The general approach is to use Tables plus some algebraic manipulations:

(a) $\frac{j\omega}{0.1+j\omega} e^{-j0.2\omega} = X_1(j\omega)e^{-j0.2\omega}$ ← use time shifting

If $X_1(j\omega) = \frac{j\omega}{0.1+j\omega}$, then $X_1(j\omega) = j\omega X_2(j\omega)$ ← use derivative

If $X_2(j\omega) = \frac{1}{0.1+j\omega} \Rightarrow x_2(t) = e^{-0.1t}u(t)$

$\Rightarrow x_1(t) = \frac{d}{dt}x_2(t) = e^{-0.1t}\delta(t) - 0.1e^{-0.1t}u(t) = \delta(t) - 0.1e^{-0.1t}u(t)$

$x(t) = x_1(t-0.2) = \delta(t-0.2) - 0.1e^{-0.1(t-0.2)}u(t-0.2)$

(b) $X(j\omega) = 2 + 2\cos\omega = 2 + e^{-j\omega} + e^{j\omega}$ ← use shifting

$x(t) = 2\delta(t) + \delta(t-1) + \delta(t+1)$

(c) use Table entry $\frac{1}{a+j\omega} \longrightarrow e^{-at}u(t)$

$x(t) = e^{-t}u(t) - e^{-2t}u(t)$

(d) use Table entry: $2\pi\delta(\omega-\omega_0) \longrightarrow e^{j\omega_0 t}$

$X(j\omega) = j\frac{2\pi}{2\pi}\delta(\omega-100\pi) - j\frac{2\pi}{2\pi}\delta(\omega-(-100\pi))$

$x(t) = \frac{j}{2\pi}e^{j100\pi t} - \frac{j}{2\pi}e^{-j100\pi t}$

$= \frac{-1}{\pi}\left\{\frac{1}{2j}e^{j100\pi t} - \frac{1}{2j}e^{-j100\pi t}\right\}$ ← use Inverse Euler

$x(t) = -\frac{1}{\pi}\sin(100\pi t)$

Problem 10.2:

(a) $x(t) = u(t) - u(t-4)$ is a shifted pulse

$$= \underset{\text{time-shift}}{\delta(t-2)} * \underbrace{[u(t+2) - u(t-2)]}_{\text{F.T.} = \frac{\sin(2\omega)}{\omega/2}}$$

$$X(j\omega) = e^{-j2\omega} \frac{\sin(2\omega)}{\omega/2}$$

(b) Each impulse in ω inverts to a complex exponential

$$S(j\omega) = 4\pi\delta(\omega) + 2\pi\delta(\omega - 10\pi) + 2\pi\delta(\omega + 10\pi)$$

$$s(t) = 2e^{j0} + e^{j10\pi t} + e^{-j10\pi t}$$

$$= 2 + 2\cos(10\pi t)$$

(c) $R(j\omega) = \frac{1}{2} - \frac{2}{4 + j2\omega} = \frac{1}{2} - \frac{1}{2 + j\omega}$

$$r(t) = \frac{1}{2}\delta(t) - e^{-2t}u(t)$$

(d) $y(t) = \delta(t+1) + 2\delta(t) + \delta(t-1)$

$$Y(j\omega) = e^{j\omega} + 2 + e^{-j\omega}$$

$$= 2 + 2\cos(\omega)$$

Prob. 10.3

(a) $h(t) = \dfrac{\sin[4\pi(2t-1)]}{2t-1}$

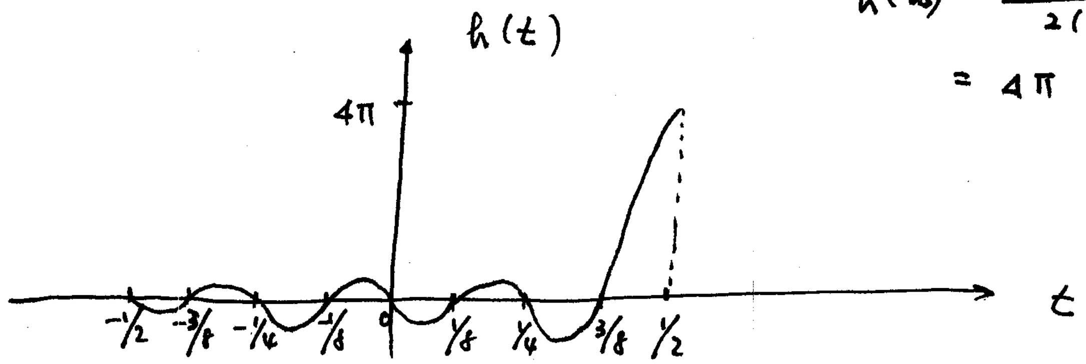
$= \dfrac{\sin[8\pi(t-.5)]}{\pi(t-0.5)} \cdot \dfrac{\pi}{2}$

A sinc function centers at $t = 1/2$, and have zero crossings at $8\pi(t-.5) = n\pi$, $n = \pm1, \pm2, \ldots$

$t = \frac{n}{8} + \frac{1}{2}$

$h(.5) = \dfrac{8\pi(t-.5)}{2(t-.5)}$

$= 4\pi$

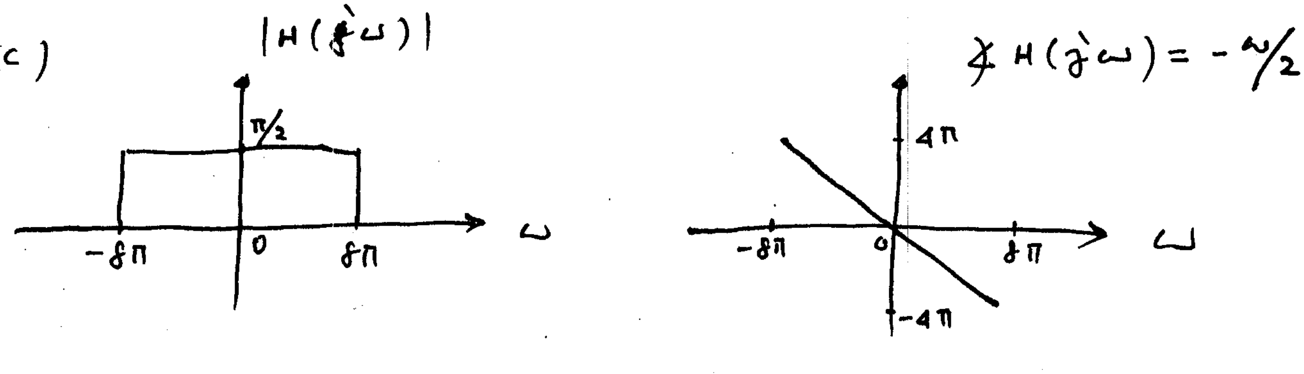


(b) The FT of $\dfrac{\sin \omega_c t}{\pi t}$ is $u(\omega+\omega_c) - u(\omega-\omega_c)$

A time shift t_0 results in $e^{-j\omega t_0}$ in FT.

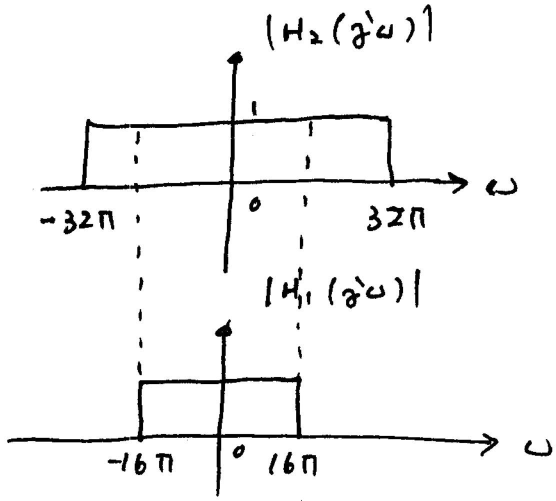
so $H(j\omega) = \frac{\pi}{2} \cdot [u(\omega+8\pi) - u(\omega-8\pi)] \cdot e^{-\frac{j\omega}{2}}$

(c)



Prob. 10.4

(a)



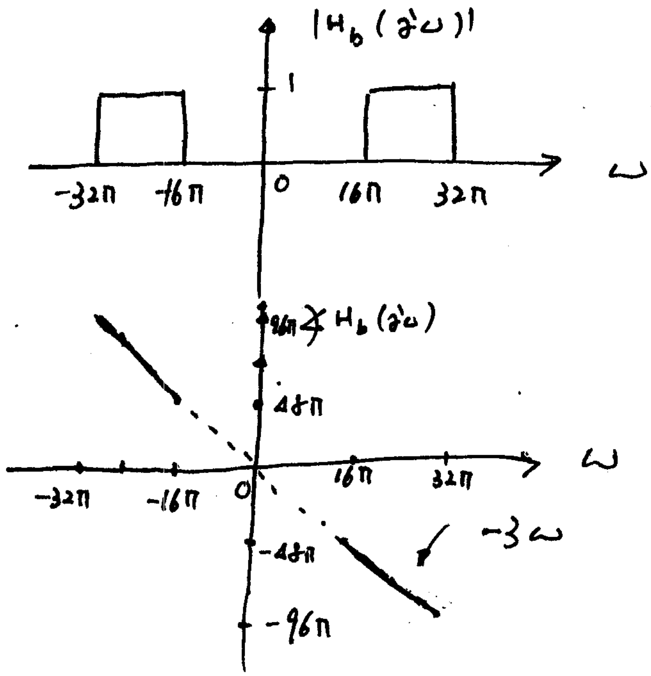
$$\measuredangle H_1(j\omega) = \measuredangle H_2(j\omega) = -3\omega$$

for $-16\pi < \omega < 16\pi$

Then $$H_b(j\omega) = H_2(j\omega) - H_1(j\omega)$$

$$= \left[u(\omega+32\pi) - u(\omega+16\pi)\right] e^{-j3\omega} +$$

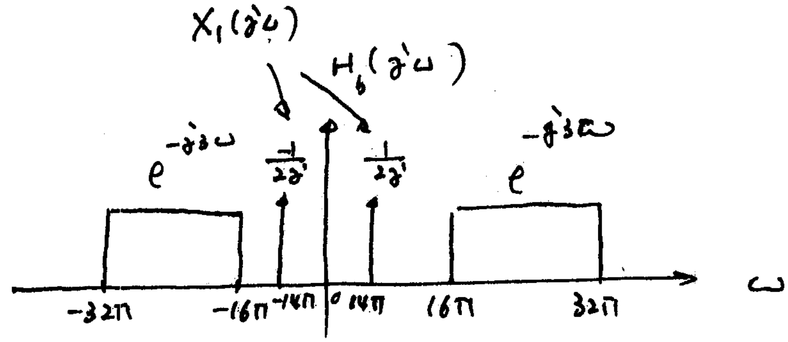
$$\left[u(\omega-16\pi) - u(\omega-32\pi)\right] e^{-j3\omega}$$



(b)

$$x(t) = x_1(t) + x_2(t).$$

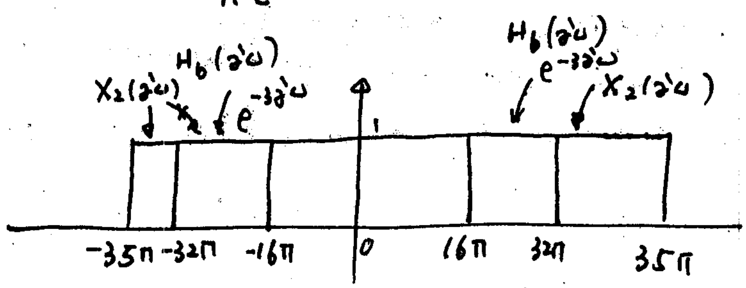
$$x_1(t) = \sin(14\pi t) \xrightarrow{FT} X_1(j\omega) = \frac{1}{2j}\left[\delta(\omega - 14\pi) - \delta(\omega + 14\pi)\right]$$



so the output of the filter $y_1(t)$ due to $x_1(t)$ is

$$y_1(t) = 0$$

$$x_2(t) = \frac{\sin(35\pi t)}{\pi t} \xrightarrow{FT} X_2(j\omega) = u(\omega + 35\pi) - u(\omega - 35\pi)$$



$$Y_2(j\omega) = X_2(j\omega) \cdot H_b(j\omega) = H_b(j\omega)$$

$y_2(t) = h_b(t)$, where $h_b(t)$ is the impulse response

$= y(t)$ $\quad (y(t) = y_1(t) + y_2(t))$

(c) since $H_b(j\omega) = H_2(j\omega) - H_1(j\omega)$

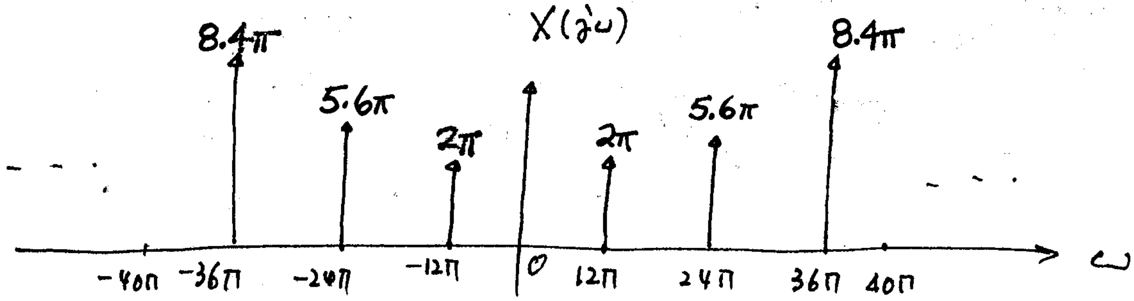
$$h_b(t) = h_2(t) - h_1(t)$$

$$= \frac{\sin(32\pi(t-3))}{\pi(t-3)} - \frac{\sin(16\pi(t-3))}{\pi(t-3)}$$

Prob. 10.5

(a) $f_0 = 6, \quad \omega_0 = 12\pi$

$2\pi f_0 \cdot k = 12k\pi$



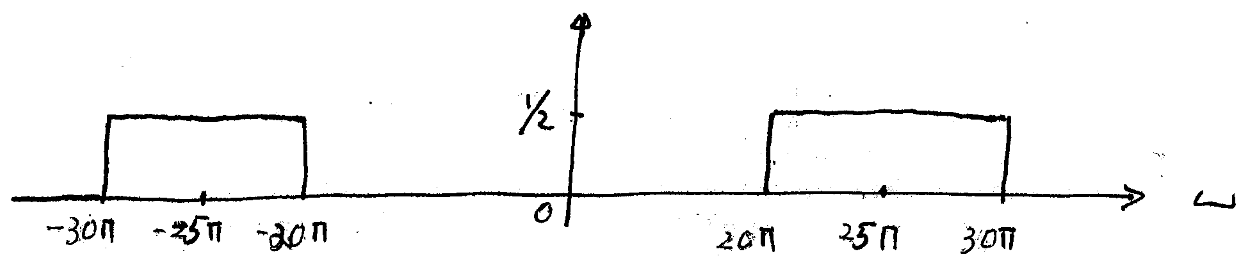
(b) $\frac{\sin \omega_{co} t}{\pi t} \xrightarrow{FT} H_1(j\omega) = u(\omega + \omega_{co}) - u(\omega - \omega_{co})$

$\cos(5\omega_{co} t) \xrightarrow{FT} H_2(j\omega) = \frac{2\pi}{2}\left[\delta(\omega - 5\omega_{co}) + \delta(\omega + 5\omega_{co})\right]$

$h(t) \xrightarrow{FT} \frac{1}{2\pi} H_1(j\omega) * H_2(j\omega)$

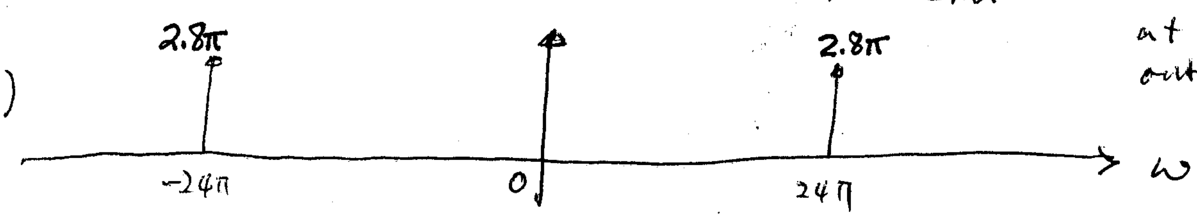
$= \frac{1}{2} H_1(j(\omega - 5\omega_{co})) + \frac{1}{2} H_1(j(\omega + 5\omega_{co}))$

$|H(j\omega)|$ $\qquad \omega_{oc} = 5\pi$



$\measuredangle H(j\omega) = 0$

(c)



The 2nd harmonic in X(t) at the filter output

(d) $y(t) = 2.8\cos(24\pi t)$