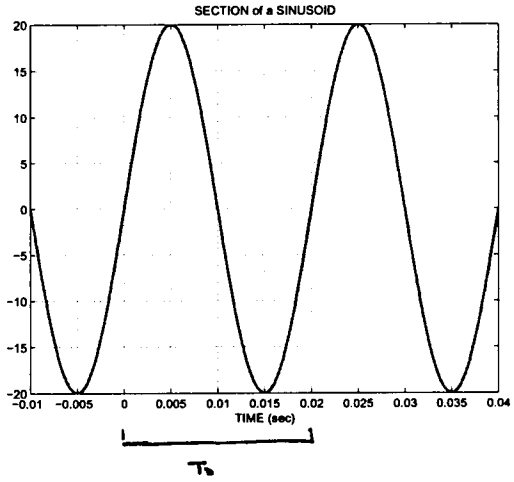
Problem 2.1 (b), View 1



Problem 2.1 (b), View 2



Problem 2.2



Problem 2.4