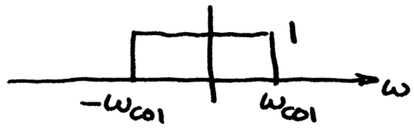
$$h_{bp}(t) = \left[\delta(t) - \frac{\sin(\omega_{co1}t)}{\pi t}\right] * \frac{\sin(\omega_{co2}t)}{\pi t}$$

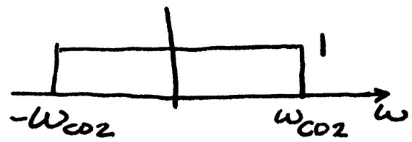
F.T.

1

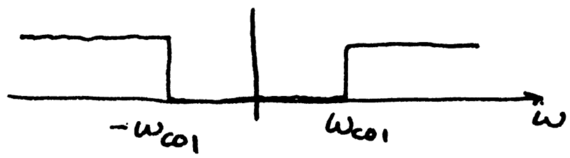
Rectangle



FT Rectangle.



1 − Rectangle



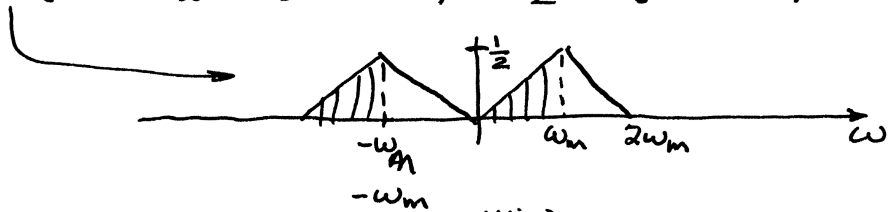
Convolution becomes multiplication in freq. domain

Multiply $(1 - \text{Rect}_1) \cdot \text{Rect}_2$

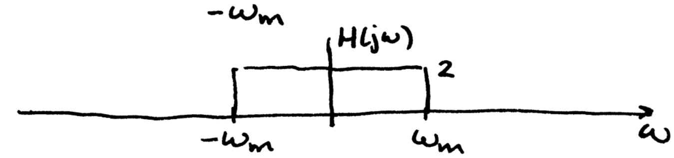
Only get 1 for $\omega > \omega_{co1}$ <u>and</u> $\omega < \omega_{co2}$

Problem 11.2:

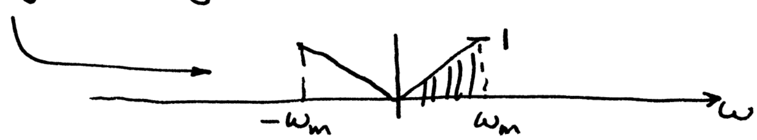
(a) $V(j\omega) = \frac{1}{2} X(j(\omega - \omega_m)) + \frac{1}{2} X(j(\omega + \omega_m))$



(b)

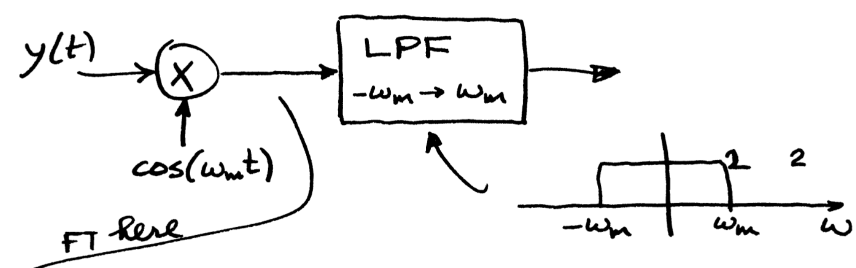


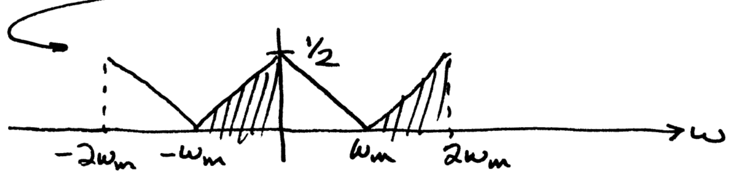
$Y(j\omega) = H(j\omega) V(j\omega)$



(c) The negative-frequency components are moved into the positive frequency region, and vice versa. Also, the high-frequency components are moved to the low frequency region (near $\omega = 0$). Likewise, low frequency components are moved to the high frequency region.

(d) Demodulator would be





To recover $x(t)$ we need an ideal LPF that will extract the spectrum from $-\omega_m$ to ω_m.

Problem 11.3:

a)

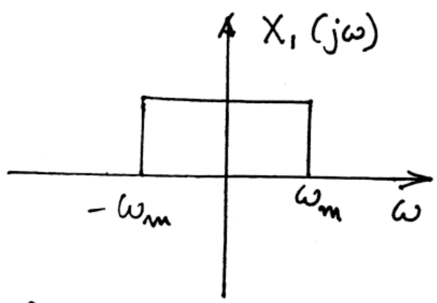
$$x_1(t)\cos(\omega_c t) \longleftrightarrow \frac{1}{2}X_1[j(\omega-\omega_c)] + \frac{1}{2}X_1[j(\omega+\omega_c)]$$

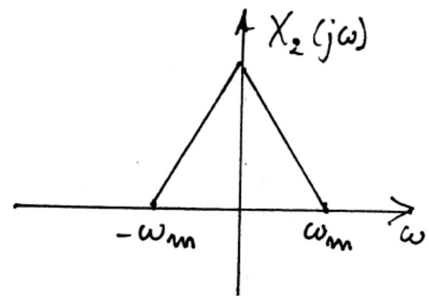
$$x_2(t)\sin(\omega_c t) \longleftrightarrow \frac{1}{2j}X_2[j(\omega-\omega_c)] - \frac{1}{2j}X_2[j(\omega+\omega_c)]$$

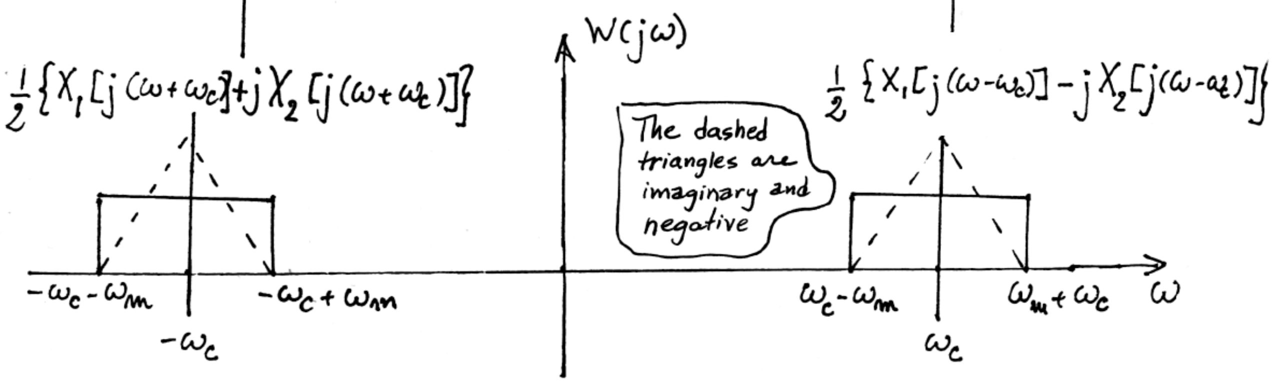
$$w(t) = x_1(t)\cos(\omega_c t) + x_2(t)\sin(\omega_c t) \longleftrightarrow$$

$$W(j\omega) = \frac{1}{2}\left\{X_1[j(\omega-\omega_c)] - jX_2[j(\omega-\omega_c)]\right\} +$$

$$+\frac{1}{2}\left\{X_1[j(\omega+\omega_c)] + jX_2[j(\omega+\omega_c)]\right\}$$







Problem 11.3 (more):

b) $\omega_a = \omega_c - \omega_m$

$\omega_b = \omega_c + \omega_m$

c)

$$v(t) = w(t)\sin(\omega_c t) = x_1(t)\sin(\omega_c t)\cos(\omega_c t) +$$

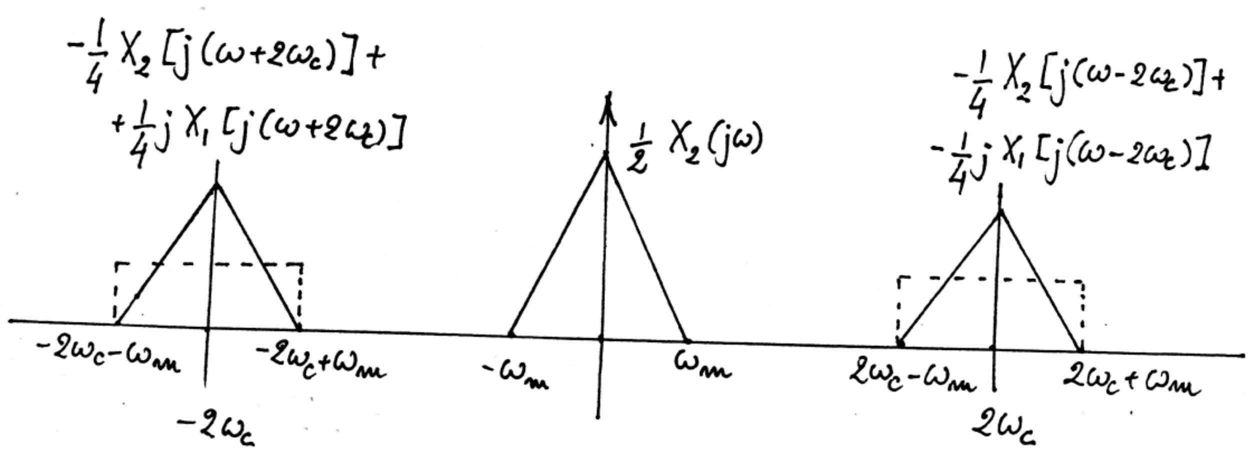
$$+ x_2(t)\sin^2(\omega_c t) = \frac{1}{2}x_1(t)\sin(2\omega_c t) +$$

$$+ \frac{1}{2}x_2(t)\left[1 - \cos(2\omega_c t)\right] = \frac{1}{2}x_2(t) +$$

$$+ \frac{1}{2}x_1(t)\sin(2\omega_c t) - \frac{1}{2}x_2(t)\cos(2\omega_c t)$$

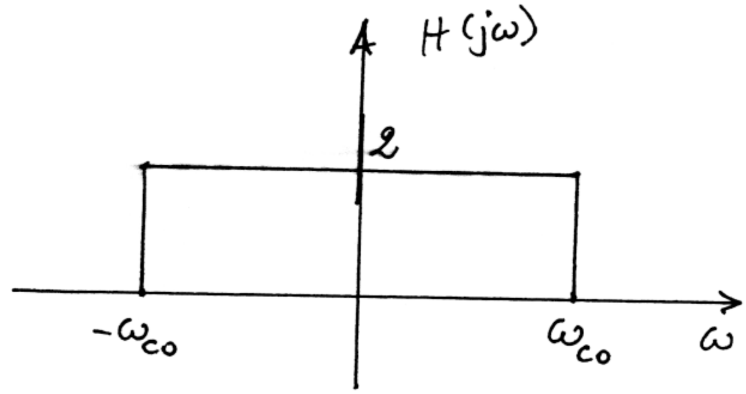
$$V(j\omega) = \frac{1}{2}X_2[j\omega] + \frac{1}{4j}\left\{X_1[j(\omega - 2\omega_c)] +\right.$$

$$\left. - X_1[j(\omega + 2\omega_c)]\right\} - \frac{1}{4}\left\{X_2[j(\omega - 2\omega_c)] + X_2[j(\omega + 2\omega_c)]\right\}$$



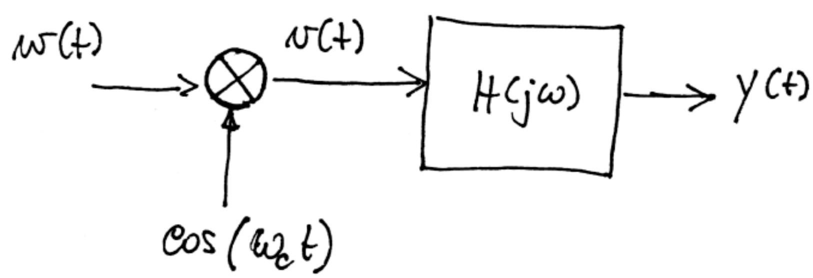
Problem 11.4:

d)



Ideal low-pass filter with gain equal to 2 and $\omega_m \leq \omega_{co} \leq 2\omega_c - \omega_m$ (in fact, a non-ideal low-pass filter will also work).

e)



$$v(t) = x_1(t)\cos^2(\omega_c t) + x_2(t)\sin(\omega_c t)\cos(\omega_c t) =$$

$$= \frac{1}{2}x_1(t)[1 + \cos(2\omega_c t)] + \frac{1}{2}x_2(t)\sin(2\omega_c t) =$$

$$= \frac{1}{2}x_1(t) + \frac{1}{2}x_1(t)\cos(2\omega_c t) + \frac{1}{2}x_2(t)\sin(2\omega_c t)$$

Problem 11.5:

a) $$\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}$$

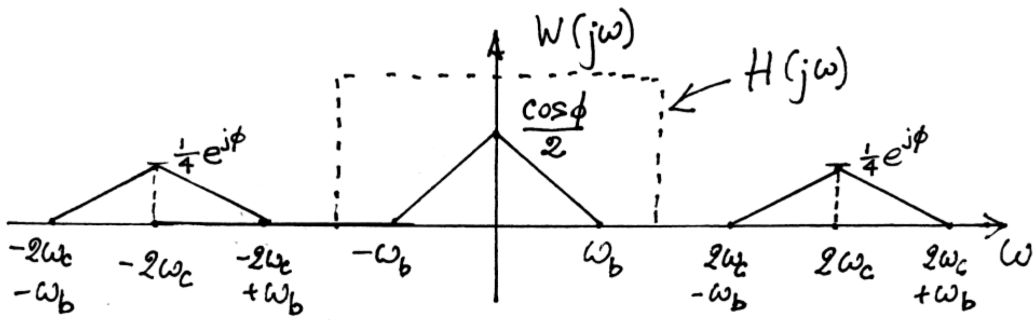
$$\cos(\omega_c t)\cos(\omega_c t+\phi) = \frac{1}{2}\cos(2\omega_c t+\phi) + \frac{1}{2}\cos\phi$$

$$w(t) = x(t)\cos(\omega_c t+\phi)\cos(\omega_c t) =$$

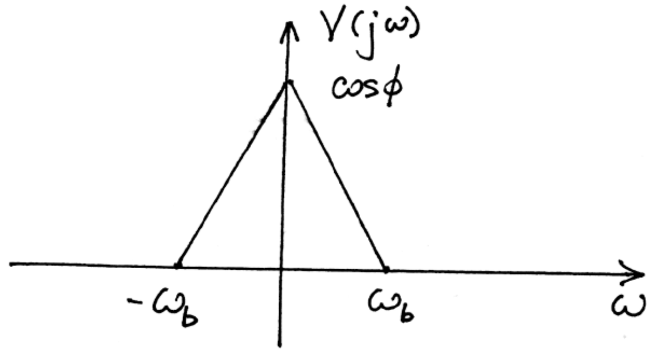
$$= \frac{1}{2}x(t)\cos(2\omega_c t+\phi) + \frac{1}{2}x(t)\cos\phi =$$

$$= \frac{1}{4}x(t)e^{j\phi}e^{j2\omega_c t} + \frac{1}{4}x(t)e^{-j\phi}e^{-j2\omega_c t} + \frac{1}{2}x(t)\cos\phi$$

b)
$$W(j\omega) = \frac{1}{4}e^{j\phi}X[j(\omega-2\omega_c)] + \frac{1}{4}e^{-j\phi}X[j(\omega+2\omega_c)] + \frac{\cos\phi}{2}X(j\omega)$$



c)



d) $$v(t) = x(t)\cos\phi$$