

**ECE 2025 Fall 2004**  
**Lab #2: Introduction to Complex Exponentials**

Date: 30 Aug. – 2 Sept. 2004

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**You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section before your assigned lab time.** You **MUST** complete the online Pre-lab exercise on Web-CT at the beginning of your lab session. You can use MATLAB or any notes you might have, but you cannot discuss the exercises with any other students. You will have approximately 20 minutes at the beginning of your lab session to complete the online Pre-Lab exercise. The Pre-Lab exercise for this this lab also includes some questions about concepts from the *previous* report.

The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. One of the laboratory instructors must verify the appropriate steps by signing the **Instructor Verification** line. When you have completed a step that requires verification, simply raise your hand and demonstrate the step to the TA or instructor. After completing the warm-up section, turn in the verification sheet to your TA.

It is only necessary to turn in Section 4 as the lab report for this lab. More information on the lab report format can be found on Web-CT under the ‘Information’ link. You are asked to **label** the axes of your plots and include a title for every plot. In order to reduce missing plots, include your plot as a figure *embedded* within your report. For more information on how to include figures and plots from MATLAB in your report file, consult the ‘information’ link on Web-CT. If you still do not know how to do so, ask your TA.

*Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports but the submitted work should be original and it should be your own work.*

**The lab report and verification will be graded out of 100 points. The Pre-Post-Lab questions will be graded separately.**

The lab report will be **due during the period 7-Sept. through 13-Sept. at the start of your lab.**

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**PRINTING BUDGET:** For the printers in the ECE labs, you have a quota. Please limit your printing to essential items for the labs. If you need to print lecture slides and other large documents, use the central (OIT) printing facilities.

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## 1 Introduction and Overview

The goal of this laboratory is to gain familiarity with complex numbers and their use in representing sinusoidal signals such as  $x(t) = A \cos(\omega t + \phi)$  as complex exponentials  $z(t) = Ae^{j\phi} e^{j\omega t}$ . The key is to use the complex amplitude and then the real part operator applied to Euler’s formula:

$$x(t) = A \cos(\omega t + \phi) = \Re\{Ae^{j\phi} e^{j\omega t}\}$$

Manipulating sinusoidal functions using complex exponentials turns trigonometric problems into simple arithmetic and algebra. In this lab, we first review the complex exponential signal and the phasor addition property needed for adding cosine waves. Then we will use MATLAB to make plots of phasor diagrams that show the vector addition needed when combining sinusoids.

## 1.1 Complex Numbers in MATLAB

MATLAB can be used to compute complex-valued formulas and also to display the results as vector or “phasor” diagrams. For this purpose several new MATLAB functions have been written and are available on the *SP First CD-ROM*. Make sure that this toolbox has been installed<sup>1</sup> by doing `help` on the new M-files: `zvect`, `zcat`, `ucplot`, `zcoords`, and `zprint`. Each of these functions can plot (or print) several complex numbers at once, when the input is formed into a vector of complex numbers. For example, try the following function call and observe that it will plot five vectors all on one graph:

```
zvect( [ 1+j, j, 3-4*j, exp(j*pi), exp(2j*pi/3) ] )
```

Here are some of MATLAB’s complex number operators:

<code>conj</code>	Complex conjugate
<code>abs</code>	Magnitude
<code>angle</code>	Angle (or phase) in radians
<code>real</code>	Real part
<code>imag</code>	Imaginary part
<code>i, j</code>	pre-defined as $\sqrt{-1}$
<code>x = 3 + 4i</code>	<code>i</code> suffix defines imaginary constant (same for <code>j</code> suffix)
<code>exp(j*theta)</code>	Function for the complex exponential $e^{j\theta}$

Each of these functions takes a vector (or matrix) as its input argument and operates on each element of the vector. Notice that the function names `mag()` and `phase()` do not exist in MATLAB.<sup>2</sup>

Finally, there is a complex numbers drill program called:

```
zdrill
```

which uses a GUI to generate complex number problems and check your answers. *Please spend some time with this drill since it is very useful in helping you to get a feel for complex arithmetic.*

When unsure about a command, use `help`.

## 1.2 Sinusoid Addition Using Complex Exponentials

Recall that sinusoids may be expressed as the real part of a complex exponential:

$$x(t) = A \cos(2\pi f_0 t + \phi) = \Re \left\{ A e^{j\phi} e^{j2\pi f_0 t} \right\} \quad (1)$$

The *Phasor Addition Rule* presented in Section 2.6.2 of the text shows how to add several sinusoids:

$$x(t) = \sum_{k=1}^N A_k \cos(2\pi f_0 t + \phi_k) \quad (2)$$

<sup>1</sup>Correct installation means that the `spfirst` directory will be on the MATLAB path. Try `help path` if you need more information.

<sup>2</sup>In the latest release of MATLAB a function called `phase()` is defined in a seldom used toolbox; it does more or less the same thing as `angle()` but also attempts to add multiples of  $2\pi$  when processing a vector.



assuming that each sinusoid in the sum has the *same* frequency,  $f_0$ . This sum is difficult to simplify using trigonometric identities, but it reduces to an algebraic sum of complex numbers when solved using complex exponentials. If we represent each sinusoid with its *complex amplitude*

$$X_k = A_k e^{j\phi_k} \quad (3)$$

Then the complex amplitude of the sum is

$$X_s = \sum_{k=1}^N X_k = A_s e^{j\phi_s} \quad (4)$$

Based on this complex number manipulation, the *Phasor Addition Rule* implies that the amplitude and phase of  $x(t)$  in equation (2) are  $A_s$  and  $\phi_s$ , so

$$x(t) = A_s \cos(2\pi f_0 t + \phi_s) \quad (5)$$

We see that the sum signal  $x(t)$  in (2) and (5) is a single sinusoid that still has the same frequency,  $f_0$ , and it is periodic with period  $T_0 = 1/f_0$ .

### 1.3 Harmonic Sinusoids

There is an important extension where  $x(t)$  is the sum of  $N$  cosine waves whose frequencies ( $f_k$ ) are *different*. If we concentrate on the case where the frequencies ( $f_k$ ) are all multiples of one basic frequency  $f_0$ , i.e.,

$$f_k = k f_0 \quad (\text{HARMONIC FREQUENCIES})$$

then the sum of  $N$  cosine waves given by (2) becomes

$$x_h(t) = \sum_{k=1}^N A_k \cos(2\pi k f_0 t + \phi_k) = \Re \left\{ \sum_{k=1}^N X_k e^{j2\pi k f_0 t} \right\} \quad (6)$$

This particular signal  $x_h(t)$  has the property that it is also periodic with period  $T_0 = 1/f_0$ , because each of the cosines in the sum repeats with period  $T_0$ . The frequency  $f_0$  is called the *fundamental frequency*, and  $T_0$  is called the *fundamental period*. (Unlike the single frequency case, there is no phasor addition theorem here to combine the harmonic sinusoids.)

## 2 Pre-Lab

You should do all the exercises in this section to prepare for the on-line pre-lab questions.

### 2.1 Complex Numbers

This section will test your understanding of complex numbers when plotted as vectors. Use  $z_1 = 2e^{j\pi/4}$  and  $z_2 = -\sqrt{3} + j$  for all parts of this section.

- (a) Enter the complex numbers  $z_1$  and  $z_2$  in MATLAB and plot them with `zvect()`, and print them with `zprint()`.

When unsure about a command, use `help`.

Whenever you make a plot with `zvect()` or `zcat()`, it is helpful to provide axes for reference. An  $x$ - $y$  axis and the unit circle can be superimposed on your `zvect()` plot by doing the following:  
`hold on, zcoords, ucplot, hold off`

- (b) Compute the conjugate  $z^*$  and the inverse  $1/z$  for both  $z_1$  and  $z_2$  and plot the results as vectors. In MATLAB, see `help conj`. Display the results numerically with `zprint`.
- (c) The function `zcat()` can be used to plot vectors in a “head-to-tail” format. Execute the statement `zcat([1+j, -2+j, 1-2j]);` to see how `zcat()` works when its input is a vector of complex numbers.
- (d) Compute  $z_1 + z_2$  and plot the sum using `zvect()`. Then use `zcat()` to plot  $z_1$  and  $z_2$  as 2 vectors head-to-tail, thus illustrating the vector sum. Use `hold on` to put all 3 vectors on the same plot. If you want to see the numerical value of the sum, use `zprint()` to display it.
- (e) Compute  $z_1 z_2$  and  $z_2/z_1$  and plot the answers using `zvect()` to show how the angles of  $z_1$  and  $z_2$  determine the angles of the product and quotient. Use `zprint()` to display the results numerically.
- (f) Make a  $2 \times 2$  subplot that displays four plots in one window: similar to the four operations done previously: (i)  $z_1$ ,  $z_2$ , and the sum  $z_1 + z_2$  on a single plot; (ii)  $z_2$  and  $z_2^*$  on the same plot; (iii)  $z_1$  and  $1/z_1$  on the same plot; and (iv)  $z_1 z_2$ . Add a unit circle and  $x$ - $y$  axis to each plot for reference.

## 2.2 Z-Drill

Work a few problems generated by the complex number drill program. To start the program simply type `zdrill` (if necessary, install the GUI and add `zdrill` to MATLAB’s path). Use the buttons on the graphical user interface (GUI) to produce different problems.

## 2.3 The Notebook Option

The purpose of this section is to introduce you to the notebook capability that links MATLAB and Microsoft Word under Windows. This option allows you to execute MATLAB commands, scripts and functions from inside a Microsoft Word document and then have results (including graphs) displayed automatically inside that same MS-Word document. This should make it easy to create lab reports by reducing the problems (and pain) associated with importing figures and results into MS-Word. You are not required to use this feature to write your lab reports, but many students find it useful.<sup>3</sup>

Note that GUI items such as `zdrill` will not work from inside MS-Word. Also note that MATLAB functions cannot be defined from inside a MS-Word document.

## 2.4 Notebook setup

Depending on whether or not you have “Administrator privileges” on your Windows machine, there are two ways to set up the notebook functionality (which works in versions 5.x and 6.x of MATLAB).

1. This *easy* setup works if your Windows account has administrator privileges: From inside MATLAB, type `notebook -setup`, and follow the instructions given.<sup>4</sup>
2. This *harder* setup that works even for accounts with non-administrative privileges (i.e., it works in ECE Labs such as BH-216).

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<sup>3</sup>In MATLAB 7 there is a new feature that allows you to publish MATLAB code to a Word document. We will not use this during the lab, but an introduction to this new MATLAB feature has been written and is available as a link on Web-CT.

<sup>4</sup>Once the the setup is complete, click on the start menu and find the `New Office Document` option. Under the `General` tab you should find `m-book.dot`. Click on it to start MS-Word. This might also start up MATLAB if it is not already running.

- (a) Open MS-Word and then go to the menu `Tools->Macro->Security` and select medium instead of high. Click the OK button.
  - (b) Now go to the `File` menu in MS-Word, and open the file `m-book.dot` which is in the folder `C:\App1\Matlab\R14\Notebook\PC\`.
  - (c) A dialog box comes up where you type the path to the `matlab.exe` file. Type in `C:\App1\MatlabR14\Bin\win32\`.
  - (d) MATLAB will start up and once it has loaded up, return to MS-Word and select the `File` menu and pick `New M-Book`.
- A new MS-Word window will come up and the setup is complete. MS-Word will have a new menu called `Notebook` which contains a number of new commands for communicating with MATLAB.

### 2.4.1 Using MATLAB from MS-Word

Once the `notebook` capability has been installed you can start it by either: (1) typing `notebook` at the MATLAB command prompt, or (2) by selecting `New M-Book` in the `File` menu of MS-Word.

- To verify that the `notebook` capability works, from inside MS-Word, type `var_A=1+1i-6`, and then press the `Ctrl-Enter` keys together. This should give you the answer in the MS-Word document.
- To create a plot inside MS-Word, type the the line below (followed by `Ctrl-Enter`)  
`t=0:0.1:pi; y=sin(3*t+0.05); plot(t,y); title('My First Plot')`
- To call one of your functions or MATLAB script files, just type in the name followed by `Ctrl-Enter`. Built-in MATLAB functions are called in the same way, for example  
`[V,lam] = eig([1 2 3; 3 3 3; -1 2 3])` followed by `Ctrl-Enter`
- Take some time to investigate the other items and more advanced features under the `Notebook` menu that was installed in MS-Word. Additional help can be found in MATLAB by typing `helpdesk` and searching for `Notebook`.
- If you dislike the way that figures get inserted inside MS-Word with a grey background, then you can change to a white background by resetting one of MATLAB's defaults. One way is to execute the following at the beginning of the MS-Word document  
`set(0,'DefaultFigureColor',[1 1 1]);set(0,'DefaultAxesColor',[1 1 1]);`

## 2.5 Vectorization

The power of MATLAB comes from its matrix-vector syntax. In most cases, loops can be replaced with vector operations because functions such as `exp()` and `cos()` are defined for vector inputs, e.g.,

$$\cos(vv) = [\cos(vv(1)), \cos(vv(2)), \cos(vv(3)), \dots, \cos(vv(N))]$$

where `vv` is an  $N$ -element row vector. Vectorization can be used to simplify your code. If you have the following code that plots a certain signal,

```

M = 200;
for k=1:M
    x(k) = k;
    y(k) = cos( 0.001*pi*x(k)*x(k) );
end
plot( x, y, 'ro-' )

```

then you can replace the `for` loop with one line and get the same result with 3 lines of code:

```

M = 200;
y = cos( 0.001*pi*(1:M).*(1:M) );
plot( 1:M, y, 'ro-' )

```

Run these two programs to see that they give identical results. Use the notebook capability to put the plots into a MS-Word document.

## 2.6 Functions

Functions are a special type of M-file that can accept inputs (matrices and vectors) and also return outputs. The keyword `function` must appear as the first word in the ASCII file that defines the function, and the first line of the M-file defines how the function will pass input and output arguments. The file extension must be lower case “m” as in `my_func.m`. See Section B-6 in Appendix B of the text for more discussion.

The following function has several mistakes. Before looking at the correct one below, try to find these mistake(s) (there are at least four):

```

matlab mfile [xx,tt] = badcos(ff,dur)
%BADCOS Function to generate a cosine wave
% usage:
%     xx = badcos(ff,dur)
%     ff = desired frequency
%     dur = duration of the waveform in seconds
%
tt = 0:1/(100*ff):dur;    %-- gives 100 samples per period
badcos = real(exp(2*pi*freeq*tt));

```

The corrected function should look something like:

```

function [xx,tt] = goodcos(ff,dur)
tt = 0:1/(100*ff):dur;    %-- gives 100 samples per period
xx = real(exp(2i*pi*ff*tt));

```

Notice the word `function` in the first line, and the exponential needs to have an imaginary exponent. Also, the variable `freeq` has not been defined before being used. Finally, the function has `xx` as an output and hence the variable `xx` should appear in the left-hand side of at least one assignment line within the function body. In other words, the function name is *not* used to hold values produced in the function.

## 3 Warm-Up: Complex Exponentials

In the Pre-Lab part of this lab, you learned how to write function M-files. In this section, you will write two functions that can generate sinusoids, or sums of sinusoids. Use MATLAB’s notebook capability to put the plots into a MS-Word document when doing the Instructor Verifications.

### 3.1 Vectorization

Use the vectorization idea to write 2 or 3 lines of code that will perform the same task as the following MATLAB script without using a `for` loop.

```
%--- make a plot of a weird signal
N = 200;
for k=1:N
    xk(k) = k/60;
    rk(k) = sqrt( xk(k)*xk(k) - 1 );
    sig(k) = exp(j*2*pi*rk(k));
end
plot( xk, real(sig), 'mo-', xk, imag(sig), 'go-' )
```

*Note:* there is a difference between the two operations `rr*rr` and `rr.*rr` when `rr` is a vector.

**Instructor Verification** (separate page)

### 3.2 M-file to Generate One Sinusoid

Write a function that will generate a **single** sinusoid,  $x(t) = A \cos(\omega t + \phi)$ , by using input arguments for complex amplitude ( $X = Ae^{j\phi}$ ), frequency ( $\omega$ ), duration (`dur`) and starting time (`tstart`). The function should return two outputs: the values of the sinusoidal signal ( $x$ ) and corresponding times ( $t$ ) at which the sinusoid values are known. Make sure that the function generates exactly 32 values of the sinusoid per period. Call this function `onecos()`. *Hint: use `goodcos()` from the Pre-Lab part as a starting point.* Plot the result from the following call to test your function.

```
[xx0,tt0] = onecos( [5*exp(j*pi/4i)], [2], 2, -1);
```

Use the `notebook` feature to make this plot inside a MS-Word document.

### 3.3 Sinusoidal Synthesis with an M-file: Different Frequencies

Since we will generate many functions that are a “sum of sinusoids,” it will be convenient to have a MATLAB function for this operation. To be general, we will allow the frequency of each component ( $f_k$ ) to be different. The following expressions are equivalent if we define the complex amplitude  $X_k$  as  $X_k = A_k e^{j\phi_k}$ .

$$x(t) = \Re \left\{ \sum_{k=1}^N (A_k e^{j\phi_k}) e^{j2\pi f_k t} \right\} \quad (7)$$

$$x(t) = \sum_{k=1}^N A_k \cos(2\pi f_k t + \phi_k) \quad (8)$$

#### 3.3.1 Write the Sum of Sinusoids M-file

Write an M-file called `sum_Cexp.m` that will synthesize a waveform in the form of (7) using  $X_k$  defined in (3). Although `for` loops are rather inefficient in MATLAB, *you must write the function with one outer loop in this lab.* The inner loop should be vectorized. The first few statements of the M-file are the comment lines—they should look like:

```

function [xx,tt] = sum_Cexp(Xk, fk, dur, tstart)
%SUM_CEXP Synthesize sinusoid from sum of complex exponentials
% usage:
% [xx,tt] = sum_Cexp(Xk, fk, dur, tstart)
% Xk = vector of COMPLEX amplitudes
% fk = vector of frequencies (all positive)
% dur = total time duration of the signal
% tstart = starting time
% xx = vector of sinusoidal values
% tt = vector of times, for the time axis
%
% Note: fk and Xk must be the same length.
% Xk(1) corresponds to frequency fk(1),
% Xk(2) corresponds to frequency fk(2), etc.
% The tt vector should be generated with a small time increment that
% creates 32 samples per period. Use the period corresponding to
% the highest frequency in the fk vector.

```

The MATLAB syntax `length(fk)` returns the number of elements in the vector `fk`, so we do not need a separate input argument for the number of frequencies. On the other hand, the programmer (that's you) should provide error checking to make sure that the lengths of `fk` and `Xk` are all the same. See `help error`.

### 3.3.2 Testing

In order to verify that this M-file can synthesize sinusoids, try the following test and plot the result in MS-Word via the notebook capability.

```
[xx0,tt0] = sum_Cexp( [5*exp(pi/3j),3i,-4], [6,4,0], 2, -1); %-Period = ?
```

Measure the DC value (which is also the average value) and also the period of `xx0` on the plot. Then write an explanation on the verification sheet of why the measured values are correct. Notice that when this M-file is used to synthesize harmonic waveforms, you must choose the entries in the frequency vector to be integer multiples of the fundamental frequency.

**Instructor Verification** (separate page)

## 4 Lab Exercises: Direction Finding

Why do humans have two ears? One answer is that the brain can process acoustic signals received at the two ears and determine the direction to the source of the acoustic energy. Using sinusoids, we can describe and analyze a simple scenario that demonstrates “direction finding” in terms of phase differences. This same principle is used in many other applications including radars that locate and track airplanes.

### 4.1 Direction Sensitivity with Microphones

Consider a simple measurement system that consists of two microphones both hearing the same source signal. If the microphones are placed some distance apart, then the sound must travel different paths from the source to the receivers. When the travel paths have different lengths, the two signals will arrive at different times.

The received signal at a receiver, called  $r(t)$ , is a delayed copy of the transmitter signal  $s(t)$ . Thus we could write

$$r(t) = s(t - t_d)$$

where  $t_d$  is the amount of time it takes for the signal to travel from the source to the receiver and  $s(\cdot)$  is the transmitted (sinusoidal) signal.<sup>5</sup> The travel time  $t_d$  can be computed easily once we know the speed of sound and the locations of the source and receiver.

If we assume that the source signal is a sinusoid, we can combine the signals from two receivers in many different ways. Consider the case of one source transmitting the signal  $s(t)$  to two receivers. The received signals could be label as

$$\text{Receiver \#1: } r_1(t) = s(t - t_1)$$

$$\text{Receiver \#2: } r_2(t) = s(t - t_2)$$

where  $t_1$  is the propagation time from the source to Receiver #1, and  $t_2$  the propagation time from the source to Receiver #2. One simple method is to *add* the two received signals.

$$y(t) = r_1(t) + r_2(t)$$

In this case, the output of the sum would have its maximum amplitude when the two signals are identical, which happens when  $t_1 = t_2$ . Figure 1 shows the two-receiver system; when the transmitter position is on the  $x$ -axis, we get the largest output.

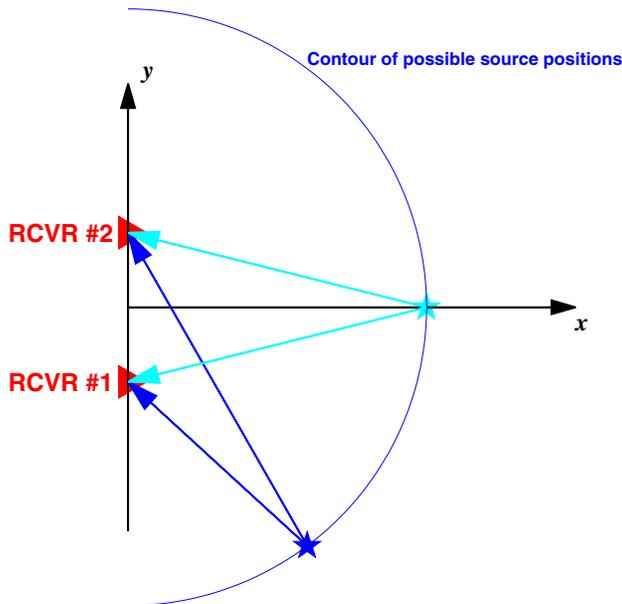


Figure 1: Scenario for a two-receiver system. Two possible locations for the transmitter (source) are shown as stars. If the received signals are summed, then the output will be largest whenever the transmitter is located on the centerline that bisects the line segment between the two receivers.

The goal in this part of the lab will be to produce a plot of the *receiver summation output* as a function of the  $(x, y)$  locations of the transmitter that lie on the circle shown in Fig. 1.

- (a) The amount of the delay (in seconds) can be computed for both propagation paths. First of all, consider the path from the transmitter to Receiver #1. The time delay is the distance from the transmitter location  $(x_t, y_t)$  to the receiver at  $(0, y_1)$ , divided by the speed of sound which is assumed to be  $c = 315$  m/s. Write a mathematical expression for this time delay, which will be called  $t_1$ .

<sup>5</sup>For simplicity we will ignore propagation losses that reduce the amplitude. Usually, the amplitude of an acoustic signal that propagates over a distance  $R$  will be reduced by an amount that is inversely proportional to  $R$ .

- (b) Now write a mathematical formula for the time delay of the signal that travels the path from the transmitter at  $(x_t, y_t)$  to Receiver #2 at  $(0, y_2)$ . Call this delay time  $t_2$  and write a mathematical expression for  $t_2$  in terms of  $d_2$ , the distance from transmitter to receiver.
- (c) The received signals at the receivers, called  $r_1(t)$  and  $r_2(t)$ , are delayed copies of the transmitted signal. Assume that the source signal  $s(t)$  is a sinusoid with a frequency of  $f_0 = 45$  Hz; and also assume that the complex amplitude of the transmitted signal is  $7e^{j\pi/4}$ . Make a plot of  $r_1(t)$  and  $r_2(t)$  when  $x_t = 40$  meters, and  $y_t = 0$ . Assume that the receivers are located at  $y_1 = -9$  meters, and  $y_2 = 9$  meters. Use `subplot` to put both signals on the same figure. Plot only 3 periods of the received sinusoids and then measure the *relative time-shift* between the two received signals by comparing peak locations. Explain why the relative time-shift turns out to be zero.
- (d) Repeat the operations of the previous part for a different transmitter location:  $x_t = 32$  meters, and  $y_t = -24$  meters. Assume the same transmitter frequency and the same receiver locations. Measure the peak amplitude of the output sinusoid that you will have in  $y(t)$ . In addition, describe how this calculation of peak amplitude could have been done *with a single phasor addition*.
- (e) Compute the distance  $R$  and the direction  $\theta$  from the origin to the transmitter located at  $(32, -24)$ . Using the result from part (d), define  $y_p(\theta)$  to be the peak amplitude of  $y(t)$  in the direction  $\theta$ .
- (f) The objective in the rest of this lab is to write a MATLAB function that will determine  $y_p(\theta)$  versus  $\theta$ , i.e., the peak amplitude of the output for changing transmitter locations (in angle). When doing this, the received signals should be manipulated as complex amplitudes, not as sinusoidal waveforms.

The MATLAB function that you write should be similar to the vectorization example in Section 3.1 in the warmup. The result of your function should be a plot that shows the output amplitude versus angle, where angle is measured counter-clockwise from the positive  $x$ -axis and the distance from the origin to the transmitter is always equal to 40 meters, i.e., the transmitter positions  $(x, y)$  satisfy:

$$x + jy = 40e^{j\theta} \quad \text{where} \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

*Note:* The complex amplitudes at the receivers  $R_1 = A_1e^{j\phi_1}$  and  $R_2 = A_2e^{j\phi_2}$  can be found by calculating the distances from the transmitter to each receiver, and using the speed of sound ( $c$ ) to convert to time delay.

- (g) *Vectorization:* It is likely that your previous programming skills would lead you to write one or two loops to do this implementation. The loop(s) would run over the set of possible locations of the transmitter  $(x_t, y_t)$ , and would do the addition of the complex amplitudes at the two receivers for each transmitter position, one at a time. Instead, you should vectorize your program in a fashion similar to what was done for the example in Section 3.1.
- (h) If we define the centerline (in Fig. 1) to be at an angle of zero degrees measured with respect to the  $x$ -axis, use the plot of output amplitude versus angle to determine other angles where the output is maximal; find all such angles.
- In addition, there will also be transmitter locations where the output from this “summation processor” is zero; find the angles to these. Express your answers for the angles in degrees.
- (i) Finally, show that you get (nearly) the same results if you change the distance from the origin to the transmitter. For example, redo the plot (output amplitude versus angle) for a range of 100 meters instead of 40. Try to explain why the result is independent of range (for large ranges).

**Lab #2**

**ECE-2025**

**Fall-2004**

**INSTRUCTOR VERIFICATION SHEET**

Turn this page in to your TA before the end of your lab period.

Name: \_\_\_\_\_

Date of Lab: \_\_\_\_\_

Part 3.1 Replace the `for` loop with a couple of lines of vectorized MATLAB code. Write the MATLAB code in the space below:

Verified: \_\_\_\_\_

Date/Time: \_\_\_\_\_

Part 3.3.2 Show that your `sum_Cexp.m` function is correct by running the test in Section 3.3.2 and plotting the result. Measure the DC value and the period of `xx0` and explain why the measured values are correct. Write your explanations in the space below.

Verified: \_\_\_\_\_

Date/Time: \_\_\_\_\_

General: Use MATLAB's `notebook` capability to create and save the plots when doing the Instructor Verifications. Show the MS-Word document to your TA.

Verified: \_\_\_\_\_

Date/Time: \_\_\_\_\_