GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2025   Fall 2004
Lab #4: FM Synthesis for Musical Instruments

Date: 14 – 20 Sept 2004

FORMAL Lab Report: You must write a formal lab report that describes your approach to FM synthesis (Section 4). *This lab report will be worth 150 points.*

You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section *before your assigned lab time.* You must complete the online Pre-lab exercise on Web-CT at the *beginning* of your scheduled lab session. You can use MATLAB and also consult your lab report or any notes you might have, but you cannot discuss the exercises with any other students. The duration of the online Pre-Post-Lab exercise is approximately 20 minutes. The Pre-Post-Lab exercise for this lab includes some questions about concepts from the previous Lab report as well as questions on the Pre-Lab section of this lab.

The Warm-up section of each lab must be completed *during your assigned Lab time* and the steps marked *Instructor Verification* must also be signed off *during the lab time.* One of the laboratory instructors must verify the appropriate steps by signing on the *Instructor Verification* line. When you have completed a step that requires verification, simply raise your hand and demonstrate the step to the TA or instructor. After completing the warm-up section, turn in the verification sheet to your TA.

The exercises in Section 4 should written up as a *FORMAL* lab report. More information on the lab report format can be found on Web-CT under the "Information" link. You should *label* the axes of your plots and include a title and Figure number for every plot. Every plot should be referenced by Figure number in your text discussion. In order to make it easy to find all the plots, include each plot *inlined* within your report. This can be done easily with MATLAB's `notebook` capability.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports but the submitted work should be original and it should be your own work.

The *FORMAL* lab report will be *due during the period 21 to 27-Sept. at the start of your lab.*

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1   Introduction

The objective of this lab is to introduce more complicated signals that are related to the basic sinusoid. These signals which implement frequency modulation (FM) and amplitude modulation (AM) are widely used in communication systems such as radio and television, but they also can be used to create interesting sounds that mimic musical instruments. There are a number of demonstrations on the CD-ROM that provide examples of these signals for many different conditions.

CD-ROM

FM Synthesis

2   Pre-Lab

We have spent a lot of time learning about the properties of sinusoidal waveforms of the form:

$$x(t) = A \cos(2\pi f_0 t + \phi) = \Re e \left\{ A e^{j\phi} e^{j 2\pi f_0 t} \right\} \tag{1}$$

In this lab, we will extend our treatment of sinusoidal waveforms to more complicated signals composed of sums of sinusoidal signals, or sinusoids with changing frequency.

2.1 Frequency Modulated Signals

We will also look at signals in which the frequency varies as a function of time. In the constant-frequency sinusoid (1) the argument of the cosine is $(2\pi f_0 t + \phi)$ which is also the exponent of the complex exponential. We will refer to the argument of the cosine as the ***angle function***. In equation (1), the *angle function* changes *linearly* versus time, and its time derivative is $2\pi f_0$ which equals the constant frequency of the cosine.

A generalization is available if we adopt the following notation for the class of signals with time-varying angle functions:

$$x(t) = A\cos(\psi(t)) = \Re e\{Ae^{j\psi(t)}\} \tag{2}$$

The time derivative of the angle function $\psi(t)$ in (2) gives a frequency

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{(rad/sec)}$$

but if we prefer units of hertz, then we divide by 2π to define the *instantaneous frequency:*

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\psi(t) \qquad \text{(Hz)} \tag{3}$$

2.2 Wideband FM

The instantaneous frequency is a useful measure of frequency content when the rate of change of the frequency is slow relative to the frequency analysis of the signal. For example, it is easy to concoct examples where FM signals with various modulation rates sound quite different "to the ear." The examples given on the *SP-First* CD-ROM in Chapter 3 demonstrate this effect.

Please view the examples on the CD-ROM

The fact that we hear different effects depending on the rate of the modulating signal can be explained by modeling the human hearing system as a spectrum analyzer. There is good physiological evidence that the cochlea (part of the inner ear) performs a decomposition of sound signals into frequency bands. Thus, we can think of the first stage of the human hearing system as equivalent to a spectrogram. If so, then the hearing system will analyze the sound signals over some time interval which seems to be in the range of 20 to 50 millisec. If we calculate a window length for the spectrogram that would have the same duration, we get lengths between 160 and 400 at 8000 samples per sec. In the examples below, we use a window length of 512, but 256 should also be a reasonable choice to show the same perceptual phenomenon.

2.3 FM Sound Synthesis

Frequency modulation (FM) can be used to make interesting sounds that mimic musical instruments, such as bells, woodwinds, drums, etc. The goal in this lab is to implement one or two of these FM schemes and hear the results.

We have already seen that FM defines the signal $x(t)$ to have a time-varying phase $\psi(t)$, and that the instantaneous frequency changes according to the derivative of $\psi(t)$. If $\psi(t)$ is linear, $x(t)$ is a constant-frequency sinusoid; whereas, if $\psi(t)$ is quadratic, $x(t)$ is a chirp signal whose frequency changes linearly in time. FM music synthesis uses a more interesting $\psi(t)$, one that is sinusoidal. Since the derivative of a sinusoidal $\psi(t)$ is also sinusoidal, the instantaneous frequency of $x(t)$ will oscillate. This is useful for synthesizing instrument sounds because the proper choice of the modulating frequencies will produce a fundamental frequency and several overtones, as many instruments do.

The general equation for an FM sound synthesizer is:

$$x(t) = A(t)\cos\left(2\pi f_c t + I(t)\cos(2\pi f_m t + \phi_m) + \phi_c\right) \tag{4}$$
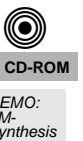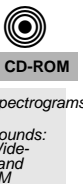
where $A(t)$ is the signal's amplitude. It is a function of time so that the instrument sound can be made to fade out slowly or cut off quickly. Such a function is called an *envelope*. The parameter f_c is called the *carrier* frequency. Note that when you take the derivative of $\psi(t)$ to find $f_i(t)$,

$$
\begin{aligned}
f_i(t) &= \frac{1}{2\pi}\frac{d}{dt}\psi(t) \\
&= \frac{1}{2\pi}\frac{d}{dt}\left(2\pi f_c t + I(t)\cos(2\pi f_m t + \phi_m) + \phi_c\right) \\
&= f_c - f_m I(t)\sin(2\pi f_m t + \phi_m) + \frac{1}{2\pi}\frac{dI}{dt}\cos(2\pi f_m t + \phi_m)
\end{aligned}
\tag{5}
$$

f_c will be a constant in that expression. It is the frequency that would be produced without any frequency modulation. The parameter f_m is called the *modulating* frequency. It expresses the rate of oscillation of the instantaneous frequency, $f_i(t)$. The parameters ϕ_m and ϕ_c are arbitrary phase constants, usually both set to $-\pi/2$ so that $x(0) = 0$.

The function $I(t)$ has a less obvious purpose than the other FM parameters in (4). It is technically called the *modulation index envelope.* To see what it does, examine the expression for the instantaneous frequency (5). The quantity $I(t)f_m$ multiplies a sinusoidal variation of the frequency. If $I(t)$ is constant or $\dfrac{dI}{dt}$ is relatively small, then $I(t)f_m$ gives the maximum amount by which the instantaneous frequency deviates from f_c. Beyond that, however, it is difficult to relate $I(t)$ to the sound made by $x(t)$ without some rather tedious mathematical analysis.

In our study of signals, we would like to characterize $x(t)$ as the sum of several constant-frequency sinusoids instead of a single sinusoidal signal whose frequency changes. In this regard, the following comments are relevant: when $I(t)$ is small (e.g., $I \approx 1$), low multiples of the carrier frequency (f_c) have high amplitudes. When $I(t)$ is large ($I > 4$), both low and high multiples of the carrier frequency have high amplitudes. The net result is that $I(t)$ can be used to vary the harmonic content of the instrument sound (called overtones). When $I(t)$ is small, mainly low frequencies will be produced. When $I(t)$ is large, higher harmonic frequencies can also be produced. Since $I(t)$ is a function of time, the harmonic content will change with time. For more details see the paper by Chowning.[1]

2.4 MATLAB Synthesis of Wideband FM

The following MATLAB code will synthesize FM signals:

```
fsamp = 8000;
dt = 1/fsamp;
dur = 1.1;
tn = 0 : dt : dur;
fc = 900;   fm = 3;   Im = 100/pi;   % <==== FM parameters
psi = 2*pi*fc*tn + Im*cos(2*pi*fm*tn-pi/2) - pi/2;
xx = real( 7.7*exp(j*psi) );
soundsc( xx, fsamp );
plotspec( xx+j*1e-14, fsamp, 512),   grid on
```

(a) In MATLAB signals can only be synthesized by evaluating the signal's defining formula at discrete instants of time. These are called *samples* of the signal. For FM we do the following:

$$
x(t_n) = A\cos(\psi(t_n))
$$

---

[1]Ref: John M. Chowning, "The Synthesis of Complex Audio Spectra by means of Frequency Modulation," *Journal of the Audio Engineering Society*, vol. 21, no. 7, Sept. 1973, pp. 526–534.

where t_n is the nth time sample. From the MATLAB code above, identify the mathematical formula for $\psi(t)$. In addition, write the mathematical expression for t_n as specified in the MATLAB code.

(b) Determine the formula for the instantaneous frequency of the FM signal and compare that formula to the result in the spectrogram. Notice that you can hear the frequency variation predicted by the instantaneous frequency. We will refer to this case as *slowly-varying FM*.

(c) Now change the "modulating parameters" f_m and I_m to be $f_m = 100$ Hz and $I_m = 1/\pi$. Listen to the signal and notice that it is no longer possible to hear the instantaneous frequency. This is called *wideband FM*. In addition, try to interpret the spectrogram in terms of what you hear. Think about the following: (i) is $x(t)$ periodic? If so, determine its fundamental period. (ii) does the spectrum appear to be harmonic? If so, determine the spacing of the spectral lines and the fundamental frequency.

3 Warm-up

During the warm-up you will need a general function for synthesizing FM signals with sinusoidal modulation. Therefore, write an M-file **fmsinus.m** that implements Eq. (4) with $A(t) = 1$, $\phi_c = \phi_m = -\pi/2$, and $I(t) = I_m$. Use the following calling arguments:

```
function  [xx,tt] = FMsinus(fc, fm, Im, dur, fsamp)
%FMSINUS     produce FM signal with sinusoidal modulation
%
%      usage:  [xx,tt] = fmsinus(fc, fm, Im, dur, fsamp)
%
%      where:  fc = carrier frequency in Hz
%              fm = modulation frequency in Hz
%              Im = scale factor for modulation index
%             dur = duration (in sec.) of the output signal
%           fsamp = sampling rate
```

Instructor Verification (separate page)

Your function will generate any FM signal with sinusoidal modulation according to the following formula:

$$x(t) = \cos(2\pi f_c t + I_m \cos(2\pi f_m t - \pi/2) - \pi/2)$$

3.1 Slowly Varying FM

Generate an FM signal with $f_c = 900$ Hz, $f_m = 2$ Hz and $I_m = 20$ over the time interval $0 \le t \le 1.25$ secs. Use a sampling rate of 8000 Hz, i.e., fsamp = 8000.

(a) Plot the FM signal's spectrogram and listen to the signal to see if it corresponds to the spectrogram.

(b) Assuming that you used a window length of 512 in the spectrogram, then the window length in seconds is equal to $512 \times (1/8000) = 0.064$ secs. Determine how much the instantaneous frequency moves during a time interval of this duration. You will need a formula for the instantaneous frequency, or you can estimate this frequency movement from the "steps" visible in the spectrogram.

(c) Make a sketch (by hand) of the instantaneous frequency for the signal. Determine the period of the signal, and the minimum and maximum values of the instantaneous frequency.

Instructor Verification (separate page)

4

3.2 Wideband FM

Generate a wideband FM signal with $f_c = 900$ Hz, $f_m = 200$ Hz and $I_m = 1/\pi$ over the time interval $0 \le t \le 1.25$ secs. Use a sampling rate of 8000 Hz, i.e., `fsamp = 8000`.

(a) Plot the FM signal's spectrogram and listen to the signal to see if it corresponds to the spectrogram.

(b) Determine the fundamental period of the FM signal, and then explain why the spectrogram has equally spaced spectral lines, and how that spacing is related to the period.

(c) Derive a formula for the instantaneous frequency for the signal defined above, and then make a sketch (by hand) of $f_i(t)$ versus t.

(d) Assuming that you used a window length of 512 in the spectrogram, then determine how much the instantaneous frequency moves during a time interval of 0.064 secs. You will need a formula for $f_i(t)$, because you cannot estimate this frequency movement from the spectrogram.

(e) Now, change I_m to be $I_m = 1$, but keep $f_c = 900$ Hz and $f_m = 200$ Hz.. Generate the signal, plot its spectrogram and listen to the signal. Describe how the sound and spectrogram have changed.
⎡ **Instructor Verification** (separate page) ⎤

(f) For an interesting wideband FM signal, listen to the following signal and view its spectrogram:

```
tt = 0 : 1/8000 : 4.4;
xx = exp(-tt).*sin(300*pi*tt+10*exp(-tt).*sin(600*pi*tt));
```

Determine which frequency component is the strongest one in the spectrogram, and try to relate that one to what is heard. (It might only be possible to do this qualitatively.)

4 Lab: FM Synthesis of Instrument Sounds

In the lab exercises, the objective is to write a function that will generate bell sounds, and then use that function to play a musical scale consisting of eight successive notes on the C-major scale (see Section 3.7 in *SP-First*).

4.1 Generating the Bell Envelopes

Now we take the general FM synthesis formula (4) and specialize for the case of a bell sound. The amplitude envelope $A(t)$ and the modulation index envelope $I(t)$ for the bell are both decaying exponentials. That is, they both have the following form:

$$y(t) = e^{-t/\tau} \tag{6}$$

where τ is a parameter that controls the decay rate of the exponential. Notice that $y(0) = 1$ and $y(\tau) = 1/e$, so τ is the time it takes a signal of the form (6) to decay to $1/e = 36.8\%$ of its initial value. For this reason, the parameter τ is called the *time constant.*

Use (6) to write a MATLAB function that will generate a decaying exponential to be used later in synthesizing a bell sound. The file header should look like this:

```
function    [yy,tt] = bellenv(tau, dur, fsamp);
%BELLENV    produces envelope function for bell sounds
%
%        usage: [yy,tt] = bellenv(tau, dur, fsamp);
%
%              where tau = time constant
%                    dur = duration of the envelope
%                  fsamp = sampling frequency
%                returns:
%                     yy = decaying exponential envelope
%    note: produces exponential decay when tau is positive
```

The function will be one or two lines of MATLAB code. The first line should define your time vector based on `fsamp` and `dur`, and the second generates the exponential (6).

The bell's amplitude envelope, $A(t)$, and modulation index envelope, $I(t)$ are identical, up to a scale factor.

$$A(t) = A_0 e^{-t/\tau} \qquad \text{and} \qquad I(t) = I_0 e^{-t/\tau}$$

Hence, one call to the `bellenv` function will be sufficient to generate the shape for both envelopes.

4.2 Parameters for the Bell

Now that we have the bell's amplitude and modulation index envelopes, we can create the actual sound signal for the bell by specifying all the parameters in the general FM synthesis formula (4). The frequencies f_c and f_m must be given numerical values. The ratio of carrier to modulating frequency is important in creating the sound of a specific instrument. For the bell, a good choice for this ratio is 1:2, e.g., $f_c = 110$ Hz and $f_m = 220$ Hz, or 5:7 with $f_c = 250$ Hz and $f_m = 350$ Hz.

Now write a simple M-file `bell.m` that implements (4) to synthesize a bell sound. Your function should call `bellenv.m` to generate the exponential needed in $A(t) = A_0 e^{-t/\tau}$ and $I(t) = I_0 e^{-t/\tau}$.

```
function  xx = bell(ff, Io, tau, dur, fsamp)
%BELL      produce a bell sound
%
%     usage:   xx = bell(ff, Io, tau, dur, fsamp)
%
%     where:  ff = frequency vector (containing fc and fm)
%             Io = scale factor for modulation index
%            tau = decay parameter for A(t) and I(t)
%            dur = duration (in sec.) of the output signal
%          fsamp = sampling rate
```

4.3 The Bell Sound

Test your `bell( )` function using the parameters of case #1 in the table below. Play the synthesized signal with the `soundsc()` function at 11,025 Hz.[2] Does it sound like a bell? The value of $I_0 = 10$ for scaling the modulation index envelope is known to give a distinctive sound. Later on, you can experiment with other values to get a variety of bells.

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[2]A higher sampling rate of 11,025 Hz is used because the signal contains many harmonics, some of which might alias if a lower f_s were used. You should experiment with lower values of f_s to see if you can hear a difference, e.g., $f_s = 8000$ Hz.

CASE	f_c (Hz)	f_m (Hz)	I_0	τ (sec)	T_{dur} (sec)	f_s (Hz)
1	110	220	10	2	6	11,025
2	220	440	5	2	6	11,025
3	110	220	10	12	3	11,025
4	110	220	10	0.3	3	11,025
5	250	350	5	2	5	11,025
6	250	350	3	1	5	11,025

The frequency spectrum of the bell sound is very complicated, but it does consist of spectral lines, which can be seen with a spectrogram. Among these frequencies, one spectral line will dominate what we hear. We would call this the ***note frequency*** of the bell. It is tempting to guess that the note frequency will be equal to f_c, but you will have to experiment to find the true answer. It might be f_m, or $|f_m - f_c|$, or maybe the fundamental frequency, $\gcd(f_c, f_m)$, or maybe something else.

For case #5 in the table, try to answer the following questions (some of which are quite subjective):

(a) Listen to the sound by playing it with the `soundsc()` function, and then display a spectrogram of the signal. Point out the "harmonic" structure of the spectrogram, and calculate the fundamental frequency, f_0. Determine whether or not the signal is periodic. Explain.

(b) Explain how the fundamental frequency of the "note" is related to f_c and f_m. Explain whether or not you think it is possible to verify by listening that you have the correct fundamental frequency.

(c) Consider whether or not you can hear the frequency content changing according to $I(t)$. Plot the instantaneous frequency $f_i(t)$ versus t for comparison.

(d) Describe how the frequency content changes versus time, and how that change might be related to $I(t)$, or to $f_i(t)$. This discussion is likely to be subjective, i.e., there is no one correct answer.

4.3.1 Comments about the Bell

In preparation for the work in the next section, here are some experiments that you should do on your own.[3] Cases #3 and #4 are extremes for choosing the decay rate τ. In case #3, the waveform does not decay very much over the course of three seconds and sounds a little like a sum of harmonically related sinusoids. With a "faster" decay rate, as in case #4, we get a percussion-like sound. Modifying the fundamental frequency f_0 (determined in part (d) above) should have a noticeable effect on the tone you hear. Try some different values for f_0 by changing f_c and f_m, but still in the ratio of 1:2. Describe how the sound changes.

Finally, experiment with different carrier to modulation frequency ratios. For example, in his paper, Chowning uses a fundamental frequency of $f_0 = 40$ Hz and a carrier to modulation frequency ratio of 5:7. Try this and a few other values. Learn how the parameters affect the sound you hear.

4.4 C-Major Scale

Read Section 3.7 in the textbook to learn more about the spectrogram and its relationship to music. Then use the "FM bell" function to synthesize the eight notes that make up the C-major scale (defined in Fig. 3.23 of *SP-First*) and play the notes in succession so that it sounds like a scale. Pick parameters for the "FM bell" so that each note has the correct frequency and lasts no longer than 0.6 seconds. Use a sampling frequency of $f_s = 22050$ Hz to avoid aliasing. *Be creative.*
Show a spectrogram of the entire scale to justify that you generated the correct notes.

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[3]Nothing from this subsection has to be included in your lab report, but you will have to do some experimentation to get a satisfactory result for the C-major scale.

Lab #4
ECE-2025
Fall-2004
INSTRUCTOR VERIFICATION SHEET

Turn this page in to your TA before the end of your lab period.

Name: _____   Date of Lab: _____

Part 3: Write the MATLAB function **fmsinus.m**

Verified:_____   Date/Time:_____

Part 3.1: Slowly Varying Sinusoidal FM Signal Generation. Sketch the instantaneous frequency:

Verified:_____   Date/Time:_____

Part 3.2: Wideband Sinusoidal FM Signal Generation. Show a spectrogram and discuss its qualitative features:

Verified:_____   Date/Time:_____