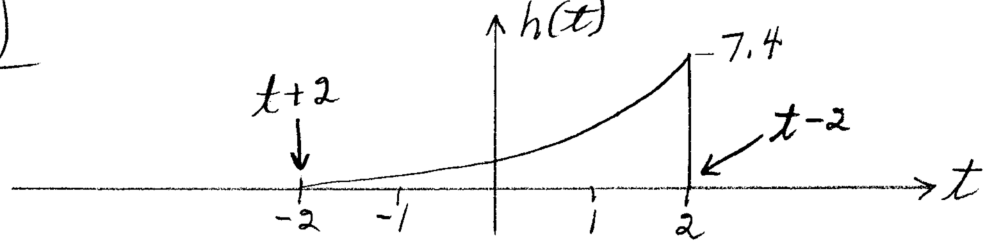
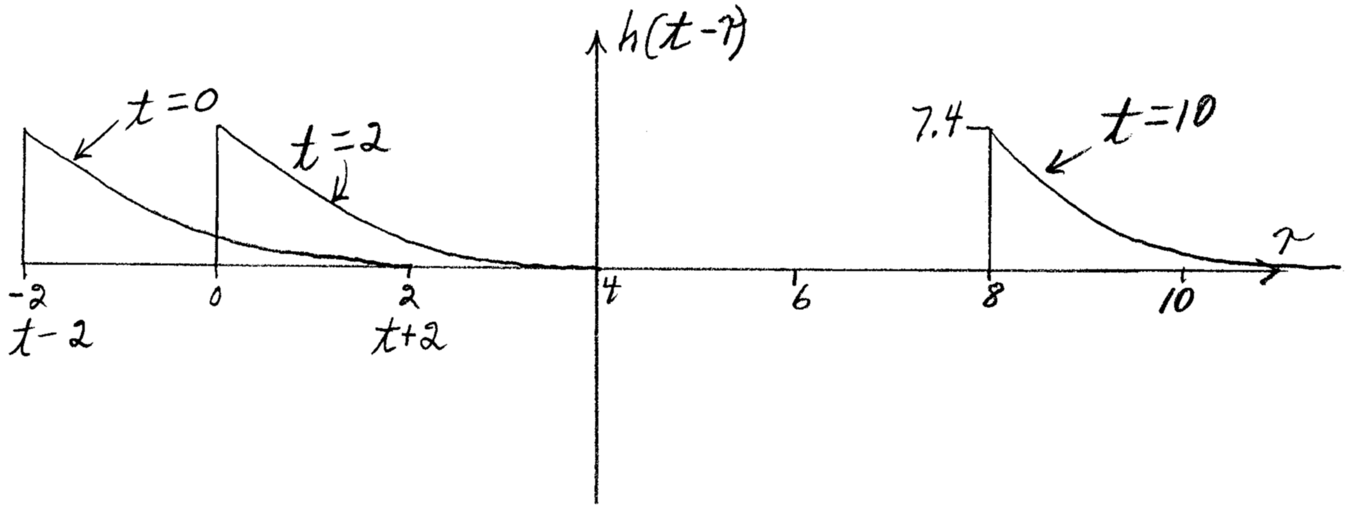
9.1(a) $h(t) = e^{0.5t}\left[u(t+2) - u(t-2)\right]$

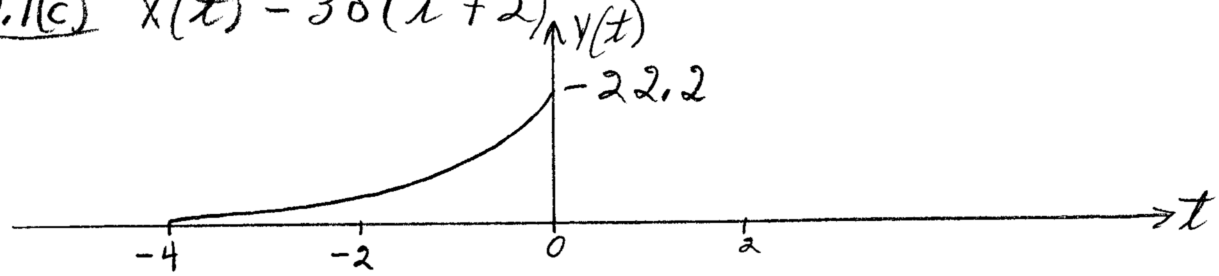
NOT causal

9.1(b)

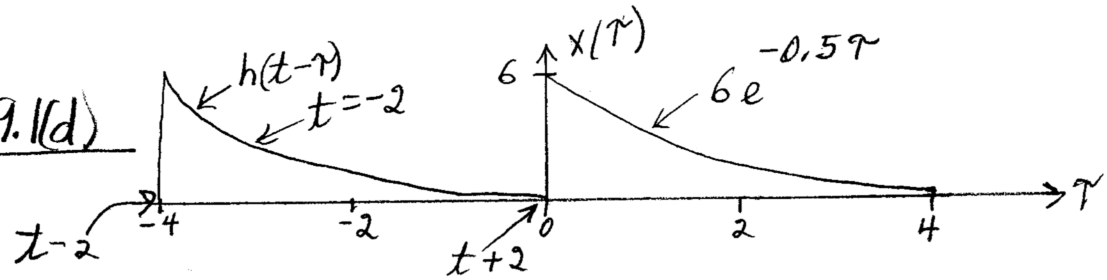




9.1(c) $x(t) = 3\delta(t+2)$



9.1(d)



There are 4 regions:

$t < -2$	$y_1(t) = 0$	1
$-2 < t < 2$	$y_2(t) = ?$	2
$2 < t < 6$	$y_3(t) = ?$	3
$6 < t$	$y_4(t) = 0$	4

Problem 9.1(d): A linear time-invariant system has impulse response:

$$h(t) = e^{0.5t}\{u(t+2) - u(t-2)\} = \begin{cases} e^{0.5t} & -2 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

Use the convolution integral to determine the output $y(t)$ when the input is

$$x(t) = 6e^{-0.5t}\{u(t) - u(t-4)\} = \begin{cases} 6e^{-0.5t} & 0 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$$

There are normally 5 regions when you convolve two finite-duration signals. In this case, the complete overlap region is just a point because both signals have a length of 4 secs, so there are only 4 regions to consider:

Region 1: No overlap for $t < -2$, so $y(t) = 0$.

Region 2: **Partial** overlap for $-2 \le t < 2$; in this region the output signal is:

$$y(t) = \int_0^{t+2} e^{-0.5\tau} 6e^{0.5(t-\tau)} d\tau$$

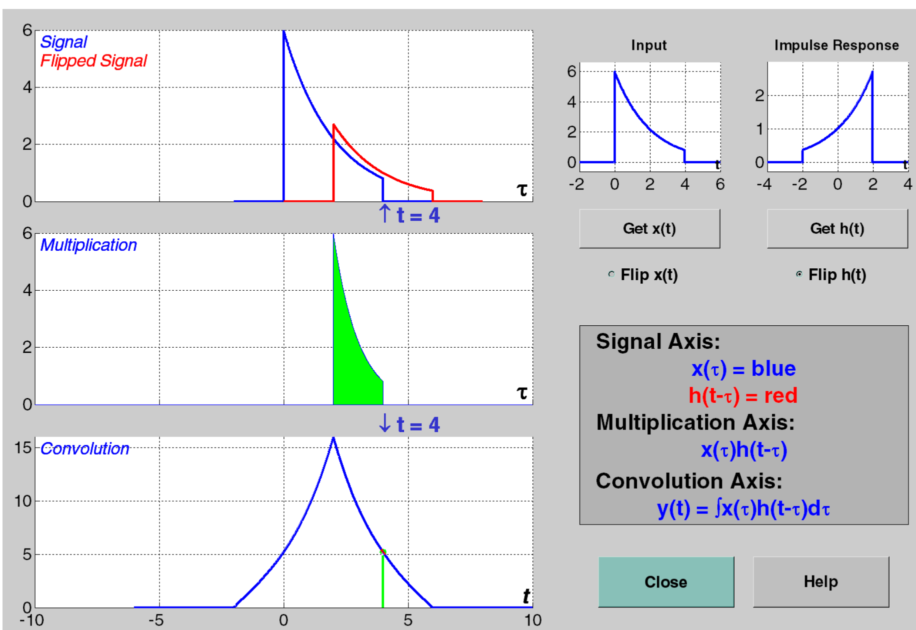
$$= 6e^{0.5t} \int_0^{t+2} e^{-\tau} d\tau = -6e^{0.5t} e^{-\tau}\Big|_0^{t+2} = 6e^{0.5t}\left(e^0 - e^{-(t+2)}\right) = 6e^{0.5t} - 6e^{-0.5t-2}$$

Region 3: **Partial** overlap for $2 \le t \le 6$; in this region the output signal is:

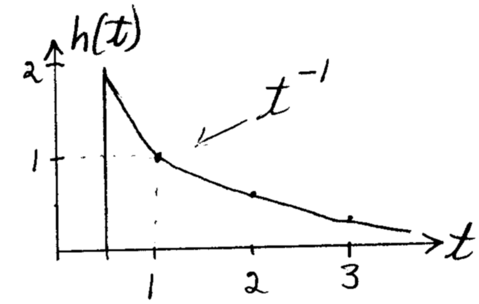
$$y(t) = \int_{t-2}^{4} e^{-0.5\tau} 6e^{0.5(t-\tau)} d\tau$$

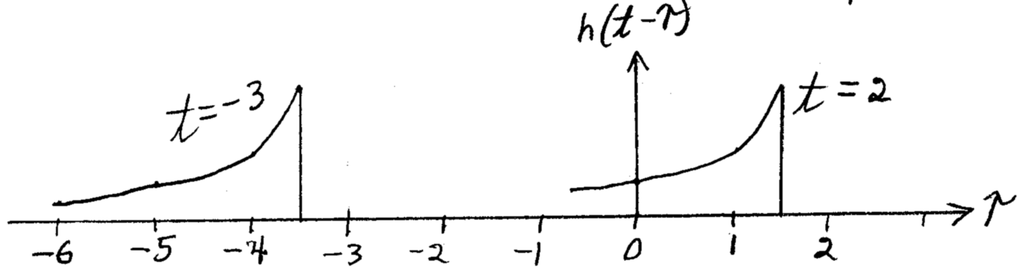
$$= 6e^{0.5t} \int_{t-2}^{4} e^{-\tau} d\tau = -6e^{0.5t} e^{-\tau}\Big|_{t-2}^{4} = 6e^{0.5t}\left(e^{-(t-2)} - e^{-4}\right) = 6e^{-0.5t+2} - 6e^{0.5t-4}$$

Region 4: No overlap for $t > 6$, so $y(t) = 0$.



9.2(a) $h(t) = t^{-1} u(t - \frac{1}{2})$



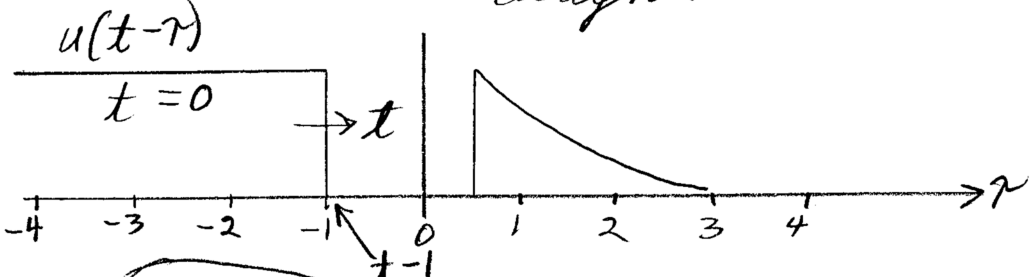


9.2(b) Yes, h(t) is causal, because there is no output before input.

9.2(c) No h(t) is not stable

$$\int_{0.5}^{\infty} t^{-1} dt = \ln \infty - \ln 0.5 \text{ which is not bounded.}$$

9.2(d) $x(t) = u(t-1)$ Draw the convolution diagram.

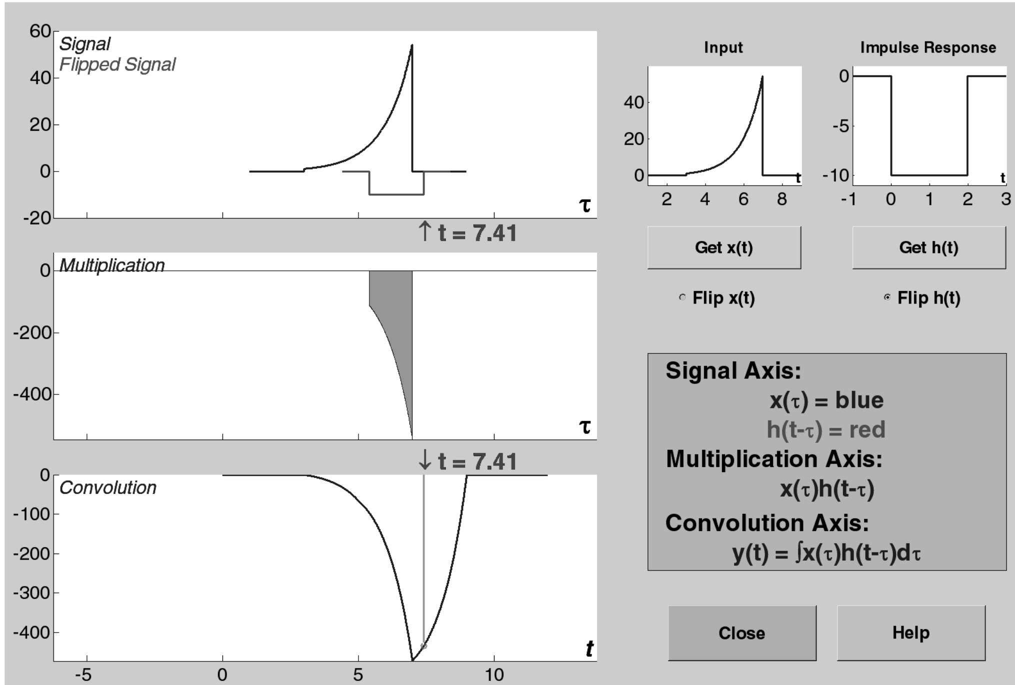


Clearly $t_1 = 1.5$

9.2(e) $$y(t) = \int_{\tau=0.5}^{\tau=t-1} \tau^{-1} d\tau = \ln \tau \Big|_{0.5}^{t-1} = \ln(t-1) - \ln 0.5$$

$$= \ln[2(t-1)]$$

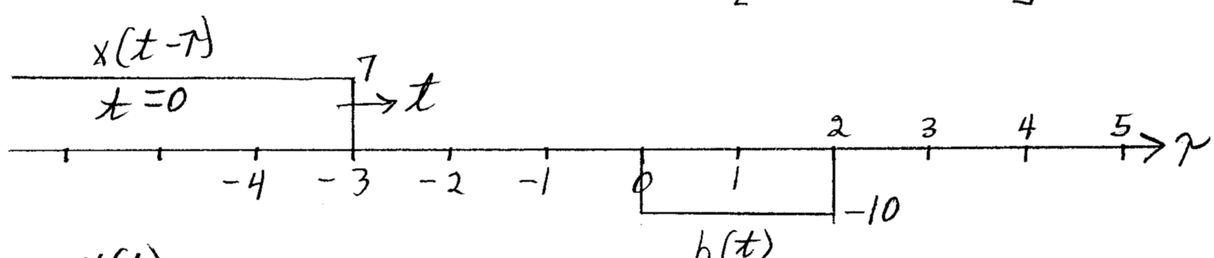
9.3(a)

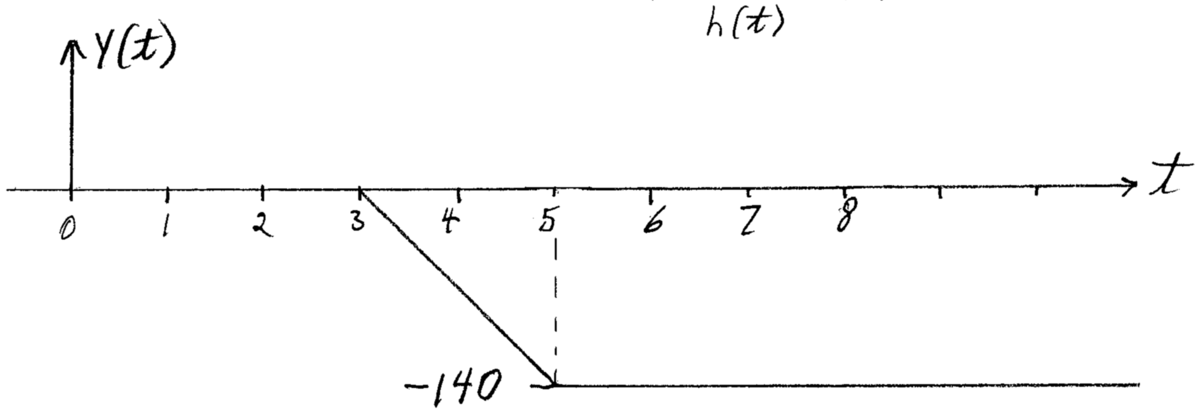


Duration $= 9-3$
$= 6$

9.3(b) The min value of $y(t)$ occurs when $t=7$

9.3(c) $x(t) = 7u(t-3)$ $\quad h(t) = 10[u(t-2) - u(t)]$





$$y(t) = \underset{-70}{B}(t - \underset{3}{T_{12}})\left[u(t - \underset{3}{T_{12}}) - u(t - \underset{5}{T_{23}})\right] + \underset{-140}{C}\,u(t - \underset{5}{T_{23}})$$

$B = -70 \quad T_{12} = 3 \quad C = -140 \quad T_{23} = 5$

9.4 $h_1(t) = e^{-t}u(t-\frac{1}{2})$ $y(t) = \frac{d}{dt}w(t-2) - \int_{-\infty}^{t-2} w(\tau)d\tau$

9.4(a)

If $x(t)=\delta(t)$, then $y(t) = h_1(t) * h_2(t)$, which is the overall system impulse response, $h(t)$. Therefore

$$y(t)\Big|_{x(t)=\delta(t)} = \frac{d}{dt}\left[e^{-(t-2)}u(t-2.5)\right] - \int_{-\infty}^{t-2} e^{-\tau}u(\tau-\tfrac{1}{2})\,d\tau$$

$$= -e^{-(t-2)}u(t-2.5) + e^{-(t-2)}\delta(t-2.5) - \int_{\tau=\frac{1}{2}}^{t-2} e^{-\tau}d\tau$$

The integral is only valid for $t-2 \geq 0.5$. $\therefore$ $t \geq 2.5$

$$= e^{-0.5}e^{-(t-2.5)}\delta(t-2.5) - e^{-0.5}e^{-(t-2.5)}u(t-2.5)$$

$$+ e^{-\tau}\Big|_{\frac{1}{2}}^{t-2} u(t-2.5)$$

$$= e^{-0.5}e^{-(t-2.5)}\delta(t-2.5) - e^{-0.5}\cancel{e^{-(t-2.5)}}u(t-2.5)$$

$$(e^{-0.5}\cancel{e^{-(t-2.5)}} - e^{-0.5})u(t-2.5)$$

$$= e^{-0.5}e^{0}\delta(t-2.5) - e^{-0.5}u(t-2.5)$$

$$= e^{-0.5}\delta(t-2.5) - e^{-0.5}u(t-2.5)$$

9.4(b) $x(t) = e^{t} u(t)$

$$y(t) = x(t) * h(t) = \left[e^{t} u(t)\right] * e^{-0.5}\left[\delta(t-2.5) - u(t-2.5)\right]$$

$$= e^{-0.5}\left[e^{t} u(t)\right] * \left[\delta(t-2.5) - u(t-2.5)\right]$$

$$= e^{-0.5}\left[e^{(t-2.5)} u(t-2.5)\right] - e^{-0.5}\left[e^{t} u(t) * u(t-2.5)\right]$$

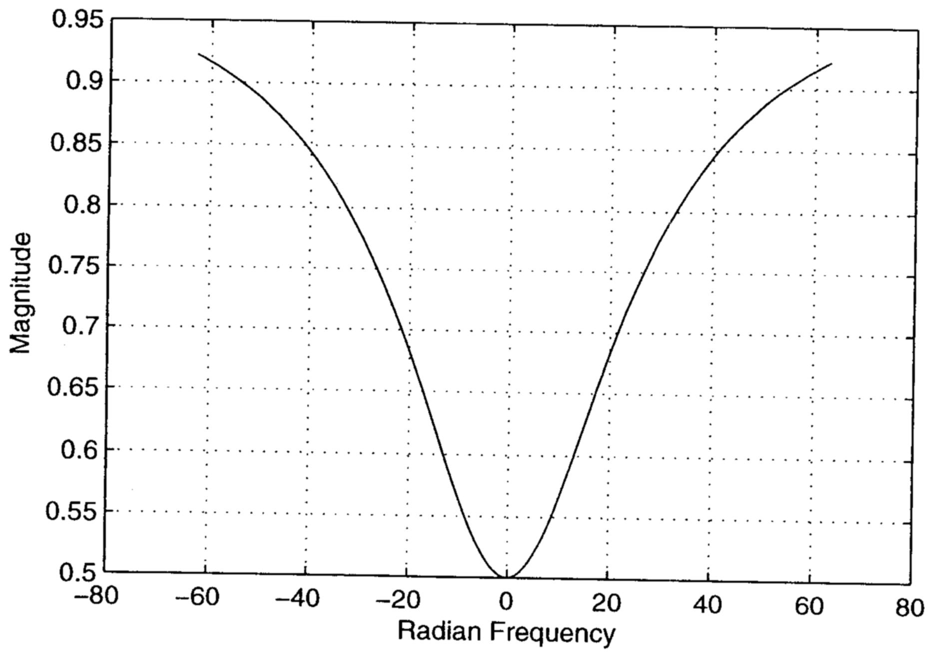
$$= e^{-0.5} e^{(t-2.5)} u(t-2.5) - e^{-0.5}\left[e^{(t-2.5)} u(t-2.5) - u(t-2.5)\right]$$

$$= e^{-0.5} u(t-2.5)$$

9.5 $h(t) = \delta(t) - \frac{1}{2} b e^{-bt} u(t)$

9.5(a) $H(\omega) = 1 - \frac{b/2}{b + j\omega}$ using the table of basic transform pairs

9.5(b) for $b = 10\pi$, $H(\omega) = 1 - \frac{5\pi}{10\pi + j\omega}$



9.5(c) The filter is a HPF.

9.5(d) $x(t) = 10 + 40\cos(90\pi t)$

$H(\omega = 0) = 0.5 \quad H(\omega = 90\pi) = 0.9954\, e^{j0.0552}$

$x(t) = 10(.5) + 40(.9954)\cos(90\pi t + 0.0552)$

$= 5 + 39.816\cos(90\pi t + 0.0552)$